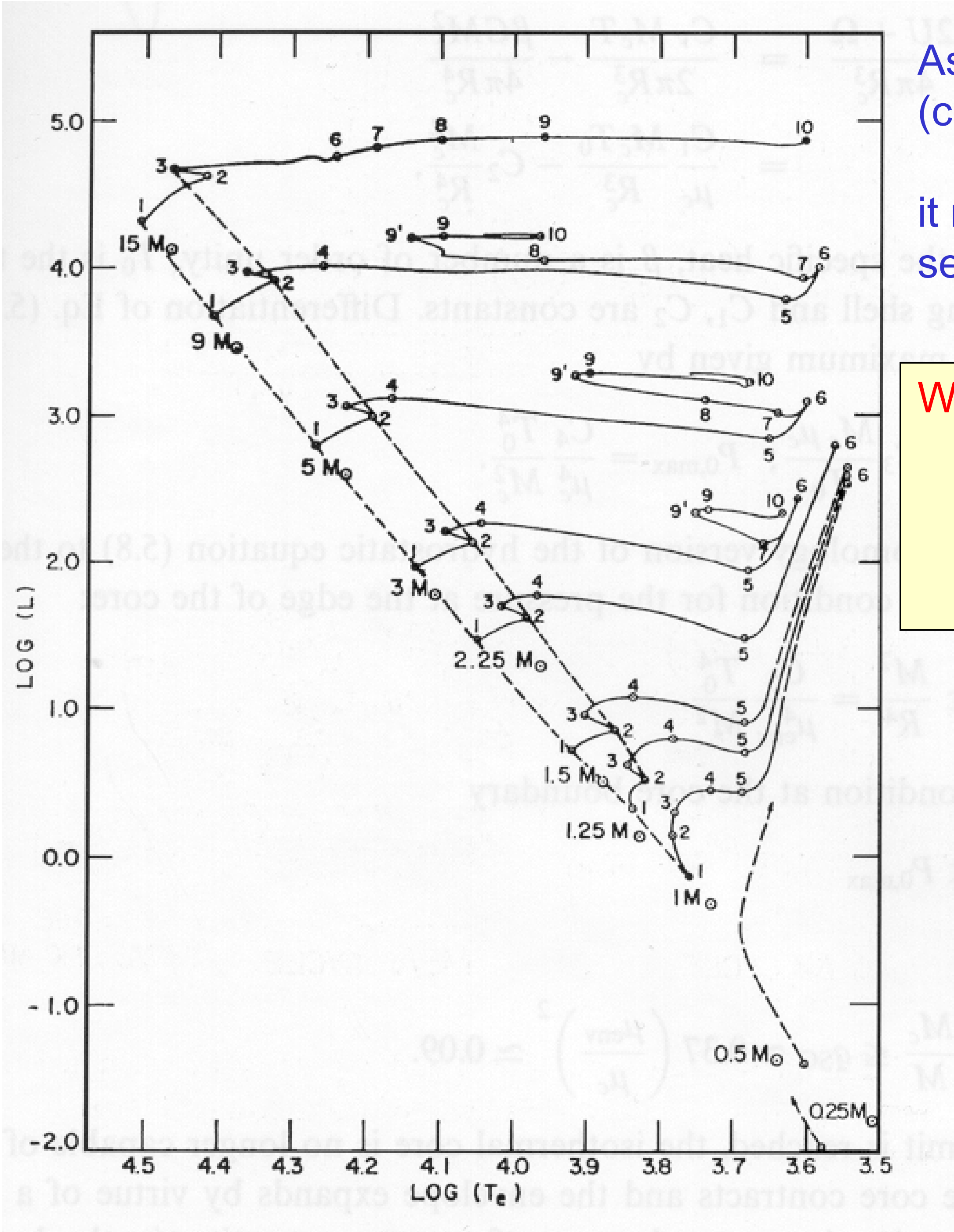


As hydrogen is exhausted in the (convective) core of a star

(point 2)

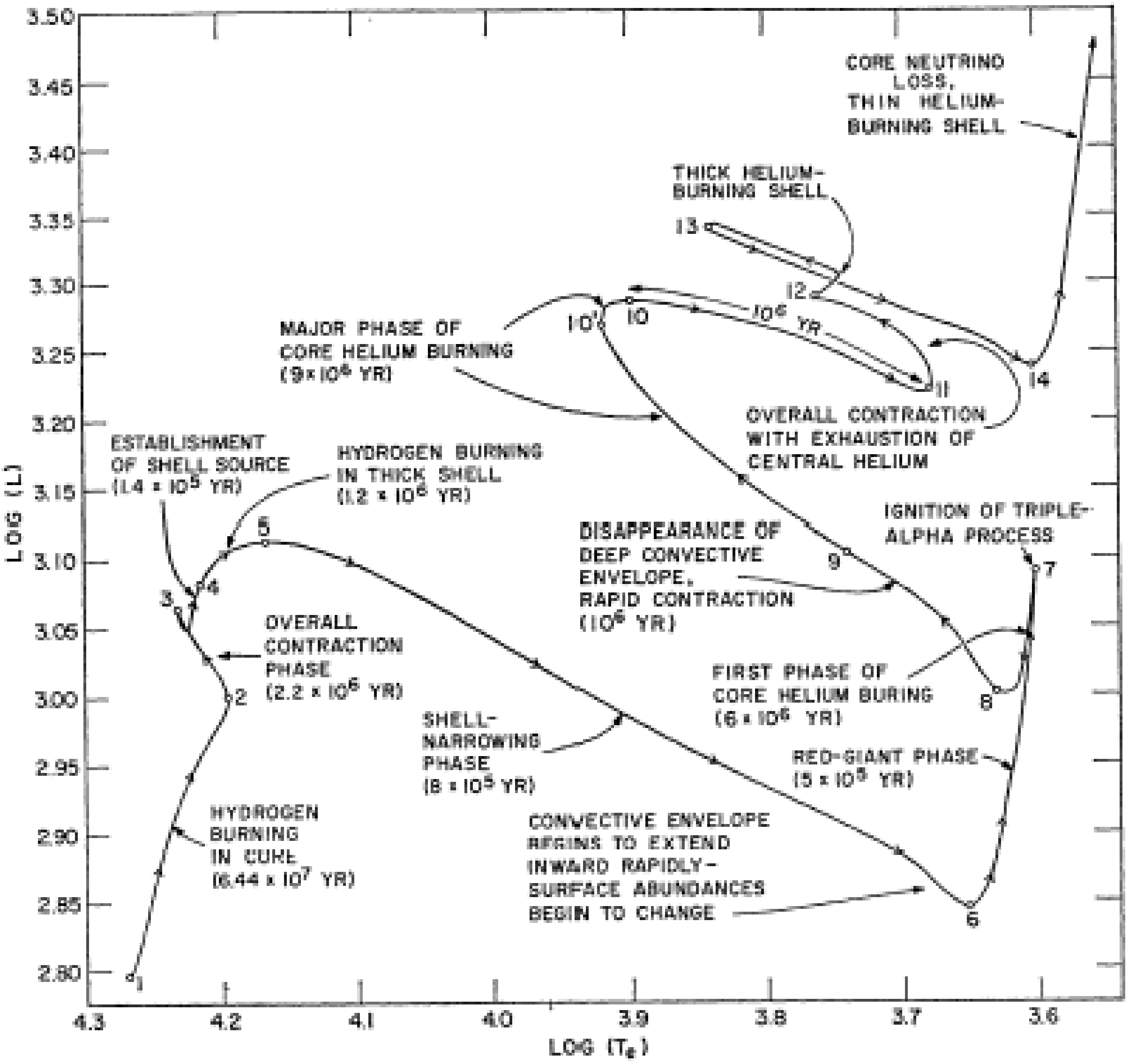
it moves away from the main sequence (point 3)

What happens to the star ?

