Topic 1: 1D Motion

PHYSICS 231

Key Concepts: 1D motion

- Particle model
- Position, Coordinate system (1 axis with positive and negative direction), Displacement
- Velocity – rate of change of position
  can have positive or negative direction
- Acceleration – rate of change of velocity
  can have positive or negative direction
- Average velocity/acceleration vs instantaneous
- Equations for X(t) and v(t) for constant acceleration
- Gravity on the surface of the earth provides an acceleration of $|g|=9.81\ m/s^2$ (if no other forces are acting – free "fall") towards the center of the earth. Concept of "free fall".

Key concepts ctd

- Displacement vs time x(t) Graph:
  - Slope is velocity (sign!)
- Velocity vs time v(t) Graph:
  - Area under curve is displacement x (sign!) (change in position)
  - Slope is acceleration (sign!)
- Acceleration vs time a(t) Graph
  - Area under the curve is change of velocity v (sign!)
- Understand how to derive one graph from another

Clicker question

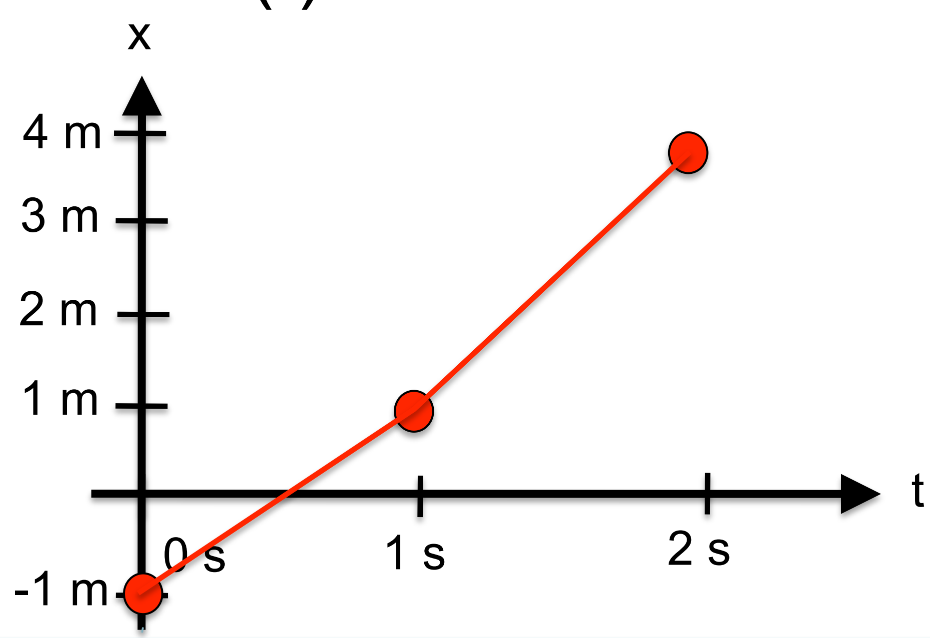
- Distance between snapshots in motion diagram
  - A increases
  - B stays the same
  - C decreases

Position

- For 1D motion we need to define a one axis coordinate system (an x-axis) along the path of motion (particle model!).
- The axis features:
  - Labeled using units of length (for example meters)
  - Zero is chosen at your convenience
  - Positive direction: we agree in this class that positive direction will always be to the right or up.
- Position of an object is the value of the axis label at its location. We use here the symbol x.
- Position may change with time. **It is a function of time x(t)**

Position vs time diagram

- A moving object has a position at each point in time.
- Graph position as a function of time: function x(t)



Displacement

- Displacement is a change in position
- Suppose there is an initial position at an earlier time x_i and a final position at a later time x_f
- The displacement Δx for this particular part of the motion is:

$$\Delta x = x_f - x_i$$

- Note this can be positive or negative (it has a **direction**)
  - If the change is towards a larger number (positive direction of the axis) x_f is larger than x_i and Δx will be positive.
  - If the change is towards a smaller number (negative direction of the axis) x_f is smaller than x_i and Δx will be negative
- Δx is ONE symbol, just like x or y

Quiz

- Maria stands at position x=30 m. 10 s later she is located at position x= -20 m. What was her displacement?

