HR Diagram for nearby stars

- **Main Sequence**
  - H burning
  - pp/CNO
  - exhaust core H

- **Contraction**
  - Density/T inc.
  - H shell burning
  - Expansion

- **Core He burning**
- **He shell burning**

HR Diagram for nearby stars
http://www.atlasoftheuniverse.com/hr.html
What happens at hydrogen exhaustion (assume star had convective core)

1. Core contracts and heats

H, He mix

He rich core

2. Core He burning sets in

H shell burning

He rich core contracts and grows from H-burning

→ red giant

→ lower mass stars become bluer
low Z stars jump to the horizontal branch
2. a \((M < 2.25 \, M_0)\) Degenerate He core

- H shell burning ignites
- degenerate, not burning He core
- onset of electron degeneracy halts contraction

then He core grows by H-shell burning until He-burning sets in.

→ He burning is initially unstable (**He flash**)

in degenerate electron gas, pressure does not depend on temperature (why ?) therefore a slight rise in temperature is not compensated by expansion

→ thermonuclear runaway:

- rise temperature
- accelerate nuclear reactions
- increase energy production
Why does the star expand and become a red giant?

Because of higher Coulomb barrier He burning requires much higher temperatures
→ drastic change in central temperature
→ star has to readjust to a new configuration

Qualitative argument:
- need about the same Luminosity – similar temperature gradient $dT/dr$
- now much higher $T_c$ – need larger star for same $dT/dr$

Lower mass stars become red giants during shell H-burning

If the sun becomes a red giant in about 5 Bio years, it will almost fill the orbit of Mars
For completeness – here’s what’s happening in detail (5 solar mass ZAMS star):

Evolution depends on stellar mass (and Z)
He burning overview

- Lasts about 10% of H-burning phase
- Temperatures: \( \sim 300 \text{ Mio K} \)
- Densities \( \sim 10^4 \text{ g/cm}^3 \)

Reactions:

\[
\begin{align*}
4\text{He} + 4\text{He} + 4\text{He} & \rightarrow 12\text{C} \quad (\text{triple } \alpha \text{ process}) \\
12\text{C} + 4\text{He} & \rightarrow 16\text{O} \quad (12\text{C}(\alpha,\gamma))
\end{align*}
\]

Main products: carbon and oxygen (main source of these elements in the universe)
Helium burning 1 – the $3\alpha$ process

First step:

$$\alpha + \alpha \rightarrow ^8\text{Be}$$

unbound by ~92 keV – decays back to 2 $\alpha$ within $2.6 \times 10^{-16}$ s!

but small equilibrium abundance is established

Second step:

$$^8\text{Be} + \alpha \rightarrow ^{12}\text{C}^*$$

would create $^{12}\text{C}$ at excitation energy of ~7.7 MeV

1954 Fred Hoyle (now Sir Fred Hoyle) realized that the fact that there is carbon in the universe requires a resonance in $^{12}\text{C}$ at ~7.7 MeV excitation energy

1957 Cook, Fowler, Lauritsen and Lauritsen at Kellogg Radiation Laboratory at Caltech discovered a state with the correct properties (at 7.654 MeV)

$\rightarrow$ Experimental Nuclear Astrophysics was born
How did they do the experiment?

- Used a deuterium beam on a $^{11}$B target to produce $^{12}$B via a (d,p) reaction.
- $^{12}$B $\beta$-decays within 20 ms into the second excited state in $^{12}$C
- This state then immediately decays under alpha emission into $^{8}$Be
- Which immediately decays into 2 alpha particles

So they saw after the delay of the $\beta$-decay 3 alpha particles coming from their target after a few ms of irradiation

**This proved that the state can also be formed by the 3 alpha process …**

- removed the major roadblock for the theory that elements are made in stars
- Nobel Prize in Physics 1983 for Willy Fowler (alone !)
Third step completes the reaction:

\[ \alpha + \alpha \rightarrow ^8\text{Be} \]

