Problem Set #5

due Friday, September 29

PHYSICS 851, FALL 2000

1. (a) Derive and solve the equations of motion for the Heisenberg operators $a(t)$ and $a^\dagger(t)$ for the harmonic oscillator.
   (b) Calculate $[a(t), a^\dagger(t')]$.

2. (a) Show that $[r_{op}, f(p_{op})] = i\hbar \nabla_p f(p_{op})$, where $f(p)$ is an arbitrary function of the momentum operator.
   (b) Using this result show that
   
   \[ e^{i\hbar \lambda / \hbar} r_{op} e^{-i\hbar \lambda / \hbar} = r_{op} + \lambda \]

3. (a) Calculate the correlation function $\langle 0 | x(t) x(t') | 0 \rangle$ where $|0\rangle$ is the harmonic oscillator ground state, and $x(t)$ is the position operator in the Heisenberg representation.
   (b) Suppose that a time dependent force $F(t)$ is applied to a particle in the oscillator potential. Show that $x(t)$ obeys the equation of motion,

   \[ m \left( \frac{d^2}{dt^2} + \omega^2 \right) x(t) = F(t) \]

   where $\omega$ is the oscillator frequency.

4. What are the matrix elements of the operator $1/|p|$ in the position representation? That is, find $\langle r | 1/|p| | r' \rangle$. Work the problem in three dimensions.