  A: 50 m
  B: 20 m
  C: 0 m
  D: -20 m
  E: -50 m

Velocity

- Velocity is rate of change of position.
- The **average** velocity for a specific part of an object's motion beginning at time t_i (initial time) ending at a later time t_f (final time) is

$$\text{Average Velocity} = \frac{\text{Displacement}}{\text{Time interval}} \quad \text{with symbols:} \quad v = \frac{\Delta x}{\Delta t} \qquad \text{With } \Delta t = t_f - t_i$$

- Unit: m/s (others: km/h, miles/h, ….)
- Velocity has a sign and therefore a direction: its the sign of Δx (because Δt is always positive)
  - Motion towards positive x-axis: positive velocity
  - Motion towards negative x-axis: negative velocity
  - Magnitude of velocity |v| is also called "speed".

$$\text{Average Speed} = \frac{\text{Distance travelled}}{\text{Time interval}}$$

Quiz

- Maria stands at position x=30 m. 10 s later she is located at position x= 10 m. What was her average velocity?

  A: 2 m/s
  B: 3 m/s
  C: 0 m/s
  D: -3 m/s
  E: -2 m/s

Quiz

- Maria stands at position x=30 m. In 10s she walks to x=50 m. In 20 s she then walks back to x=30 m. What was her average velocity?

  A: 1.5 m/s
  B: 1.33 m/s
  C: 0 m/s
  D: -1.33 m/s
  E: -1.5 m/s

Clicker question

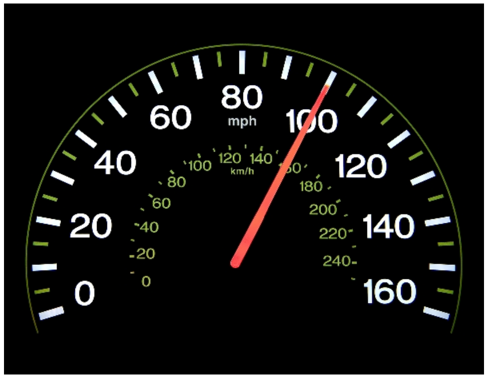
- Maria stands at position x=30 m. In 10s she walks to x=50 m. In 20 s she then walks back to x=30 m. What was her average **speed**?