- lower T → redder
- same L → larger (Stefan's L.)

star becomes a **red giant**

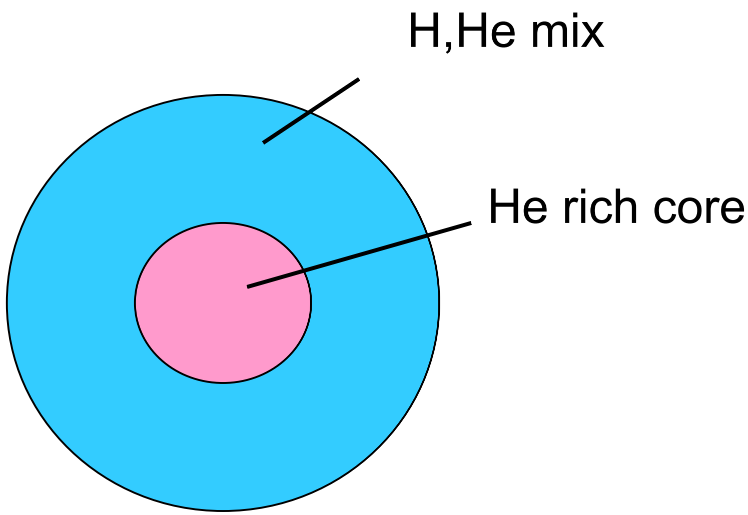
For completeness – here's what's happening in detail (5 solar mass ZAMS star):

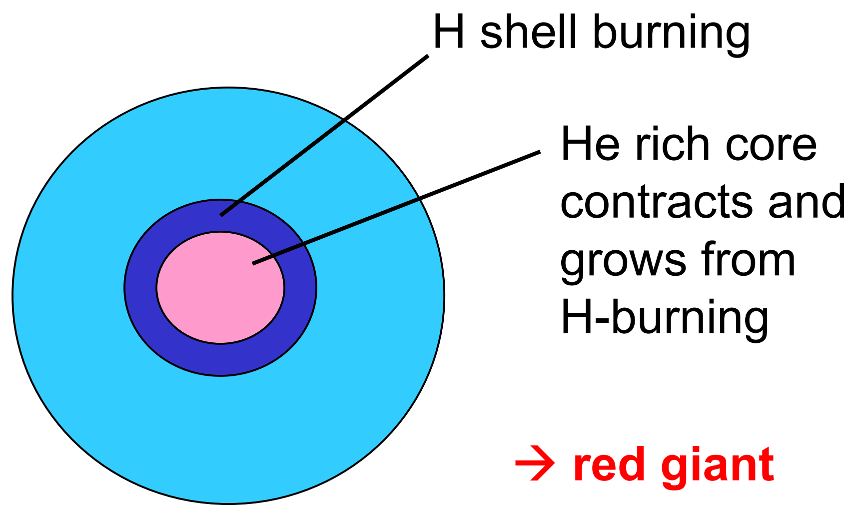


I. Iben, Ann. Rev. Astron. Astroph. Vol 5 (1967) P. 571

What happens at hydrogen exhaustion
(assume star had convective core)

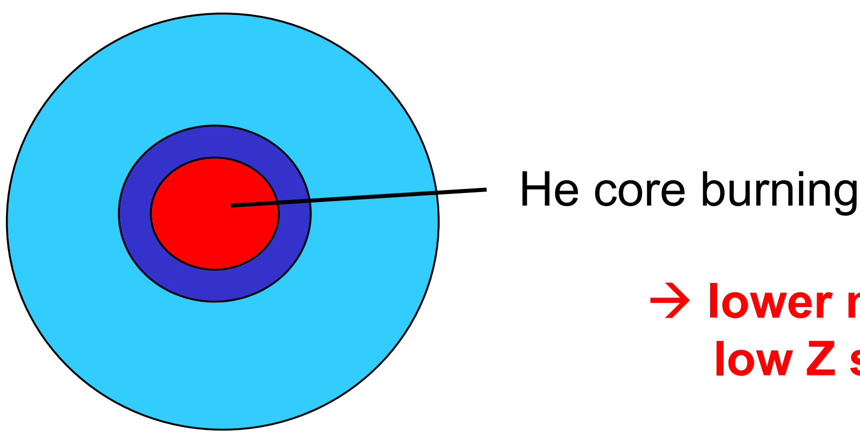
1. Core contracts and heats





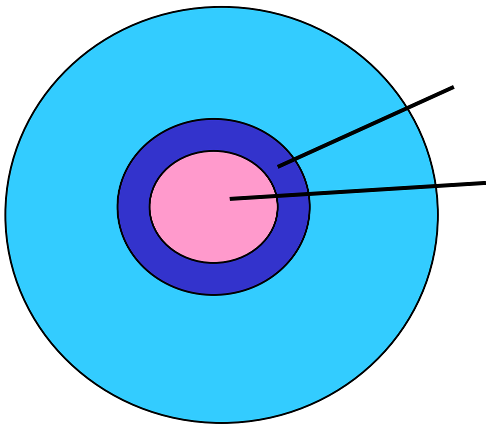
→ **red giant**

2. Core He burning sets in



→ **lower mass stars become bluer**
low Z stars jump to the horizontal branch

2. a ($M < 2.25\ M_0$) Degenerate He core



onset of electron degeneracy halts contraction

then He core grows by H-shell burning until He-burning sets in.

→ He burning is initially unstable (**He flash**)

in degenerate electron gas, pressure does not depend on temperature (why ?)
therefore a slight rise in temperature is not compensated by expansion

→ thermonuclear runaway:

- rise temperature
- accelerate nuclear reactions
- increase energy production

Why does the star expand and become a red giant ?

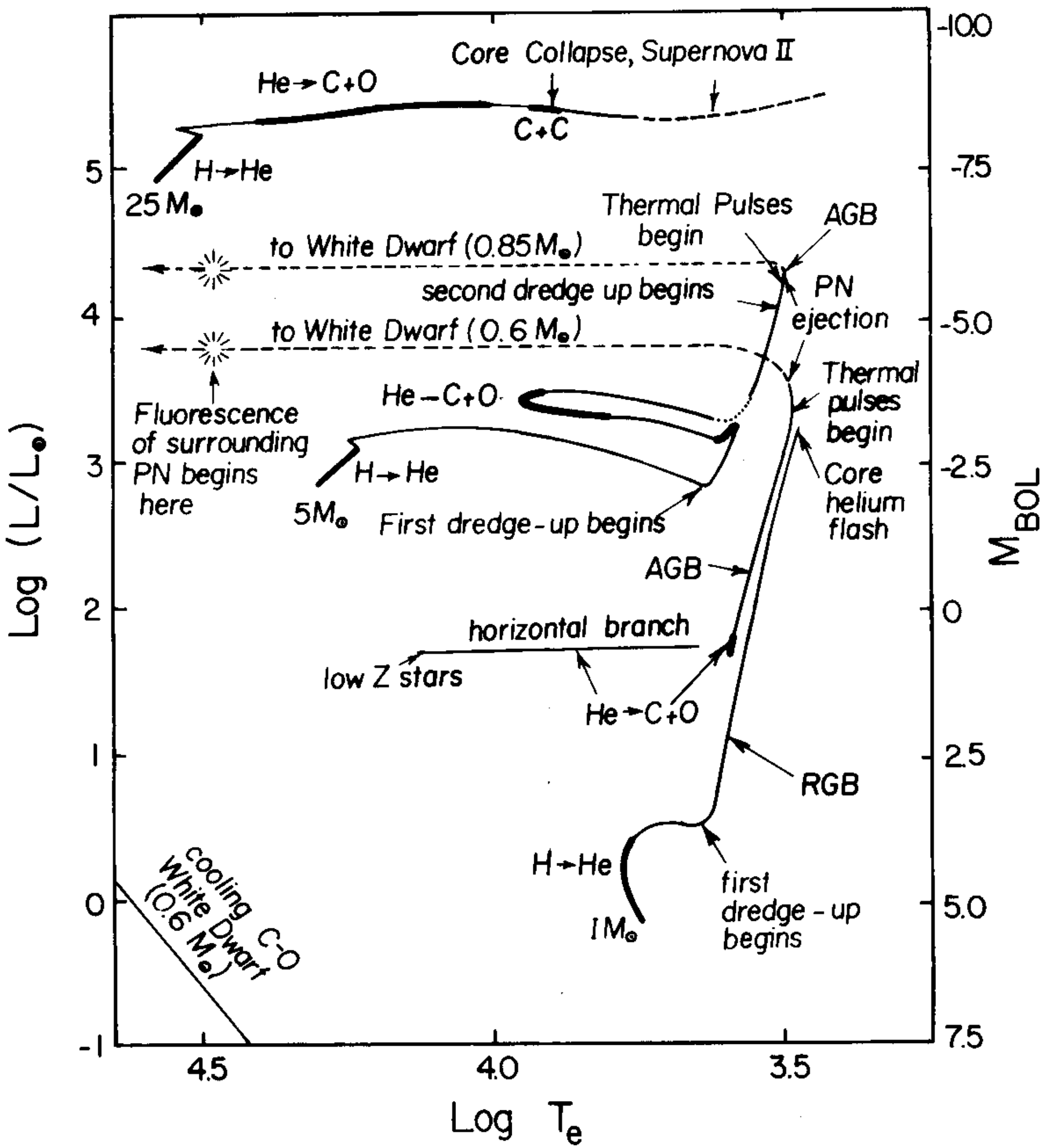
Because of higher Coulomb barrier He burning requires much higher temperatures
→ drastic change in central temperature
→ star has to readjust to a new configuration

Qualitative argument:

- need about the same Luminosity – similar temperature gradient dT/dr
- now much higher T_c – need larger star for same dT/dr

