PHY983
HW8 Solutions

HW8

1.) a) $E_0 = 300$ keV (from literature)

$$E_{cm} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{m_\alpha m_{12C}}{m_\alpha + m_{12C}} \cdot v$$

$$E_{lab} = \frac{1}{2} m_\alpha v^2$$

$$\frac{E_{cm}}{E_{lab}} = \frac{12}{4+12} \rightarrow E_{lab} = E_{cm} \cdot \frac{16}{12} = 400 \text{ keV}$$

1.) b) $S = 165$ keVb

$$\sigma = 1/E \cdot e^{-b/\sqrt{E}} \cdot S \qquad b = 31.28 \cdot Z_1 Z_2 A^{1/2} \sqrt{\text{keV}}$$

Use center of mass energy E = 300 keV !

$$A = \frac{A_{12C} \cdot A_\alpha}{A_{12C} + A_\alpha} = 3$$

$$= 2.75 \cdot 10^{-17} \cdot 10^{-24} \text{ cm}^2 \qquad \hookrightarrow b = 650.142 \sqrt{\text{keV}}$$

event rate per second R

~~$R = \frac{I_{particle}}{\alpha}$~~

$$R = I_{particle} \cdot n_n \cdot d \cdot \sigma \cdot \varepsilon$$

particle current $I_{particle} = \frac{I_{electric}}{Q_\alpha \cdot e} = 3.12 \cdot 10^{15}$ pps

target number density $n_n = \frac{\rho \cdot N_A}{m_{mole}}$ $\quad \rho = 2.25\,g/cm^3 \rightarrow n_n = 1.13 \cdot 10^{23} \frac{1}{cm^3}$

target thickness $d = 10^{-4}$ cm

efficiency $\varepsilon = 0.5$

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↳ $R = 4.85 \cdot 10^{-7}\ 1/s$

total number of counts N after beamtime t

$N = R \cdot t \rightarrow t = \frac{N}{R} = 6.5$ years

2. Sequence of core collaps supernova:

1. Inner core begins to collapes when iron core exceeds Chandrasekhar mass limit (either by adding mass or by decreasing electron fraction due to weak interaction)
2. Collapse of inner core halts when about nuclear density is reached (it overshoots) leading to a bounce, that launches an outward going shock (pressure wave).
3. Outer core falls in while shock moves through it and dissociates iron into protons and neutrons. This takes energy out of the shock so that it stalls.
4. Some mechanism revives the shock so it continues to move outward and eventually initiates a reversal of the matter flow (mass cut).
5. From there on shock moves to surface and accelerates material behind it leading to an explosion.
6. Material within the mass cut falls back on collapsed core, which begins to form a proto neutron star and later a neutron star.

2.1. $U = -\frac{3}{5} G \frac{M^2}{R} = -3.10 \cdot 10^{59} \frac{1}{R}$ [cgs]

$r_i = 10^9 cm$

$r_f = 10^6 cm$

$\Delta E = U_i - U_f = -3.1 \cdot 10^{59} \left[\frac{1}{10^9} - \frac{1}{10^6} \right] \doteq 3 \cdot 10^{53}$ erg

2.2. $X_O = 0.5$

$X_C = 0.5$

$E = \sum_i Y_i \Delta_i - \sum_i Y_i' \Delta_i$ [MeV/u] with mass excess in MeV (Δ)

this gives energy in MeV/u

(Y_i: initial abundances; Y_i': final abundances)

$= \frac{0.5}{16}(-4.737) + \frac{0.5}{12} \cdot 0 - \frac{1}{56}(-53.9)$

$= 0.814$ MeV/u

$E_{total} = E$ [MeV/u] $\cdot M \cdot N_A \cdot \underbrace{10^{13} e}_{MeV \to erg} = 2 \cdot 10^{51}$ erg

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4) same equation as for SNIa

$E[MeV/u] = \sum_i Y_{i\,initial} \Delta_i - \sum_i Y_{i\,final} \Delta_i$

$\Delta(^{54}Fe) = -56.252$

$\Delta(^1H) = 7.289$

$\Delta(n) = 8.0713$

$E = -56.252 \cdot \frac{1}{54} - \left[\frac{26}{54} 7.289 + \frac{28}{54} 8.0713 \right]$

$= -8.73$ MeV/u

$E_{total} = 0.4 M_\odot \cdot N_A \cdot E \cdot e \cdot 10^{13} \simeq 6.77 \cdot 10^{51}$ erg

($M_\odot$ in g; E in MeV/u)