  A: 1.5 m/s
  B: 1.3 m/s
  C: 0 m/s
  D: -1.3 m/s
  E: -1.5 m/s

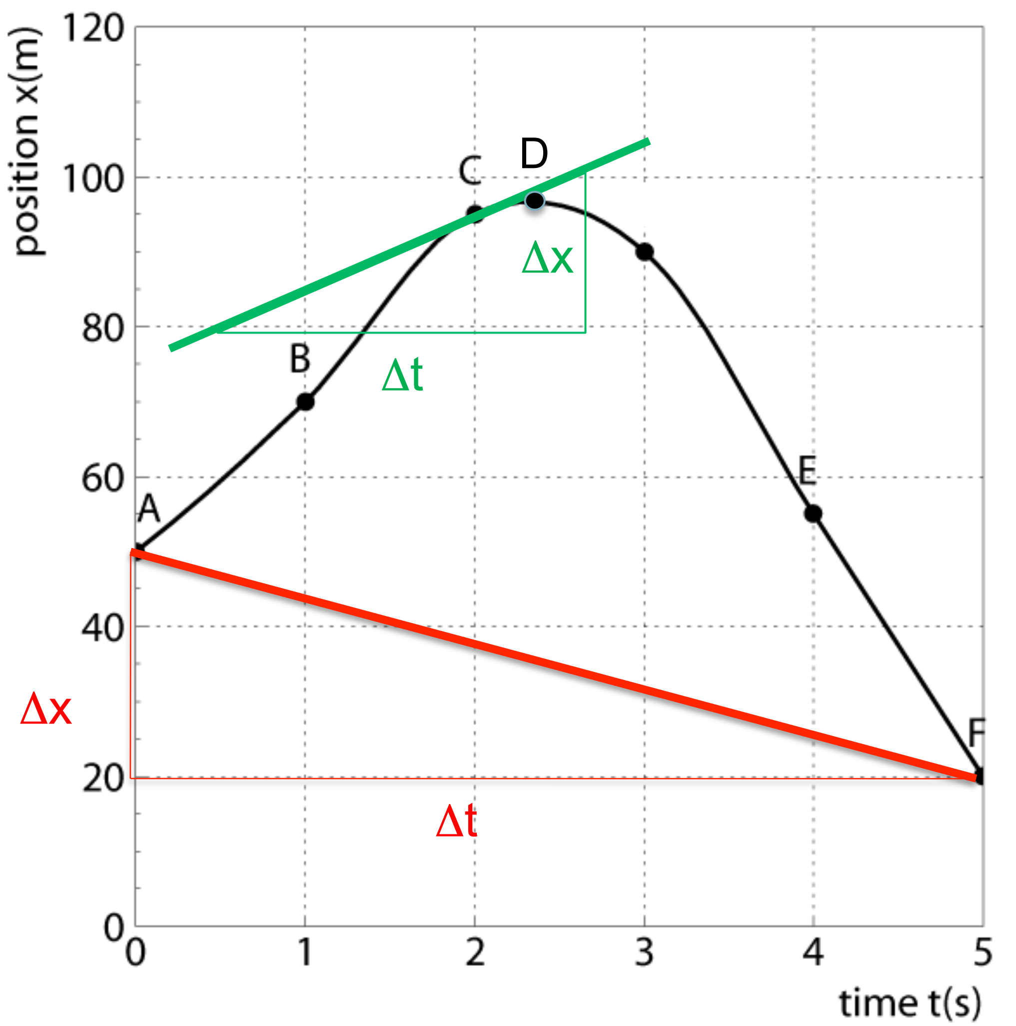
Instantaneous velocity

Officer, it took me 1 h to go 25 miles so my average speed was only 25 mph

Instantaneous speed
= speed at one point in time
(averaged over very small time interval)



Velocity in position vs time graph



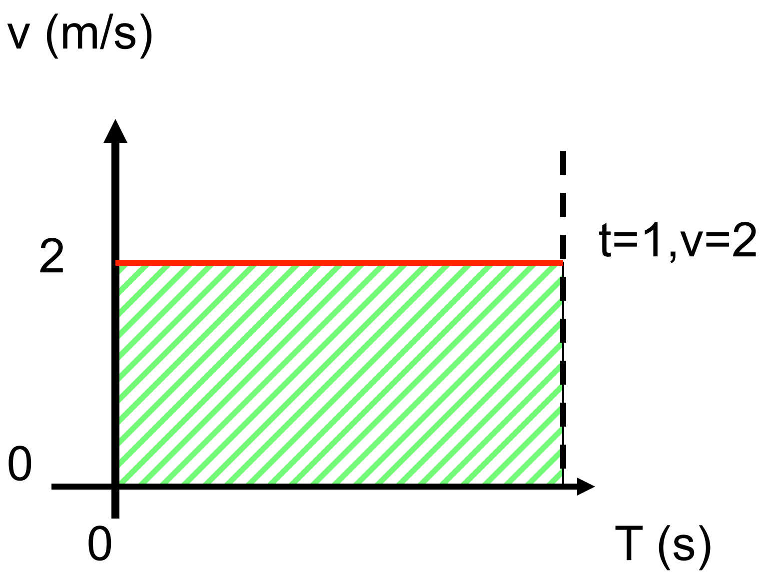
What is the instantaneous velocity at t=2.0 s?