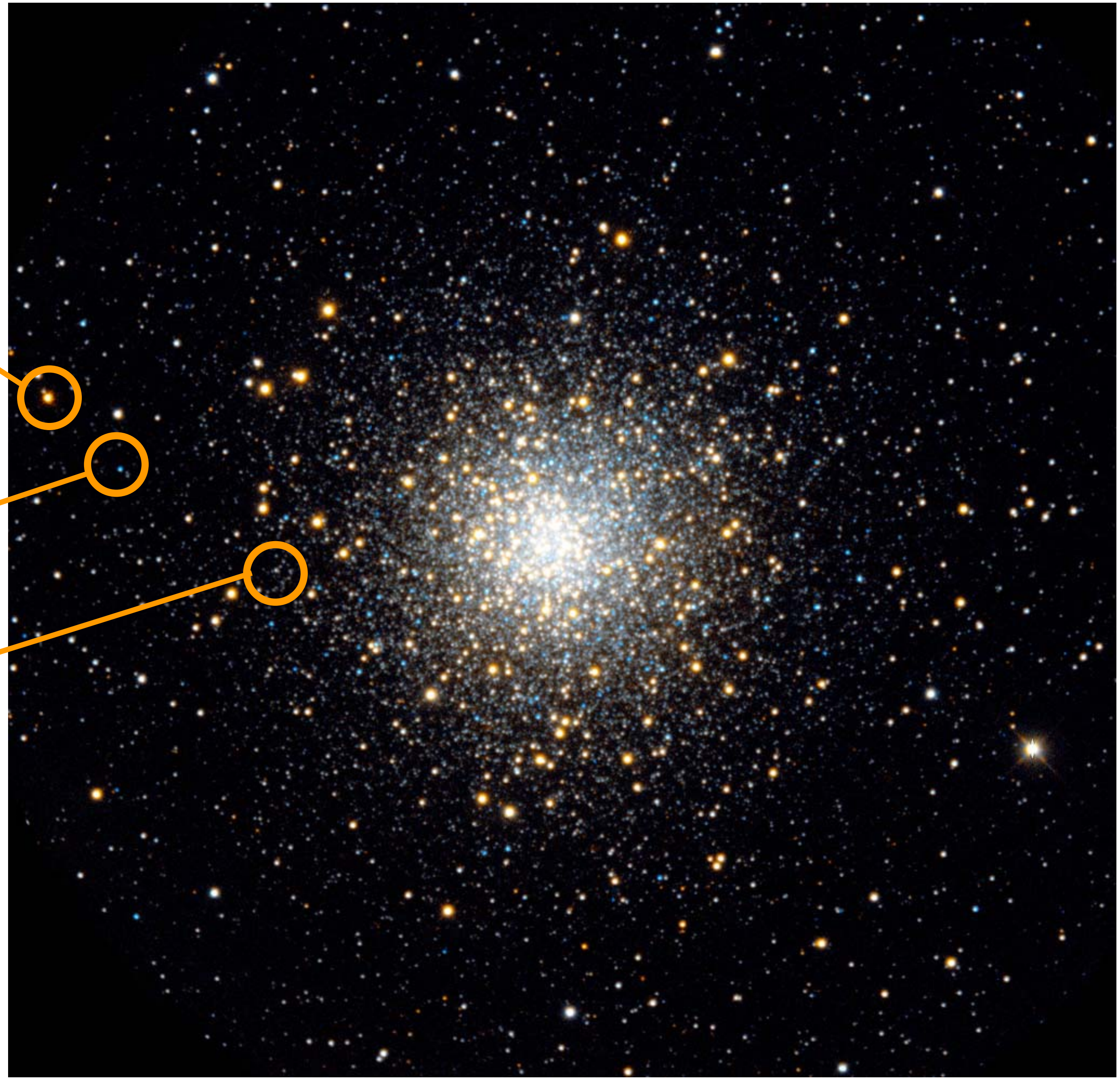
Lower mass stars become red giants during shell H-burning

If the sun becomes a red giant in about 5 Bio years, it will almost fill the orbit of Mars



Pagel, Fig. 5.14

Globular Cluster M10



He burning overview

- Lasts about 10% of H-burning phase
- Temperatures: ~300 Mio K
- Densities ~ 10^4 g/cm^3

Reactions: $^4He + ^4He + ^4He \rightarrow ^{12}C$ (triple α process)

$^{12}C + ^4He \rightarrow ^{16}O$ ($^{12}C(\alpha,\gamma)$)

Main products: carbon and oxygen (main source of these elements in the universe)

Helium burning 1 – the 3α process

First step:

$\alpha + \alpha \rightarrow {}^{8}Be$

unbound by ~92 keV – decays back to 2 α within 2.6E-16 s !

but small equilibrium abundance is established

Second step:

${}^{8}Be + \alpha \rightarrow {}^{12}C^{*}$ would create ${}^{12}C$ at excitation energy of ~7.7 MeV

1954 Fred Hoyle (now Sir Fred Hoyle) realized that the fact that there is carbon in the universe requires a resonance in ${}^{12}C$ at ~7.7 MeV excitation energy

1957 Cook, Fowler, Lauritsen and Lauritsen at Kellogg Radiation Laboratory at Caltech discovered a state with the correct properties (at 7.654 MeV)

→ **Experimental Nuclear Astrophysics was born**

How did they do the experiment ?

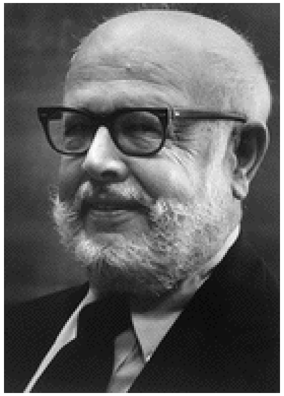
- Used a deuterium beam on a 11B target to produce 12B via a (d,p) reaction.
- 12B β-decays within 20 ms into the second excited state in 12C
- This state then immediately decays under alpha emission into 8Be
- Which immediately decays into 2 alpha particles

So they saw after the delay of the b-decay 3 alpha particles coming from their target after a few ms of irradiation

This proved that the state can also be formed by the 3 alpha process …

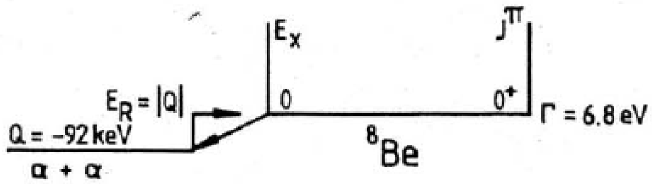
→ removed the major roadblock for the theory that elements are made in stars

→ Nobel Prize in Physics 1983 for Willy Fowler (alone !)



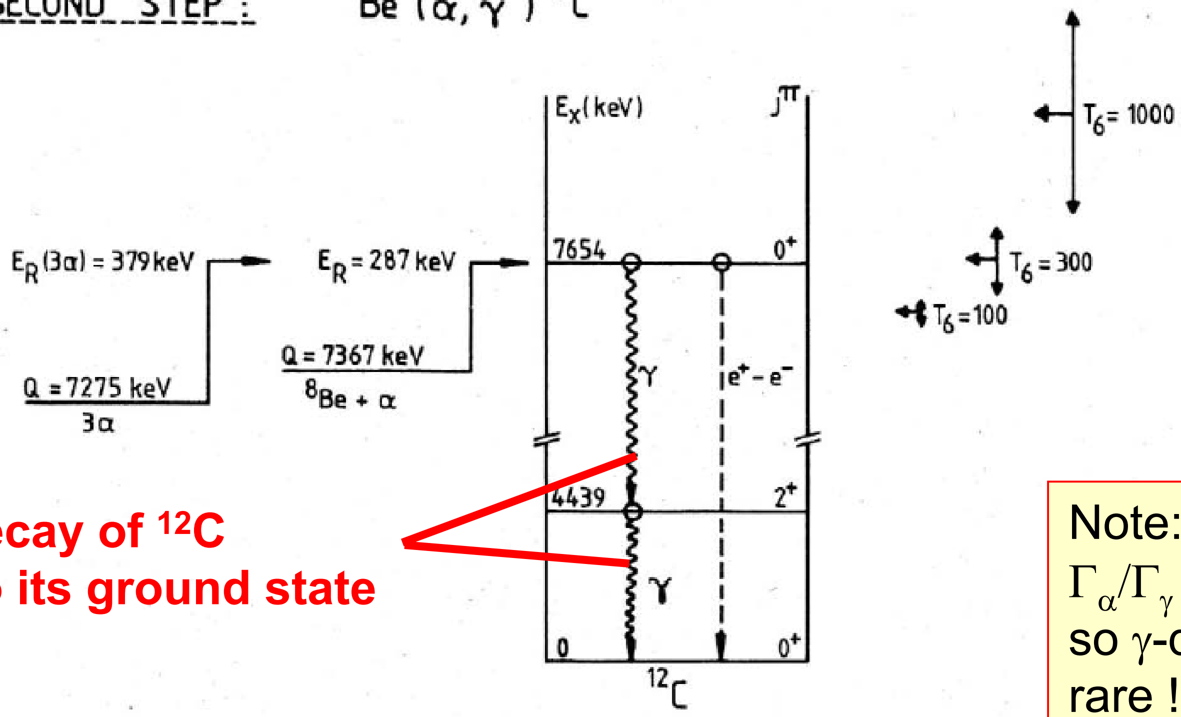
Third step completes the reaction:

FIRST STEP: $\alpha + \alpha \rightleftharpoons {}^{8}Be$



Note: ^{8}Be ground state is a 92 keV resonance for the α+α reaction

SECOND STEP: $^{8}Be\,(\alpha,\gamma)\,^{12}C$



γ decay of ^{12}C into its ground state

Note:
$\Gamma_\alpha/\Gamma_\gamma > 10^3$
so γ-decay is very rare !