\( E_R = |Q| \)

\( Q = -92 \text{ keV} \)

\( r = 6.8 \text{ eV} \)

\( ^8\text{Be} \)

Note: \(^8\text{Be}\) ground state is a 92 keV resonance for the \(\alpha + \alpha\) reaction

\( E_R (3\alpha) = 379 \text{ keV} \)

\( Q = 7275 \text{ keV} \)

\( ^8\text{Be} + \alpha \)

\( E_R = 287 \text{ keV} \)

\( E_X = 7654 \text{ keV} \)

\( ^{12}\text{C} (\alpha, \gamma) \)

\( J^\pi = 0^+ \)

\( \gamma \) decay of \(^{12}\text{C}\) into its ground state

Note: \( \Gamma_\alpha / \Gamma_\gamma > 10^3 \)
so \(\gamma\)-decay is very rare!
Helium burning 2 – the $^{12}\text{C}(\alpha,\gamma)$ rate

No resonance in Gamow window – C survives!

Resonance in Gamow window - C is made!

But some C is converted into O ...
some tails of resonances just make the reaction strong enough …

complications:
  • very low cross section makes direct measurement impossible
  • subthreshold resonances cannot be measured at resonance energy
  • Interference between the E1 and the E2 components
Therefore:

Uncertainty in the $^{12}\text{C}(\alpha,\gamma)$ rate is the single most important nuclear physics uncertainty in astrophysics

Affects:
- C/O ration $\rightarrow$ further stellar evolution (C-burning or O-burning ?)
- iron (and other) core sizes (outcome of SN explosion)
- Nucleosynthesis (see next slide)

More than 30 experiments in past 30 years …

Some current results for $S(300\text{ keV})$:

$S_{E_2}=53+13-18$ keV b (Tischhauser et al. PRL88(2002)2501
(recent new measurement by Hammer et al. 81+-22 keV b)

$S_{E_1}=79+21-21$ keV b (Azuma et al. PRC50 (1994) 1194)
(recent new measurement by Hammer et al. 77+-17 keV)
Evolution of the star that became Supernova 1987a
The Stellar Onion

http://cococubed.asu.edu/pix_pages/87a_art.shtml
Neon burning

**Burning conditions:**

- for stars > 12 M☉ (solar masses) (ZAMS)
  - T ~ 1.3-1.7 Bio K
  - ρ ~ 10^6 g/cm³

**Why would neon burn before oxygen??**

Answer:

Temperatures are sufficiently high to initiate **photodisintegration** of 20Ne

\[
\begin{align*}
20\text{Ne} + \gamma &\rightarrow 16\text{O} + \alpha \\
16\text{O} + \alpha &\rightarrow 20\text{Ne} + \gamma
\end{align*}
\]

This is followed by (using the liberated helium)

\[
20\text{Ne} + \alpha \rightarrow 24\text{Mg} + \gamma
\]

So net effect:

\[
2 \times 20\text{Ne} \rightarrow 16\text{O} + 24\text{Mg} + 4.59 \text{ MeV}
\]
Silicon burning

**Burning conditions:**

- $T \sim 3-4 \text{ Bio}$
- $\rho \sim 10^9 \text{ g/cm}^3$

**Reaction sequences:**

- Silicon burning is fundamentally different to all other burning stages.
- **Complex network of fast** $(\gamma, n)$, $(\gamma, p)$, $(\gamma, a)$, $(n, \gamma)$, $(p, \gamma)$, and $(a, \gamma)$ reactions
- The net effect of Si burning is: $2 \ ^{28}\text{Si} \rightarrow \ ^{56}\text{Ni}$,

**need new concept to describe burning:**

- Nuclear Statistical Equilibrium (NSE)
- Quasi Static Equilibrium (QSE)
Nuclear Statistical Equilibrium

**Definition:**

In **NSE**, each nucleus is in equilibrium with protons and neutrons.