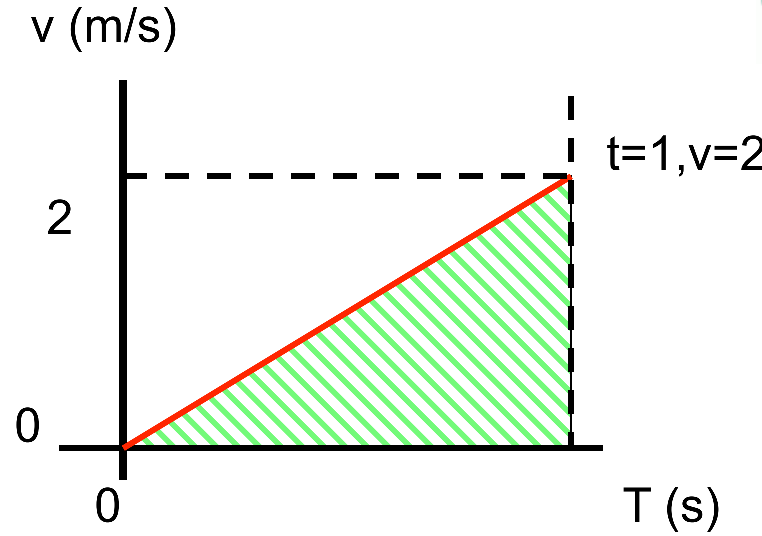
The velocity at one point in time is the slope of the tangent to the x-t curve at that time.
Calculate slope of green curve:

$$v = \frac{\Delta x}{\Delta t}$$

What is the average velocity for motion from t=0s to t=5.0s?
Calculate slope of red curve

http://www.math.umn.edu/~garrett/qy/TraceTangent.html





1)What is the distance covered in 1 second?
2)What is the area indicated by ▨ ?

Q 1. 2.

a) 1. 1.
b) 1. 2.
c) 2. 1.
d) 2. 2.

The area under the v-t curve is equal to the displacement of the object!
Note unit of area: area=2m/s*1s=2m

Acceleration

- Acceleration is the rate of change of velocity
- NOTE: This word is used differently from everyday use. Acceleration can mean increase, decrease or change of direction of velocity.
- When the velocity does not change the acceleration is zero
- If velocity changes from v_i at time t_i to v_f at time t_f then the **average** acceleration a is:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

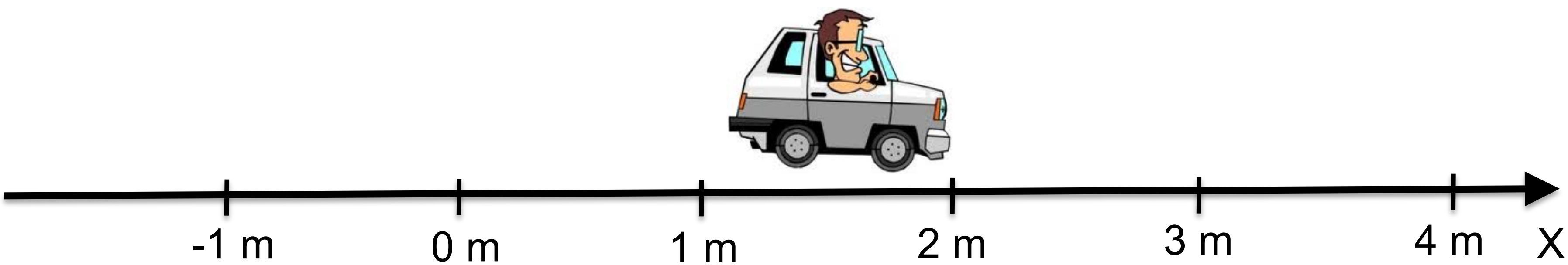
- Unit: m/s^2

Sign of acceleration

- As v_i and v_f can each be positive or negative, acceleration can also be positive or negative
- The sign indicates the direction of the change of velocity:
  – If velocity becomes smaller/more negative then the acceleration is negative ($v_f < v_i$)
  – If velocity becomes larger/more positive then the acceleration is positive ($v_f > v_i$)
- Note that negative acceleration DOES NOT mean the object gets slower.
  – Example: if a car's velocity changes from -10 m/s to -30 m/s the car is getting faster (moving in negative x direction) and the acceleration is negative as the velocity becomes smaller