Calculation of the 3α rate in stellar He burning

Under stellar He-burning conditions, production and destruction reactions for $^{12}C^{*}$(7.6 MeV) are very fast (as state mainly α-decays !)

therefore the whole reaction chain is in equilibrium:

$\alpha + \alpha \rightleftarrows {}^{8}Be \qquad {}^{8}Be + \alpha \rightleftarrows {}^{12}C^{*}(7.7\ \mathrm{MeV})$

The $^{12}C^{*}$(7.6 MeV) abundance is therefore given by the Saha Equation:

$$Y_{12C(7.6\,\mathrm{MeV})}\left(\frac{2\pi\hbar^2}{m_{12C}kT}\right)^{3/2} = Y_{4He}^{3}\rho^2 N_A^2\left(\frac{2\pi\hbar^2}{m_{4He}kT}\right)^{9/2} e^{-Q/kT}$$

with $Q/c^2 = m_{12C(7.7)} - 3m_\alpha$

using $m_{12\mathrm{C}} \approx 3m_{4\mathrm{He}}$

one obtains: $$Y_{12\mathrm{C}(7.6\,\mathrm{MeV})} = 3^{3/2} Y_{4\mathrm{He}}^3 \rho^2 N_A^2 \left(\frac{2\pi\hbar^2}{m_{4\mathrm{He}} kT} \right)^3 \mathrm{e}^{-\mathrm{Q/kT}}$$

The total 3α reaction rate (per s and cm³) is then the total gamma decay rate (per s and cm³) from the 7.6 MeV state.

This reaction represents the leakage out of the equilibrium !

Therefore for the total 3α rate r:

$$r = Y_{12\mathrm{C}(7.6\,\mathrm{MeV})} \rho N_A \frac{\Gamma_\gamma}{\hbar}$$

And with the definition

$$\lambda_{3\alpha} = \boxed{\frac{1}{6}} Y_\alpha^2 \rho^2 N_A^2 < \alpha\alpha\alpha >$$

note 1/6 because 3 identical particles !

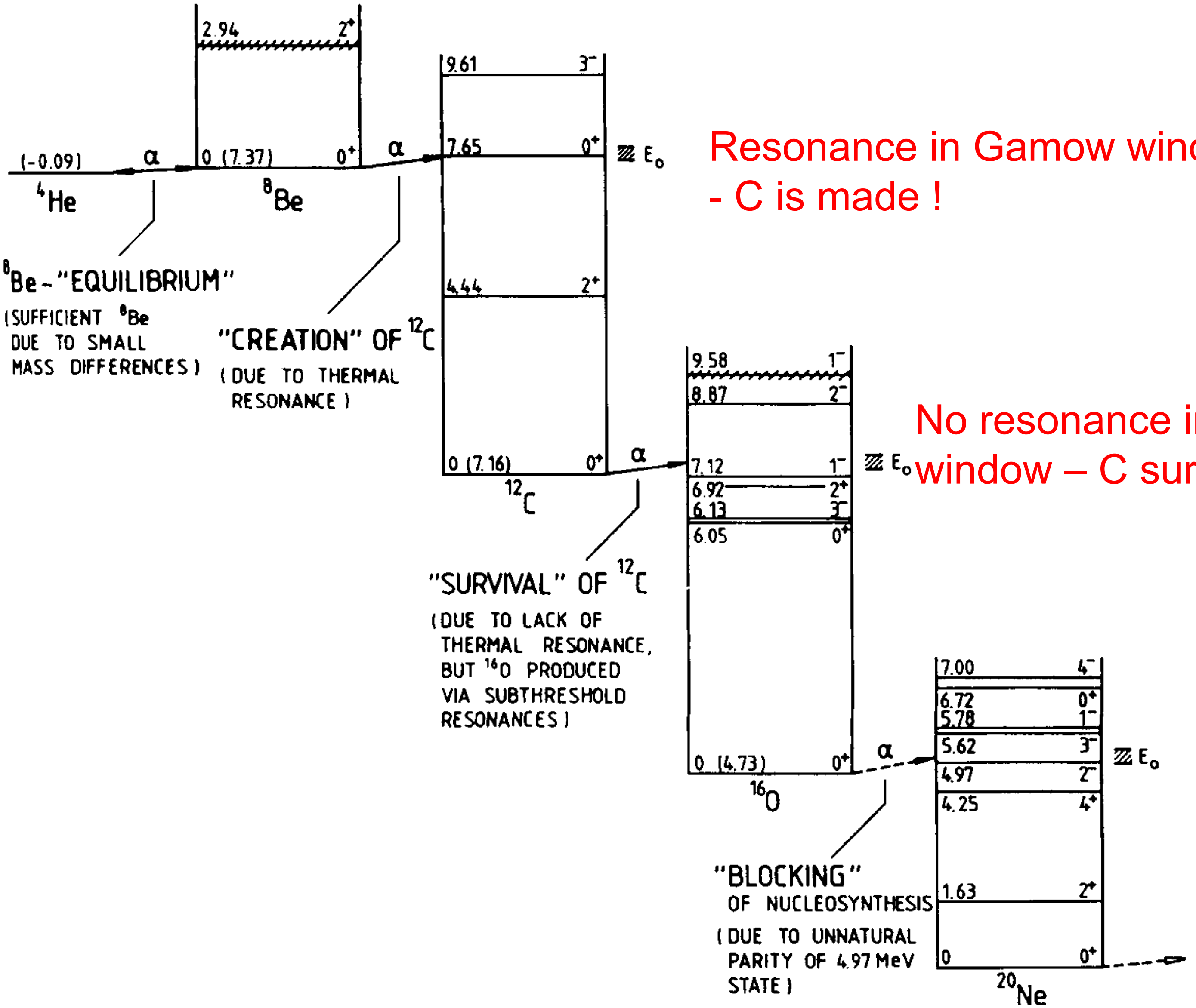
one obtains:

$$N_A^2 < \alpha\alpha\alpha >= 6 \cdot 3^{3/2} \rho^2 N_A^2 \left(\frac{2\pi\hbar^2}{m_{4\mathrm{He}} kT} \right)^3 \frac{\Gamma_\gamma}{\hbar} \mathrm{e}^{-\mathrm{Q/kT}}$$

(Nomoto et al. A&A 149 (1985) 239)

With the exception of masses, the only information needed is the gamma width of the 7.6 MeV state in ^{12}C. This is well known experimentally by now.

Helium burning 2 – the $^{12}C(\alpha,\gamma)$ rate

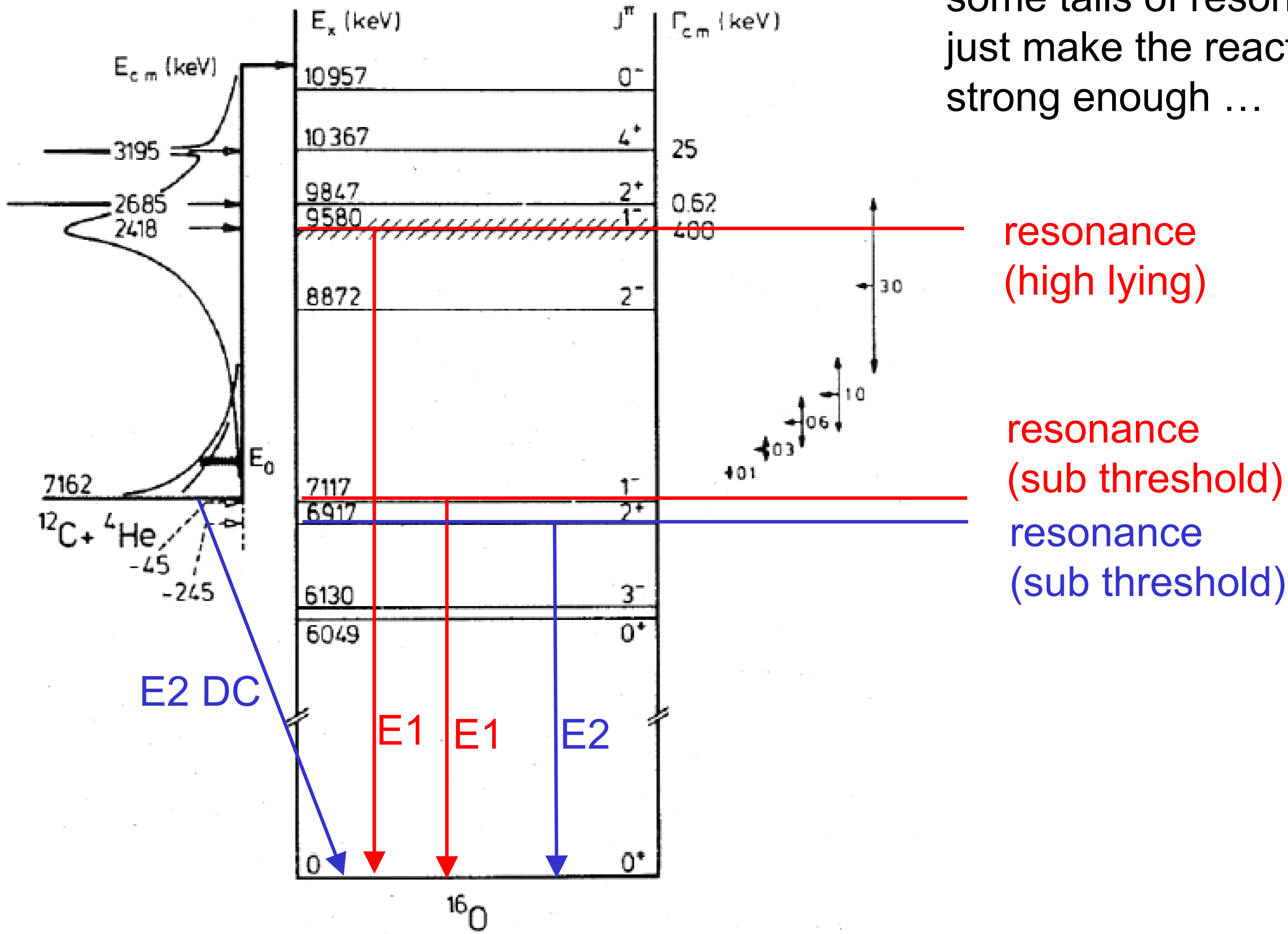


Resonance in Gamow window
- C is made !

