PHY983 - Nuclear Astrophysics - Spring 2007
Homework set 2
Due: Friday, Feb 2, 2007, 10:20 am
Keywords: Nuclear masses, energy generation by nuclear reactions

1. Problem: [10pts]
   Using the Weizsaecker liquid drop mass formula,

   a. [5 pts] determine the nucleus with the maximum binding energy per nucleon assuming equal number of protons and neutrons (neglect the asymmetry term and pairing term).

   b. [5 pts] derive an expression for the Z corresponding to the most stable nucleus within an A=const chain of nuclei (neglect pairing). Calculate the most stable Z for A=16, 56 and A=208 and compare with experimental reality. Based on your result, give a simpler approximation for small A.

2. Problem: [13pts]
   The question of the energy source of the sun was a big mystery from about the middle of the 19th century when thermodynamics showed that things can't be hot forever without a heat source. This mystery persisted until 1926 when Sir Arthur Eddington proposed a nuclear energy source. Calculate the <u>lifetime</u> of the sun in years (assuming complete consumption of all the fuel) at its current luminosity of 3.826e33 erg/s for the following hypothetical "solar models":

   a. [2pts] The sun is powered by burning coal (assume the whole sun with its current mass is made of coal). Coal typically has an energy content of about 1e4 btu/lb. 1 btu corresponds to 1055 Joule.

   b. [3pts] The sun is powered by gravitational energy release due to continuous shrinking. This was proposed by Kelvin and Helmholtz in 1887. For an order of magnitude estimate, calculate the timescale for the sun to radiate away its entire gravitational energy at the current luminosity - this is called the Kelvin-Helmholtz timescale (you can assume the sun is a homogeneous sphere, and at that level of accuracy you can also neglect any numerical prefactors).

   c. [7pts] The sun is powered by fusion of hydrogen into He via the effective reaction (of course this proceeds in several steps)
      $4\ {}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu_e$
      Use the atomic mass table from the nuclear wallet cards
      http://www.nndc.bnl.gov/wallet/walletcontent.shtml.
      You can assume that each emitted positron is annihilates with a surrounding electron and that the energy generated by this processes generates extra heat (in other words, the annihilation gamma rays are thermalized). Give in addition the energy generated by the reaction in MeV/nucleon and in erg/g.

3. Problem: [2pts]
    a. [1pt] Based on your results from 2a,b,c which model makes most sense and why ? How do we know for sure ?
    b. [1pt] in 2c, does all the energy generated in the nuclear reaction contribute to the photon luminosity ? Explain.

4. Problem: [5pts]

Masses determine whether a nucleus is stable and which kind of decays are energetically possible. For ^{56}Co, calculate the Q-values for β^+ decay, β^- decay, electron capture, proton decay, neutron decay, and α decay using the masses given in the nuclear wallet cards (see problem 2c). Based on your results, is ^{56}Co stable, and if not, what is (are) the dominant decay mode(s) ? (justify).

What is the atomic mass excess Δ for ^{56}Co (in usual units) that you would obtain from the Weizsaecker Liquid Drop formula discussed in class (neglect electron binding energies)?

How does it compare with the measured value? Give the difference in absolute MeV/c^2 and as fraction of the total ^{56}Ni mass.

In your opinion, would this kind of accuracy be sufficient to calculate reasonable estimates for your decay Q-values above?

What is the order of magnitude of a useful precision for a mass value to be useful to determine decay Q-values and decay channels, as it is needed for example in nuclear astrophysics?