Clicker question

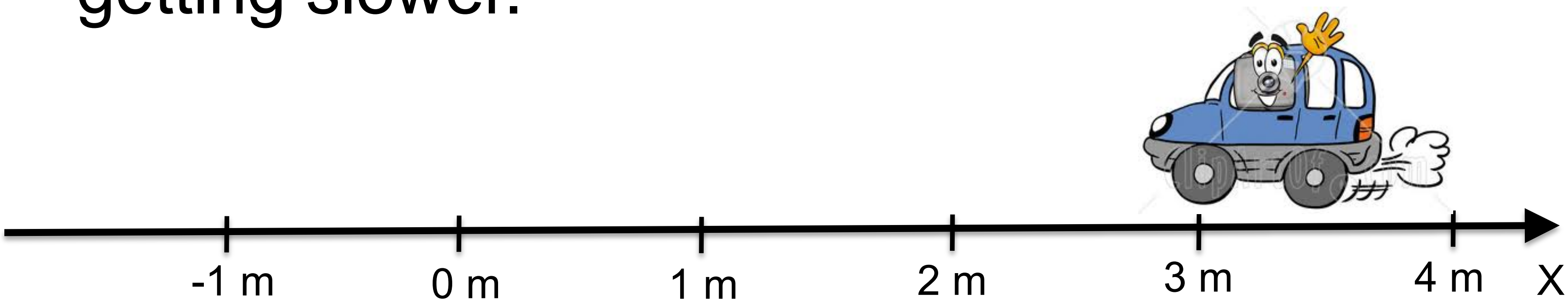
- A car is moving to the right and breaks so its getting slower.



The acceleration is
A positive
B zero
C negative
D don't know

Clicker question

- A car is moving to the left and breaks so its getting slower.



The acceleration is
A positive
B zero
C negative
D don't know

Motion with constant acceleration

- For a given acceleration a, an initial position at t=0 x_0 and an initial velocity at t=0 of v_0 we can predict
- Velocity at time t $$v(t) = v_0 + at$$
- Position at time t $$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Position changes quadratically !!!

Motion with constant acceleration 2

- Handy equation: combine

$$v_f = v(t) = v_0 + at$$

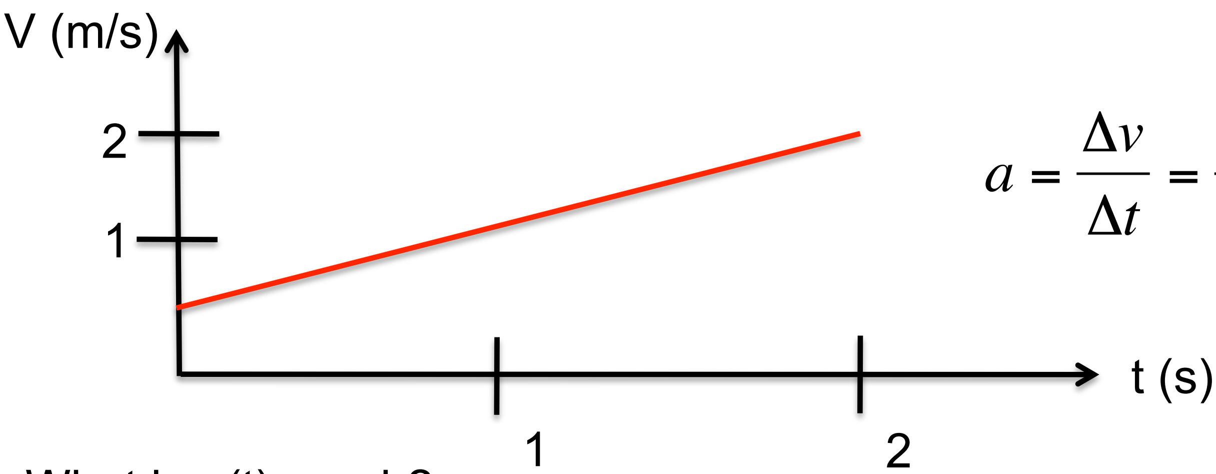
$$x_f = x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

for final velocity v_f and position x_f after constant acceleration a for some time t