No resonance in Gamow window – C survives !

But some C is converted into O …

some tails of resonances just make the reaction strong enough …



complications:

- very low cross section makes direct measurement impossible
- subthreshold resonances cannot be measured at resonance energy
- Interference between the E1 and the E2 components

Therefore:

Uncertainty in the $^{12}C(\alpha,\gamma)$ rate is the single most important nuclear physics uncertainty in astrophysics

Affects:

- C/O ration → further stellar evolution (C-burning or O-burning ?)
- iron (and other) core sizes (outcome of SN explosion)
- Nucleosynthesis (see next slide)

Some current results for S(300 keV):

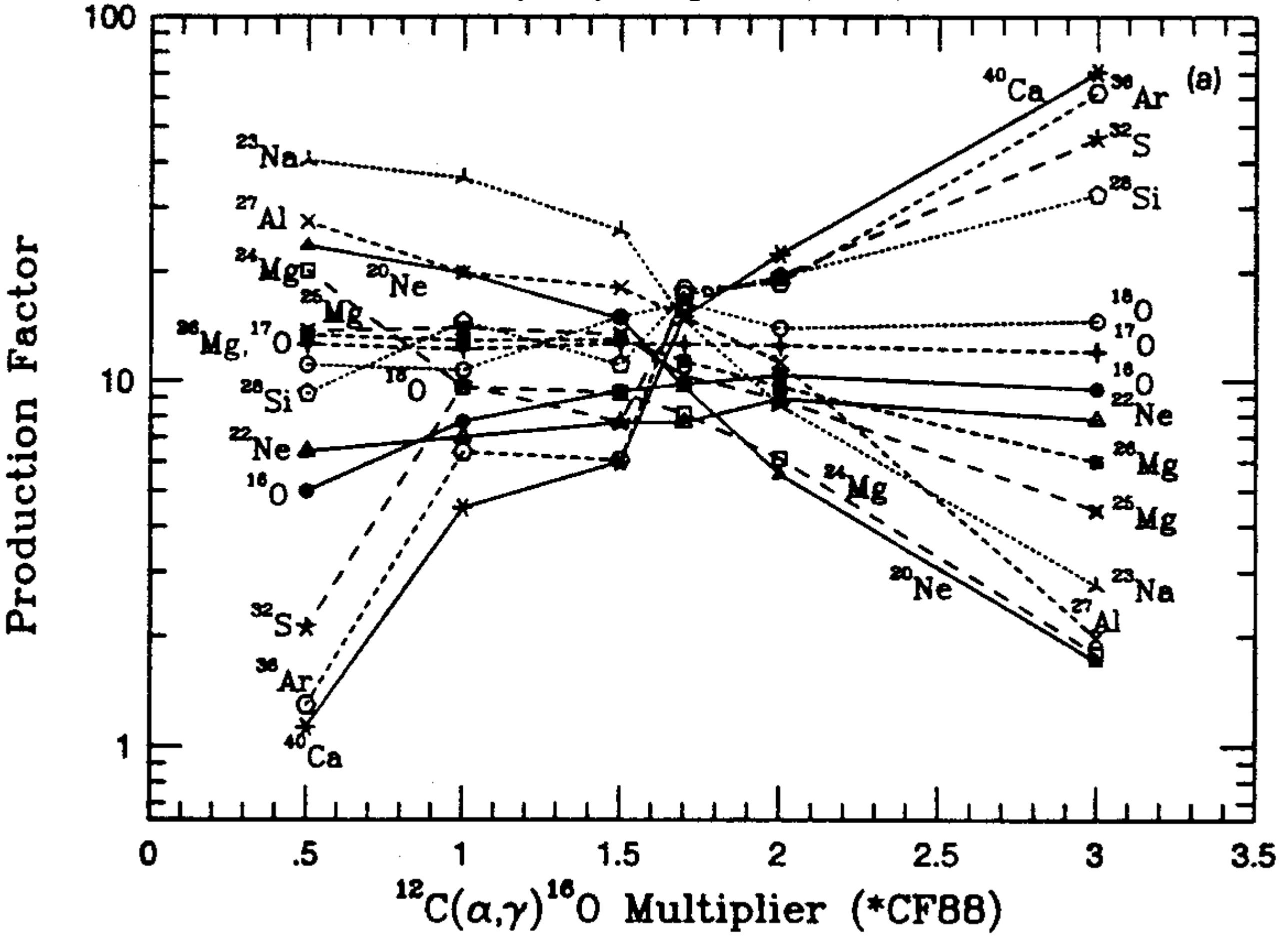
S_{E2}=53+13-18 keV b (Tischhauser et al. PRL88(2002)2501

S_{E1}=79+21-21 keV b (Azuma et al. PRC50 (1994) 1194)

But others range among groups larger !

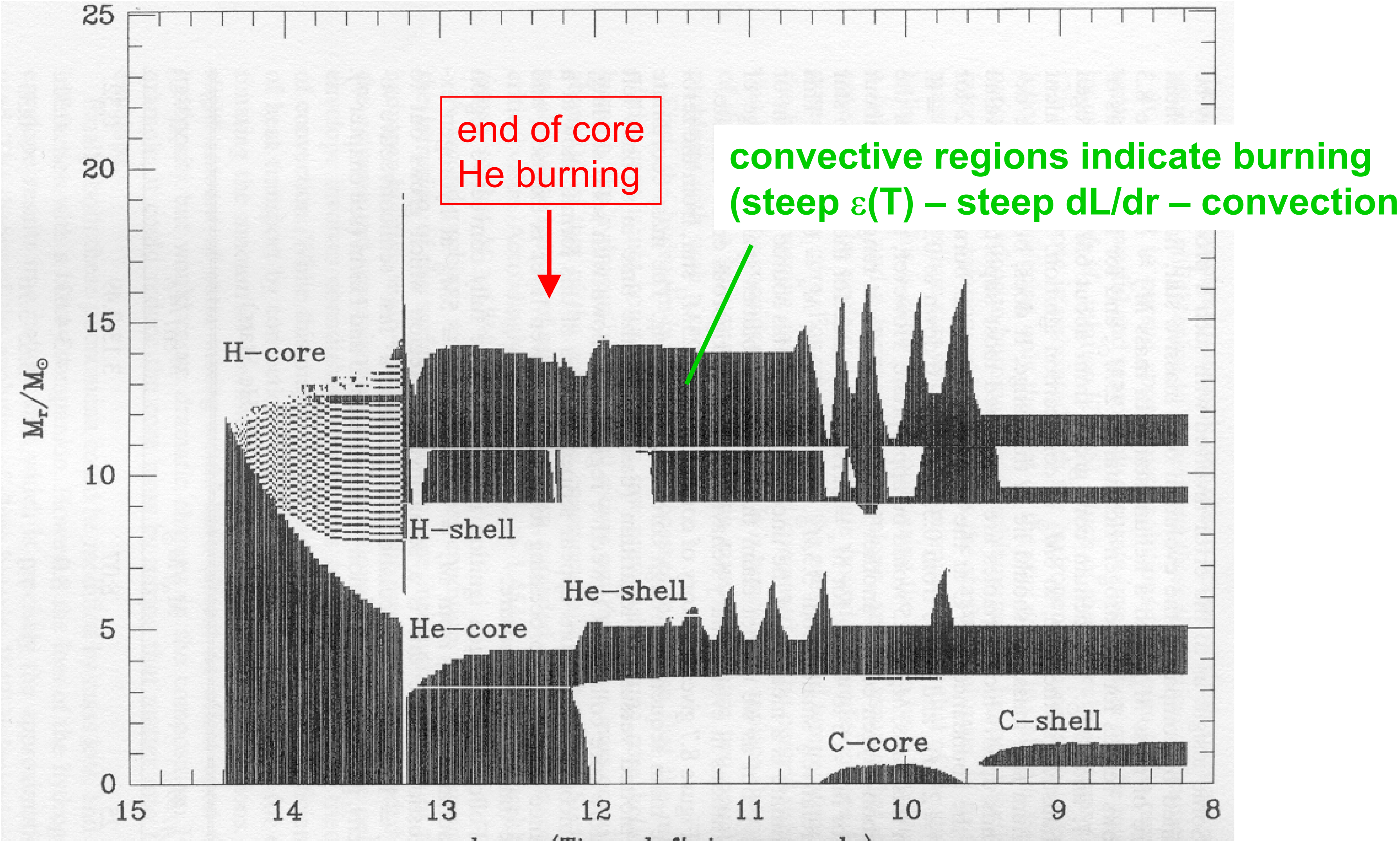
Massive star nucleosynthesis model as a function of $^{12}C(\alpha,\gamma)$ rate

