Problem Set #4
due Friday, September 22

PHYSICS 851, FALL 2000

1. Consider a proton scattering off of an attractive one-dimensional potential,

\[
V(x) = \begin{cases} 
\infty, & x < 0 \\
-V_0 \left(1 - \frac{x^2}{R^2}\right), & 0 < x < R \\
0, & r > R
\end{cases}
\]

For this example, we will consider \( R = 2.5 \) fm, and \( V_0 = 16 \) MeV.

If you wish, to make the units more natural, you may consider \( \hbar c = 197.327 \) MeV·fm, and \( m_p = 938.3 \) MeV/c\(^2\).

Consider a particle incident on the well with energy \( E \) that enters and leaves the well with energy \( E \). Far away, the solutions are of the form,

\[
\psi(x) = e^{-ipx/\hbar} - e^{2i\delta + ipx/\hbar}, \quad x >> R
\]

(a) Programming in either FORTRAN or C++, construct a program that runs and returns a listing of \( \delta \) vs. \( p \) for \( 0 < p < 600 \) MeV/c, in steps of 2.0 MeV/c. You may turn in the program through electronic mail. Please only send in the source code. It is due by 5PM Friday, Sep. 22. My email address is pratt@nscl.msu.edu.

If you have no experience writing programs in FORTRAN or C++, you may visit the 851 home page at theo15.nscl.msu.edu/phy851, and see two example programs.

A graph of the results:

(b) EXTRA CREDIT Make a graph like the one above, except for the region between \( p=0 \) and \( p=1.0 \) MeV, and consider two strengths of the potential, \( V_0 = 17.0 \) MeV and \( V_0 = 17.025 \) MeV. Be sure to calculate values for very small values of \( p \), in steps of .001 MeV. For this problem, turn in a paper copy of the graph.
2. Consider the harmonic oscillator problem, with Hamiltonian,

\[ H = \frac{P^2}{2m} + \frac{kX^2}{2} \]

a.) Express \( X \) and \( P \) in terms of \( a \) and \( a^\dagger \).

b.) Calculate the following matrix elements, (in terms of \( k \) and \( m \))

\[ \langle n | X | m \rangle, \langle n | PX | m \rangle, \langle n | P | m \rangle, \langle n | XP | m \rangle, \langle n | X^2 | m \rangle, \]

\[ \langle n | P^2 | m \rangle, \langle n | a | m \rangle, \langle n | a^\dagger | m \rangle, \langle n | H | m \rangle \]

c.) Show that the expectation value of the potential energy in an energy eigenstate of the harmonic oscillator equals the expectation value of the kinetic energy in that state.