Means: the reaction $Z \cdot p + N \cdot n \longleftrightarrow (Z,N)$ is in equilibrium.

Or more precisely:

$$Z \cdot \mu_p + N \cdot \mu_n = \mu_{(Z,N)}$$

for all nuclei $(Z,N)$.

**NSE** is established when both, photodisintegration rates of the type

- $(Z,N) + \gamma \longrightarrow (Z-1,N) + p$
- $(Z,N) + \gamma \longrightarrow (Z,N-1) + n$
- $(Z,N) + \gamma \longrightarrow (N-2,N-2) + \alpha$

and capture reactions of the types

- $(Z,N) + p \longrightarrow (Z+1,N)$
- $(Z,N) + n \longrightarrow (Z,N+1)$
- $(Z,N) + \alpha \longrightarrow (Z+2,N+2)$

are fast.
NSE is established on the timescale of these reaction rates (the slowest reaction).

A system will be in NSE if this timescale is shorter than the timescale for the temperature and density being sufficiently high.

For temperatures above ~5 GK even explosive events achieve full NSE.
Nuclear Abundances in NSE

The ratio of the nuclear abundances in NSE to the abundance of free protons and neutrons is entirely determined by

$$Z \cdot \mu_p + N \cdot \mu_n = \mu_{(Z,N)}$$

which only depends on the chemical potentials

$$\mu = mc^2 + kT \ln \left[ \frac{n}{g} \left( \frac{h^2}{2\pi mkT} \right)^{3/2} \right]$$

So all one needs are density, temperature, and for each nucleus mass and partition function (one does not need reaction rates!! - except for determining whether equilibrium is indeed established)
Solving the two equations on the previous page yields for the abundance ratio:

\[ Y(Z, N) = Y_p^Z Y_n^N G(Z, N)(\rho N_A)^{A-1} \frac{A^{3/2}}{2^A} \left( \frac{2\pi \hbar^2}{m_u kT} \right)^{3/2} (A-1) e^{B(Z,N)/kT} \]

with the nuclear binding energy \( B(Z,N) \)

Some features of this equation:

- in NSE there is a mix of free nucleons and nuclei
- higher density favors (heavier) nuclei
- higher temperature favors free nucleons (or lighter nuclei)
- nuclei with high binding energy are strongly favored but this effect is more pronounced for low temperature

For \( Z\sim Z', N\sim N' \):

\[ \frac{Y(Z, N)}{Y(Z', N')} \approx e^{\frac{B-B'}{kT}} \Rightarrow 1 \text{ for high } T \]

\( \Rightarrow \) For high temperature broad abundance distribution around maximum binding
To solve for $Y(Z,N)$ two additional constraints need to be taken into account:

**Mass conservation**

$$\sum_i A_i Y_i = 1$$

**Charge conservation**

$$\sum_i Z_i Y_i = Y_e$$  (related to proton/neutron ratio)

In general, weak interactions are much slower than strong interactions. Changes in $Y_e$ can therefore be calculated from beta decays and electron captures on the NSE abundances for the current, given $Y_e$.

In many cases weak interactions are so slow that $Y_{ei}$ is roughly fixed.
Sidebar – another view on NSE: Entropy

In Equilibrium the entropy has a maximum dS=0

• This is equivalent to our previous definition of equilibrium using chemical potentials:
  First law of thermodynamics:

  \[
  dE = TdS + \rho N_A \sum_i \mu_i dY_i - pdV
  \]

  so as long as \( dE = dV = 0 \), we have in equilibrium \( dS = 0 \):

  \[
  \sum_i \mu_i dY_i = 0
  \]

  for any reaction changing abundances by dY

For the reaction \( Zp + Nn \rightarrow (Z, N) \) this yields again

\[
Z \cdot \mu_p + N \cdot \mu_n = \mu_{(Z, N)}
\]
There are two ways for a system of nuclei to increase entropy:

1. Generate energy (more Photon states) by creating heavier, more bound nuclei
2. Increase number of free nucleons by destroying heavier nuclei

These are conflicting goals, one creating heavier nuclei around iron/nickel and the other one destroying them.