Weaver and Woosley Phys Rep 227 (1993) 65



- This demonstrates the sensitivity
- One could deduce a preference for a total S(300) of ~120-220
  (But of course we cannot be sure that the astrophysical model is right)

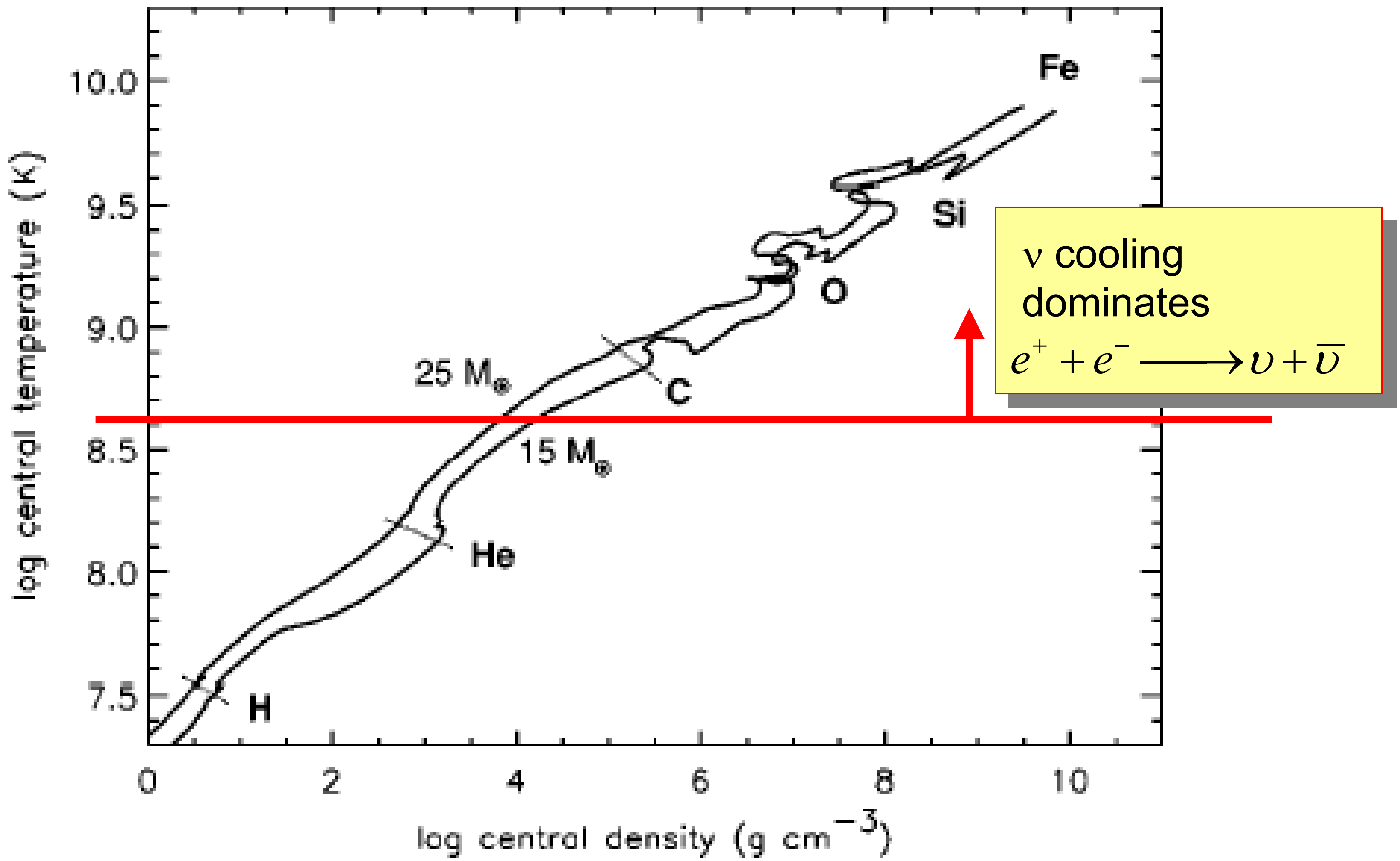
End of core helium burning and beyond



Arnett, fig 8.7

→ note complicated multiple burning layers !!!

Further evolution of burning conditions



Woosley, Heger, Weaver, Rev. Mod. Phys 74 (2002)1015

Carbon burning

Burning conditions:

for stars > 8 M_o (solar masses) (ZAMS)

T~ 600-700 Mio
ρ ~ 10^5-10^6 g/cm^3

Major reaction sequences:

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}^* &\rightarrow {}^{23}\mathrm{Mg} + n - 2.62\ \mathrm{MeV} \\
&\rightarrow {}^{20}\mathrm{Ne} + \alpha + 4.62\ \mathrm{MeV} \\
&\rightarrow {}^{23}\mathrm{Na} + p + 2.24\ \mathrm{MeV}.
\end{aligned}
$$

dominates by far

of course p's, n's, and a's are recaptured … ^{23}Mg can b-decay into ^{23}Na

Composition at the end of burning:

mainly ^{20}Ne, ^{24}Mg, with some ^{21,22}Ne, ^{23}Na, ^{24,25,26}Mg, ^{26,27}Al

of course ^{16}O is still present in quantities comparable with ^{20}Ne (not burning … yet)

Neon burning

Burning conditions:

for stars > 12 M_o (solar masses) (ZAMS)

T~ 1.3-1.7 Bio K
$\rho \sim 10^6$ g/cm^3

Why would neon burn before oxygen ???

Answer:

Temperatures are sufficiently high to initiate **photodisintegration** of ^{20}Ne

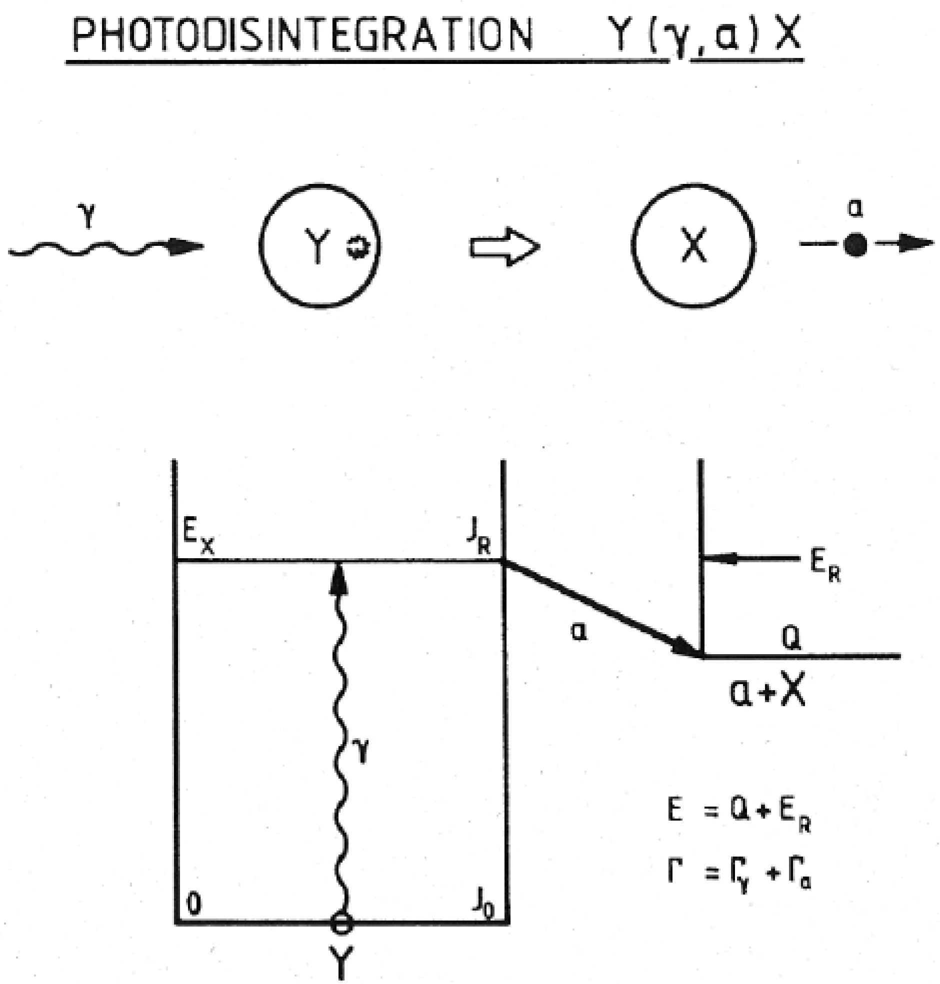
$$\left.\begin{array}{l} ^{20}\text{Ne}+\gamma \rightarrow {}^{16}\text{O} + \alpha \\ ^{16}\text{O}+\alpha \rightarrow {}^{20}\text{Ne} + \gamma \end{array}\right\}$$ equilibrium is established

this is followed by (using the liberated helium)

$$^{20}\text{Ne}+\alpha \rightarrow {}^{24}\text{Mg} + \gamma$$

so net effect: $$2\ {}^{20}\text{Ne} \rightarrow {}^{16}\text{O} + {}^{24}\text{Mg} + 4.59\ \text{MeV}.$$

Photodisintegration



(Rolfs, Fig. 8.5.)