- Eliminate t to relate directly the velocity and the displacement (for example to obtain the velocity after travelling a certain distance)

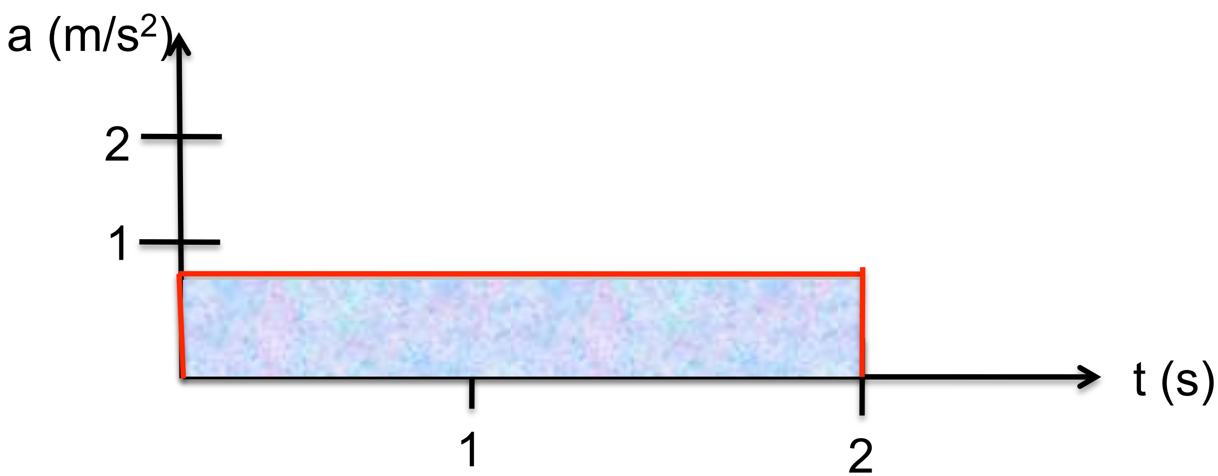
$$v_f^2 = v_i^2 + 2a\,\Delta x$$

Acceleration vs time graph



$$a = \frac{\Delta v}{\Delta t} = \frac{(2m/s - 0.5m/s)}{2s - 0s} = 0.75\frac{m}{s^2}$$

What is a(t) graph?



Area under the curve from t=0s to t=2s:
A=0.75 m/s^2 x 2s = 1.5 m/s
=1.5 m/s
is the change of velocity!

In symbols:
A=a x Δt = Δv

Example of a=const: Free Fall

- Earth's gravity causes objects on the surface of the earth to accelerate with $|a|=g=9.81\ m/s^2$ (varies a bit from place to place) IF no other force acts (Free Fall)
- Already Galileo found that this is true for any object regardless of its weight (If free fall is a good approximation – no significant air resistance)
- Note: "Free Fall" is used differently than in everyday language. The object in free fall does not need to fall – a ball thrown up in the air, once it leaves the hand, is in free fall as no other forces than gravity act, even during its upward motion
- Direction is towards the center of the earth (so if that is the negative direction of the position axis then $a=-g = -9.81\ m/s^2$)

Example: ball throw

- A ball is thrown vertically into the air with an initial speed of 2 m/s. Neglect air resistance.
  - How long does it take to reach the highest point?
  - What is the height it reaches?
  - Graph x(t), v(t) and a(t)
  - What is the velocity at the highest point?
  - What is the acceleration at the highest point?