The system settles in a compromise with a mix of nucleons and most bound nuclei.

Tendency:

- high entropy per baryon (low $\rho$, high $T$) $\rightarrow$ more nucleons
- low entropy per baryon (high $\rho$, low $T$) $\rightarrow$ more heavy nuclei

(entropy per baryon (if photons dominate): $\sim T^3/\rho$)
NSE composition ($Y_e=0.5$)

Incomplete Equilibrium - Equilibrium Cluster

Often, some, but not all nuclei are in equilibrium with protons and neutrons (and with each other).

A group of nuclei in equilibrium is called an equilibrium cluster. Because of reactions involving single nucleons or alpha particles being the mediators of the equilibrium, neighboring nuclei tend to form equilibrium clusters, with cluster boundaries being at locations of exceptionally slow reactions.

This is referred as Quasi Static Equilibrium (or QSE)

Can think of this as “local” NSE

Typical Example:

$3\alpha$ rate is slow $\rightarrow$ $\alpha$ particles are not in full NSE

HW prob:

$^{14}\text{N}/^{12}\text{C}$ fixed due to QSE

absolute abundances increase slowly as $3\alpha \rightarrow ^{12}\text{C}$
NSE during Silicon burning

• Nuclei heavier than $^{24}\text{Mg}$ are in NSE
• High density environment favors heavy nuclei over free nucleons
• $Y_e \sim 0.46$ in core Si burning due to some electron captures

→ main product $^{56}\text{Fe}$ ($26/56 \sim 0.46$)

formation of an iron core

(in explosive Si burning no time for weak interactions, $Y_e \sim 0.5$ and therefore final product $^{56}\text{Ni}$)
Summary stellar burning

**Table 8.1 Evolutionary Stages of a 25 \( M_\odot \) Star**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time Scale</th>
<th>Temperature ( T_0 )</th>
<th>Density ( g \text{ cm}^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen burning</td>
<td>( 7 \times 10^6 \text{ y} )</td>
<td>0.06</td>
<td>5</td>
</tr>
<tr>
<td>Helium burning</td>
<td>( 5 \times 10^5 \text{ y} )</td>
<td>0.23</td>
<td>( 7 \times 10^2 )</td>
</tr>
<tr>
<td>Carbon burning</td>
<td>600 y</td>
<td>0.93</td>
<td>( 2 \times 10^5 )</td>
</tr>
<tr>
<td>Neon burning</td>
<td>1 y</td>
<td>1.7</td>
<td>( 4 \times 10^6 )</td>
</tr>
<tr>
<td>Oxygen burning</td>
<td>6 months</td>
<td>2.3</td>
<td>( 1 \times 10^7 )</td>
</tr>
<tr>
<td>Silicon burning</td>
<td>1 d</td>
<td>4.1</td>
<td>( 3 \times 10^7 )</td>
</tr>
<tr>
<td>Core collapse</td>
<td>seconds</td>
<td>8.1</td>
<td>( 3 \times 10^9 )</td>
</tr>
<tr>
<td>Core bounce</td>
<td>milliseconds</td>
<td>34.8</td>
<td>( \approx 3 \times 10^{14} )</td>
</tr>
<tr>
<td>Explosive burning</td>
<td>0.1–10 s</td>
<td>1.2–7.0</td>
<td>Varies</td>
</tr>
</tbody>
</table>

Why do timescales get smaller?

**Note:** Kelvin-Helmholtz timescale for red supergiant \( \sim 10,000 \text{ years} \), so for massive stars, no surface temperature - luminosity change for C-burning and beyond
Final composition of a 25 $M_0$ star:

- up to Si burned
- up to O burned
- up to Ne-burned
- up to He burned
- up to H-burned
- unburned
http://cococubed.asu.edu/pix_pages/87a_art.shtml