Calculations of inverse reaction rates

A reaction rate for a process like $^{20}Ne+\gamma \rightarrow {}^{16}O + \alpha$ can be easily calculated from the inverse reaction rate $^{16}O+\alpha \rightarrow {}^{20}Ne + \gamma$ using the formalism developed so far.

In general there is a simple relationship between the rates of a reaction rate and its inverse process (if all particles are thermalized)

Derivation of "detailed balance principle":

Consider the reaction A+B → C with Q-value Q in thermal equilibrium.
Then the abundance ratios are given by the Saha equation:

$$\frac{n_A n_B}{n_C} = \frac{g_A g_B}{g_C}\left(\frac{m_A m_B}{m_C}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} \mathrm{e}^{-Q/kT}$$

In equilibrium the abundances are constant per definition. Therefore in addition

$$\frac{dn_C}{dt} = n_A n_B < \sigma\upsilon > - \lambda_C n_C = 0$$

or $$\frac{\lambda_C}{< \sigma\upsilon >} = \frac{n_A n_B}{n_C}$$

If <σv> is the A+B→ C reaction rate, and λ_C is the C →A+B decay rate

Therefore the rate ratio is defined by the Saha equation as well !

Using both results one finds

$$\frac{\lambda_C}{< \sigma v >} = \frac{g_A g_B}{g_C} \left(\frac{m_A m_B}{m_C} \right)^{3/2} \left(\frac{kT}{2\pi\hbar^2} \right)^{3/2} \mathrm{e}^{-Q/kT}$$

or using $m_C \sim m_A + m_B$ and introducing the reduced mass μ

Detailed balance:

$$\frac{\lambda_C}{<\sigma v>} = \frac{g_A g_B}{g_C}\left(\frac{\mu kT}{2\pi\hbar^2}\right)^{3/2} \mathrm{e}^{-Q/kT}$$

So just by knowing partition functions g and mass m of all participating particles on can calculate for every reaction the rate for the inverse process.

Partition functions:

For a particle in a given state i this is just $g_i = 2J_i + 1$

However, in an astrophysical environment some fraction of the particles can be in thermally excited states with different spins. The partition function is then given by:

$$g = \sum_i g_i e^{-E_i/kT}$$

Oxygen burning

Burning conditions:

T~ 2 Bio

$\rho \sim 10^7$ g/cm^3

Major reaction sequences:

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^* \rightarrow {}^{31}\mathrm{S} + n + 1.45\ \mathrm{MeV} \quad (5\%)$$

$$\rightarrow {}^{31}\mathrm{P} + p + 7.68\ \mathrm{MeV} \quad (56\%)$$

$$\rightarrow {}^{30}\mathrm{P} + d - 2.41\ \mathrm{MeV} \quad (5\%)$$

$$\rightarrow {}^{28}\mathrm{Si} + \alpha + 9.59\ \mathrm{MeV}. \quad (34\%)$$

plus recapture of n,p,d,α

Main products:

^{28}Si,^{32}S (90%) and some ^{33,34}S,^{35,37}Cl,$^{36.38}$Ar, ^{39,41}K, ^{40,42}Ca

Silicon burning

Burning conditions:

T~ 3-4 Bio
$\rho \sim 10^9$ g/cm^3

Reaction sequences:

- Silicon burning is fundamentally different to all other burning stages.
- **Complex network of fast (γ,n), (γ,p), (γ,a), (n,γ), (p,γ), and (a,γ) reactions**
- The net effect of Si burning is: 2 ^{28}Si --> ^{56}Ni,

need new concept to describe burning:

Nuclear Statistical Equilibrium (NSE)

Quasi Statistical Equilibrium (QSE)

Nuclear Statistical Equilibrium

Definition:

In **NSE**, each nucleus is in equilibrium with protons and neutrons

Means: the reaction Z * p + N * n <---> (Z,N) is in equilibrium

Or more precisely: $Z \cdot \mu_p + N \cdot \mu_n = \mu_{(Z,N)}$ for all nuclei (Z,N)

NSE is established when both, photodisintegration rates of the type

(Z,N) $+ \gamma$ --> (Z-1,N) + p

(Z,N) $+ \gamma$ --> (Z,N-1) + n

(Z,N) $+ \gamma$ --> (N-2,N-2) + α

and capture reactions of the types

(Z,N) + p --> (Z+1,N)

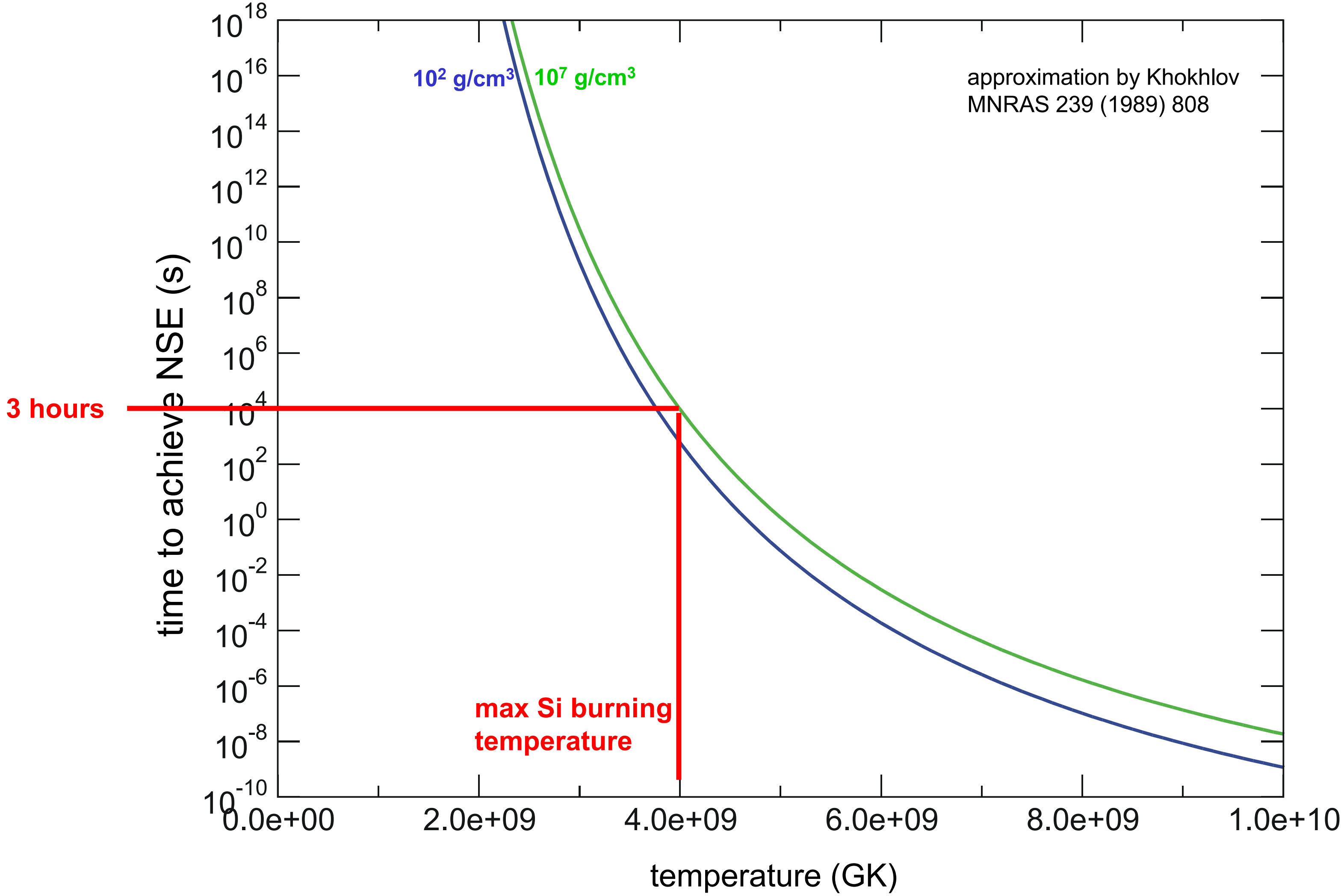
(Z,N) + n --> (Z,N+1)

(Z,N) $+ \alpha$ --> (Z+2,N+2)

are fast

NSE is established on the timescale of these reaction rates (the slowest reaction)

A system will be in NSE if this timescale is shorter than the timescale for the temperature and density being sufficiently high.



for temperatures above ~5 GK even explosive events achieve full NSE

Nuclear Abundances in NSE

The **ratio of the nuclear abundances in NSE to the abundance of free protons and neutrons** is entirely determined by

$$Z \cdot \mu_p + N \cdot \mu_n = \mu_{(\mathrm{Z,N})}$$

which only depends on the chemical potentials

$$\mu = mc^2 + kT \ln\left[\frac{n}{g} \left(\frac{h^2}{2\pi mkT} \right)^{3/2} \right]$$

So all one needs are **density**, **temperature**, and for each nucleus **mass** and **partition function** (**one does not need reaction rates** !! - except for determining whether equilibrium is indeed established)

Solving the two equations on the previous page yields for the abundance ratio:

$$Y(Z,N) = Y_p^Z Y_n^N G(Z,N)(\rho N_A)^{A-1} \frac{A^{3/2}}{2^A} \left(\frac{2\pi\hbar^2}{m_u kT} \right)^{\frac{3}{2}(A-1)} \mathrm{e}^{B(Z,N)/kT}$$

with the nuclear binding energy B(Z,N)

Some features of this equation:

- in NSE there is a mix of free nucleons and nuclei
- higher density favors (heavier) nuclei
- higher temperature favors free nucleons (or lighter nuclei)
- nuclei with high binding energy are strongly favored

To solve for Y(Z,N) two additional constraints need to be taken into account:

Mass conservation $$\sum_i A_i Y_i = 1$$

Proton/Neutron Ratio $$\sum_i Z_i Y_i = Y_e$$

In general, weak interactions are much slower than strong interactions. Changes in Y_e can therefore be calculated from beta decays and electron captures on the NSE abundances for the current, given Y_e

In many cases weak interactions are so slow that $Y_{e\ i}$ is roughly fixed.

Sidebar – another view on NSE: Entropy

In Equilibrium the entropy has a maximum dS=0

- This is equivalent to our previous definition of equilibrium using chemical potentials:
  First law of thermodynamics:

$$dE = TdS + \rho N_A \sum_i \mu_i dY_i - pdV$$

so as long as dE=dV=0, we have in equilibrium (dS=0) :

$$\sum_i \mu_i dY_i = 0$$

for any reaction changing abundances by dY

For Zp+Nn --> (Z,N) this yields again

$$Z \cdot \mu_p + N \cdot \mu_n = \mu_{(\mathrm{Z,N})}$$

There are two ways for a system of nuclei to increase entropy:

1. Generate energy (more Photon states) by creating heavier, more bound nuclei
2. Increase number of free nucleons by destroying heavier nuclei

These are conflicting goals, one creating heavier nuclei around iron/nickel and the other one destroying them

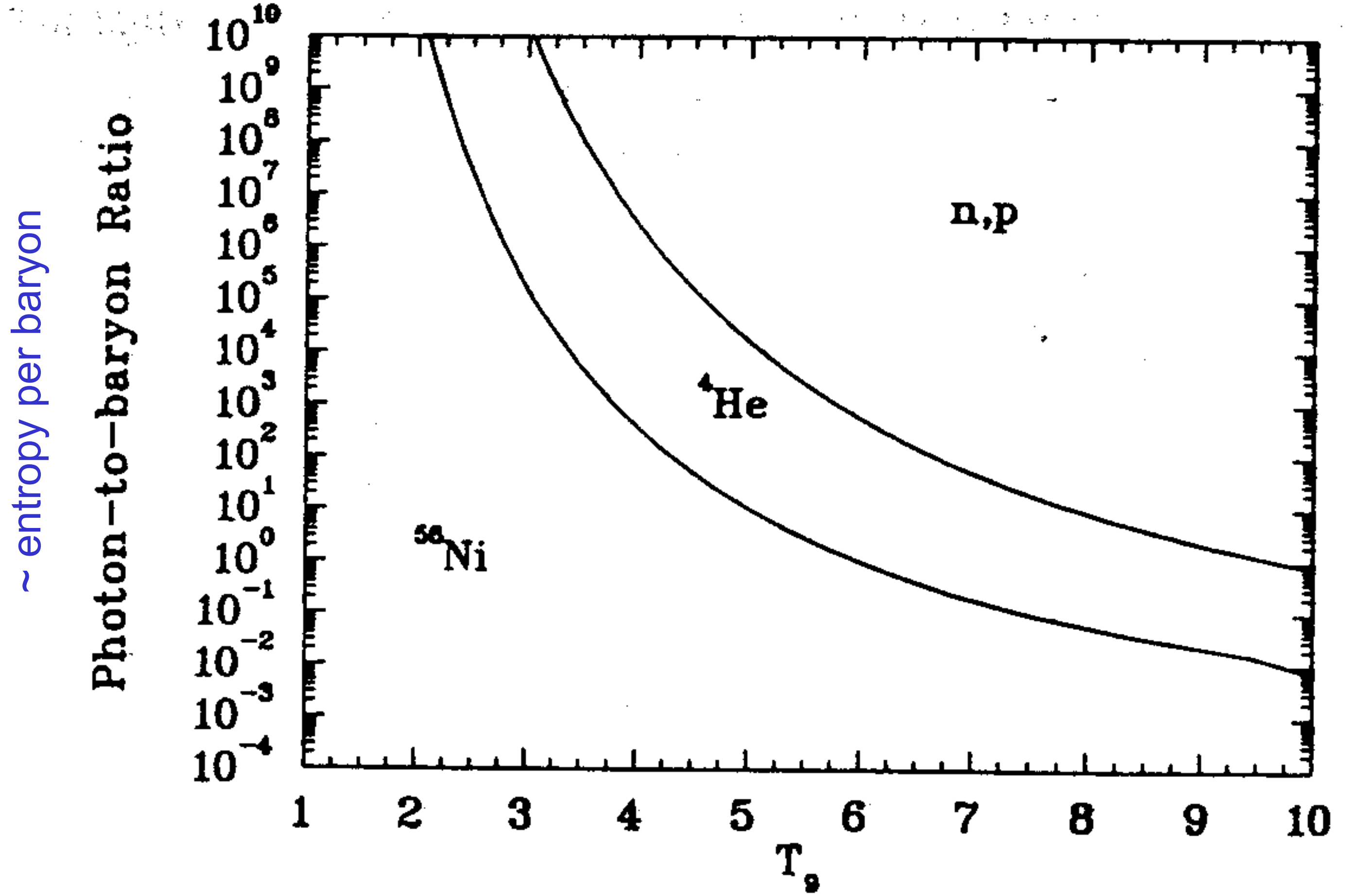
→ The system settles in a compromise with a mix of nucleons and most bound nuclei

→ for FIXED temperature:

high entropy per baryon (low ρ, high T) → more nucleons
low entropy per baryon (high ρ, low T) → more heavy nuclei

(entropy per baryon (if photons dominate): $\sim T^3/\rho$)

NSE composition (Y_e=0.5)



after Meyer, Phys Rep. 227 (1993) 257 “Entropy and nucleosynthesis”

Incomplete Equilibrium - Equilibrium Cluster

Often, some, but not all nuclei are in equilibrium with protons and neutrons (and with each other).

A group of nuclei in equilibrium is called an equilibrium cluster. Because of reactions involving single nucleons or alpha particles being the mediators of the equilibrium, neighboring nuclei tend to form equilibrium clusters, with cluster boundaries being at locations of exceptionally slow reactions.

This is referred as Quasi Statistical Equilibrium (or QSE)

Typical Example:

3α rate is slow $\longrightarrow$ α particles are not in full NSE

NSE during Silicon burning

- **Nuclei heavier than ^{24}Mg are in NSE**
- **High density environment favors heavy nuclei over free nucleons**
- **Y_e ~0.46 in core Si burning due to some electron captures**

→ **main product ^{56}Fe (26/56 ~ 0.46)**

formation of an iron core

(in explosive Si burning no time for weak interactions, Y_e~ 0.5 and therefore final product ^{56}Ni)

Summary stellar burning

TABLE 8.1 Evolutionary Stages of a 25 $M_\odot$ Star[a]

Stage	Time Scale	Temperature (T_9)	Density (g cm^{-3})
Hydrogen burning	7×10^6 y	0.06	5
Helium burning	5×10^5 y	0.23	7×10^2
Carbon burning	600 y	0.93	2×10^5
Neon burning	1 y	1.7	4×10^6
Oxygen burning	6 months	2.3	1×10^7
Silicon burning	1 d	4.1	3×10^7
Core collapse	seconds	8.1	3×10^9
Core bounce	milliseconds	34.8	$\simeq 3 \times 10^{14}$
Explosive burning	0.1–10 s	1.2–7.0	Varies

$>0.8M_0$ (Hydrogen burning – Helium burning)

$>8M_0$ (Hydrogen burning – Carbon burning)

$>12M_0$ (Hydrogen burning – Explosive burning)

Why do timescales get smaller ?

Note: Kelvin-Helmholtz timescale for red supergiant ~10,000 years, so for massive stars, no surface temperature - luminosity change for C-burning and beyond

Final composition of a 25 M_0 star: