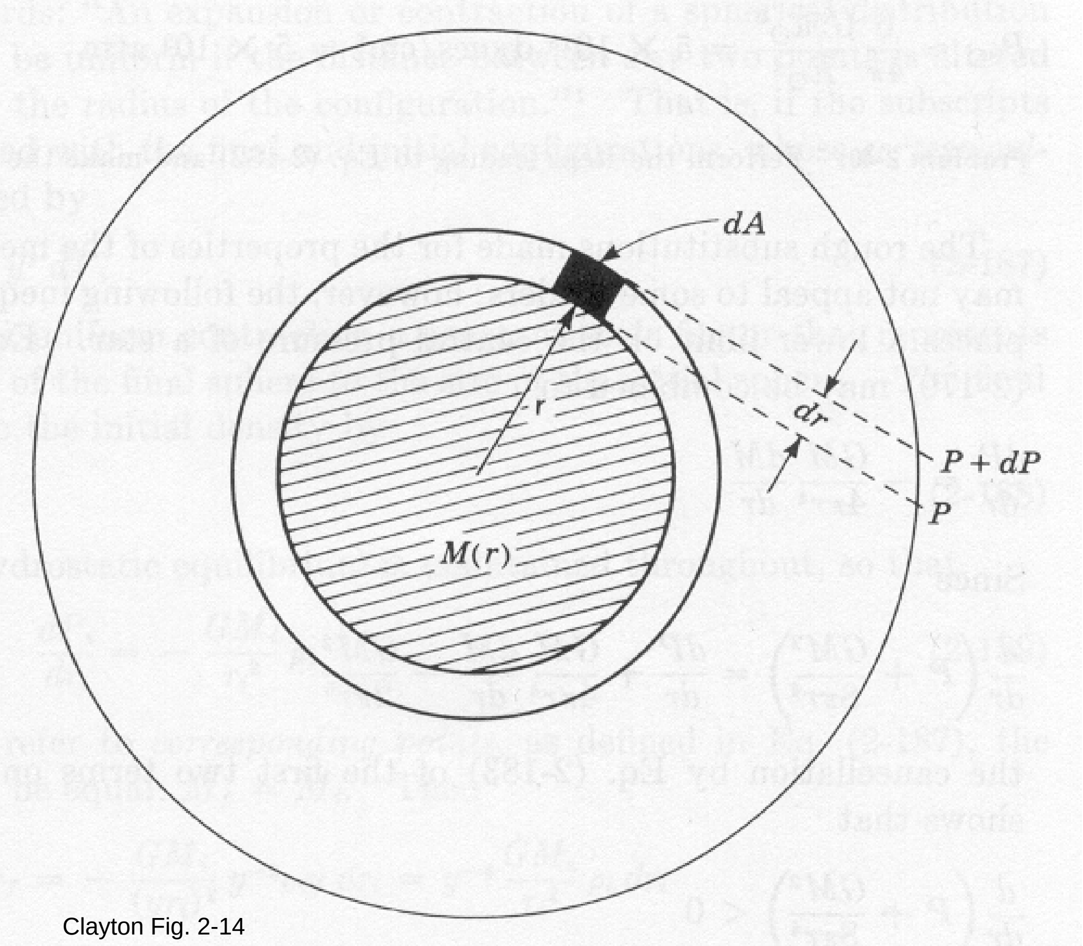
Properties of stars during hydrogen burning

Hydrogen burning is first major **hydrostatic burning phase** of a star:

Star is “stable” - radius and temperature everywhere do not change drastically with time

Hydrostatic equilibrium:

a fluid element is “held in place” by a pressure gradient that balances gravity



Clayton Fig. 2-14

Force from pressure:

$$F_p = PdA - (P + dP)dA$$
$$= -dPdA$$

Force from gravity:

$$F_G = -GM(r)\rho(r)\,dAdr / r^2$$

For balance: $F_G = F_P$ need:

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$

The origin of pressure: equation of state

Under the simplest assumption of an ideal gas: $P = \rho N_A RT / \mu_I$

→ need high temperature !

Keeping the star hot:

The star cools at the surface - energy loss is luminosity L

To keep the temperature constant everywhere luminosity must be generated

In general, for the luminosity of a spherical shell at radius r in the star:
(assuming steady state dS/dt = 0)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon$$ (energy equation)

where ε is the energy generation rate (sum of all energy sources and losses) per g and s

Luminosity is generated in the center region of the star (L(r) rises) by nuclear reactions and then transported to the surface (L(r)=const)

Energy transport to the surface - cooling:

The star will settle into a hydrostatic and thermal equilibrium, where cooling is balanced by nuclear energy generation and there is no time dependence of any state variables.

The generated heat will then exactly match the outgoing energy flow (luminosity) at any point in the star.

Heat flows from hot to cold

→ temperature gradient is required to carry the luminosity outward:

Therefore T(r) and P(r) drop towards the surface → ρ(r) also drops

Possible mechanisms of heat transport:

1. Conduction (not important at low densities in normal stars)
2. Radiative diffusion
3. Convection

Radiative energy transport:

Effectiveness depends on opacity κ :
unit cm^2/g – could call it specific cross section,
for example luminosity L in a layer r gets attenuated by photon absorption with a cross section σ:

$$L = L_0 e^{-\sigma n r} = L_0 e^{-\kappa \rho r}$$

Photon mean free path l: $$l = \frac{1}{\kappa\rho}$$ (about 1cm in the sun)

Required temperature gradient:

Luminosity per cm^2

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4acT^3}\frac{L(r)}{4\pi r^2}$$

a: radiation density constant
=7.56591e-15 $erg/cm^3/K^4$

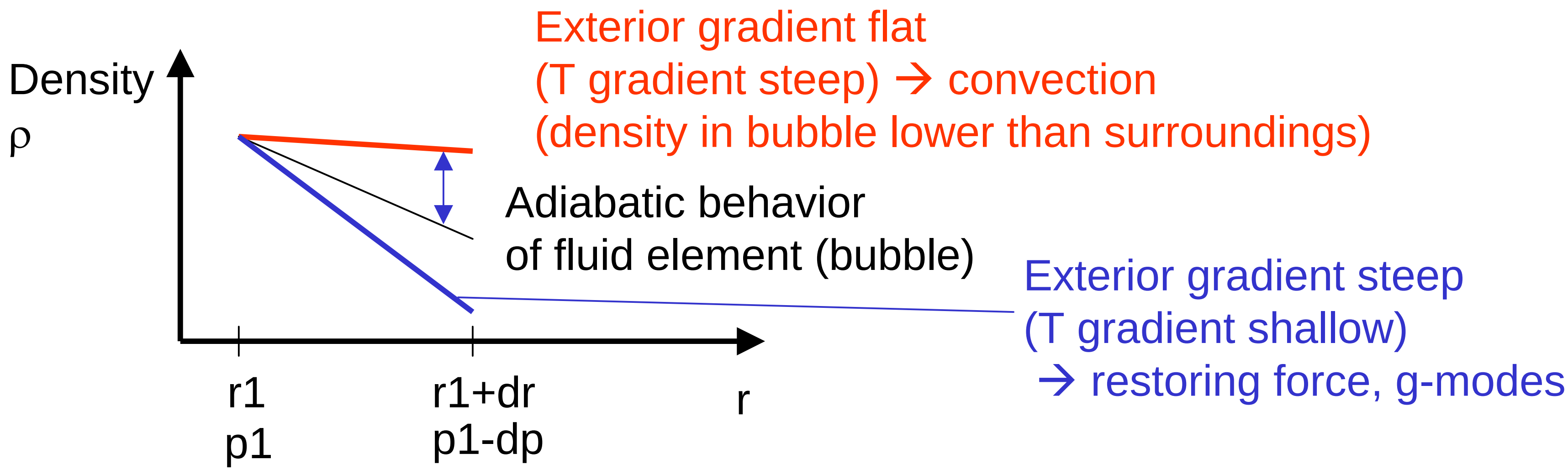
→ Large gradients needed for

- large luminosity at small r (large L/cm^2)
- large opacity

Convective energy transport:

takes over when necessary temperature gradient is too steep
hot gas moves up, cool gas moves down, within convective zone
fluid elements move adiabatically (adiabatic temperature gradient) driven by temperature dependent bouyancy

Motivational consideration:

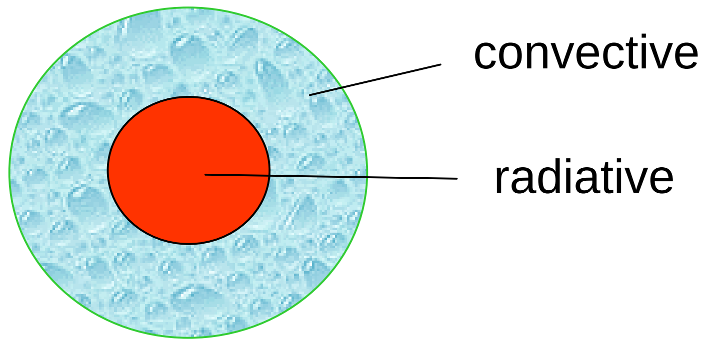


A more rapid drop in T over dr leads to a comparably lower T and higher density at same pressure p1-dp (for example ideal gas) → flatter density gradient

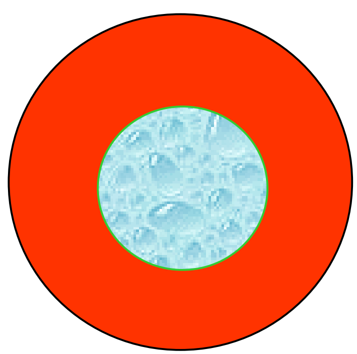
Convection occurs for flat density gradients, steep temperature gradients

Convection also mixes abundances → a convection zone has uniform composition (as long as convection timescale << nuclear reaction timescale)

Stars with $M<1.2\ M_0$ have radiative core and convective outer layer (like the sun):



Stars with $M>1.2\ M_0$ have convective core and radiative outer layer:



(convective core about 50% of mass for $15M_0$ star)

The structure of a star in hydrostatic equilibrium:

Equations so far:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$ (hydrostatic equilibrium)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho\varepsilon$$ (energy)

$$L(r) = -4\pi r^2 \frac{4acT^3}{3\kappa\rho}\frac{dT}{dr}$$ (radiative energy transfer)

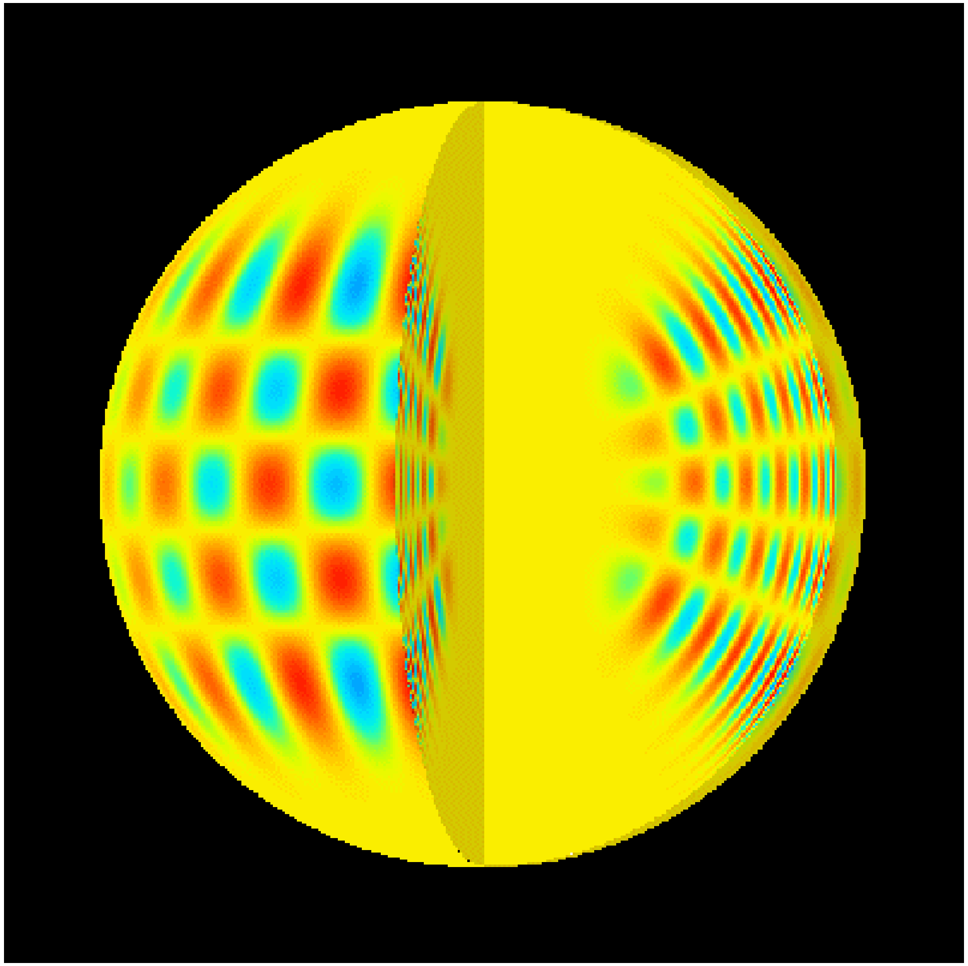
In addition of course: $$\frac{dM(r)}{dr} = 4\pi\rho r^2$$ (mass)

and an equation of state

BUT: solution not trivial, especially as ε,κ in general depend strongly on composition, temperature, and density

Example: The sun

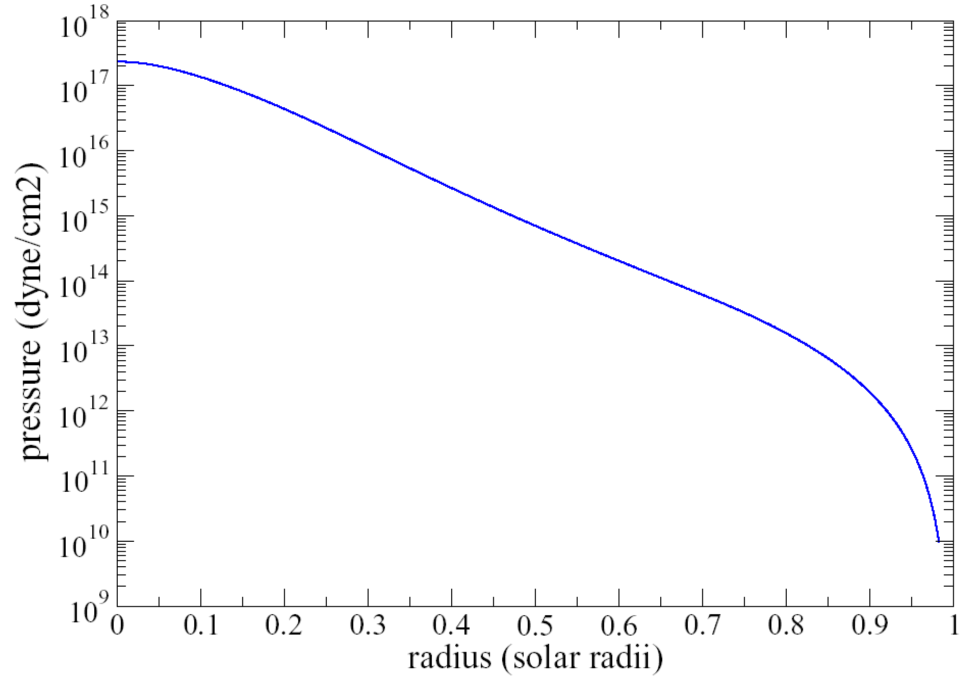
But - thanks to **helioseismology** one does not have to rely on theoretical calculations, but can directly measure the internal structure of the sun

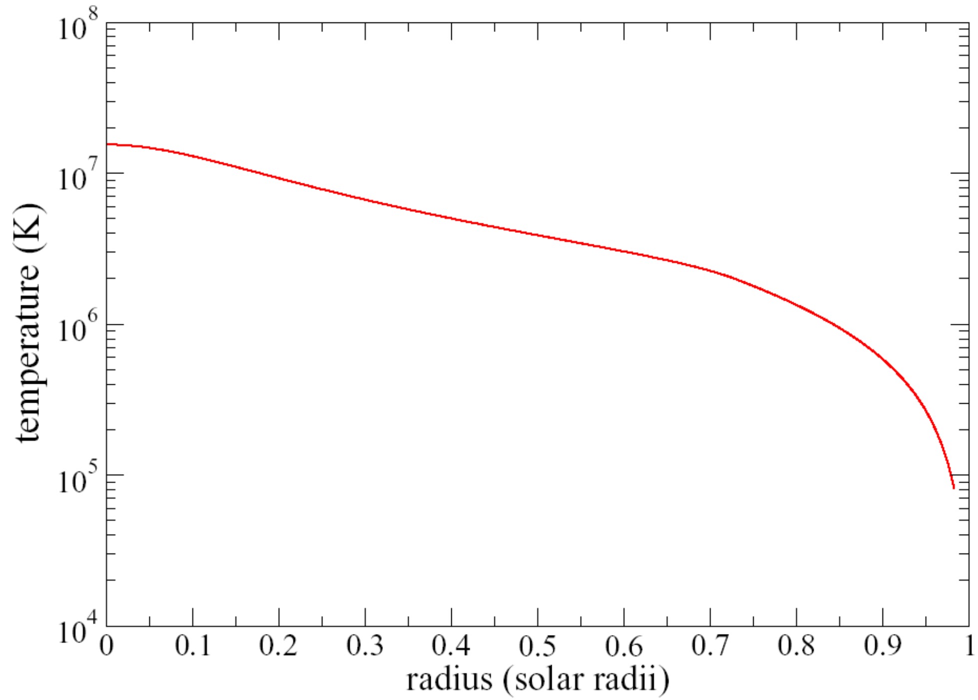


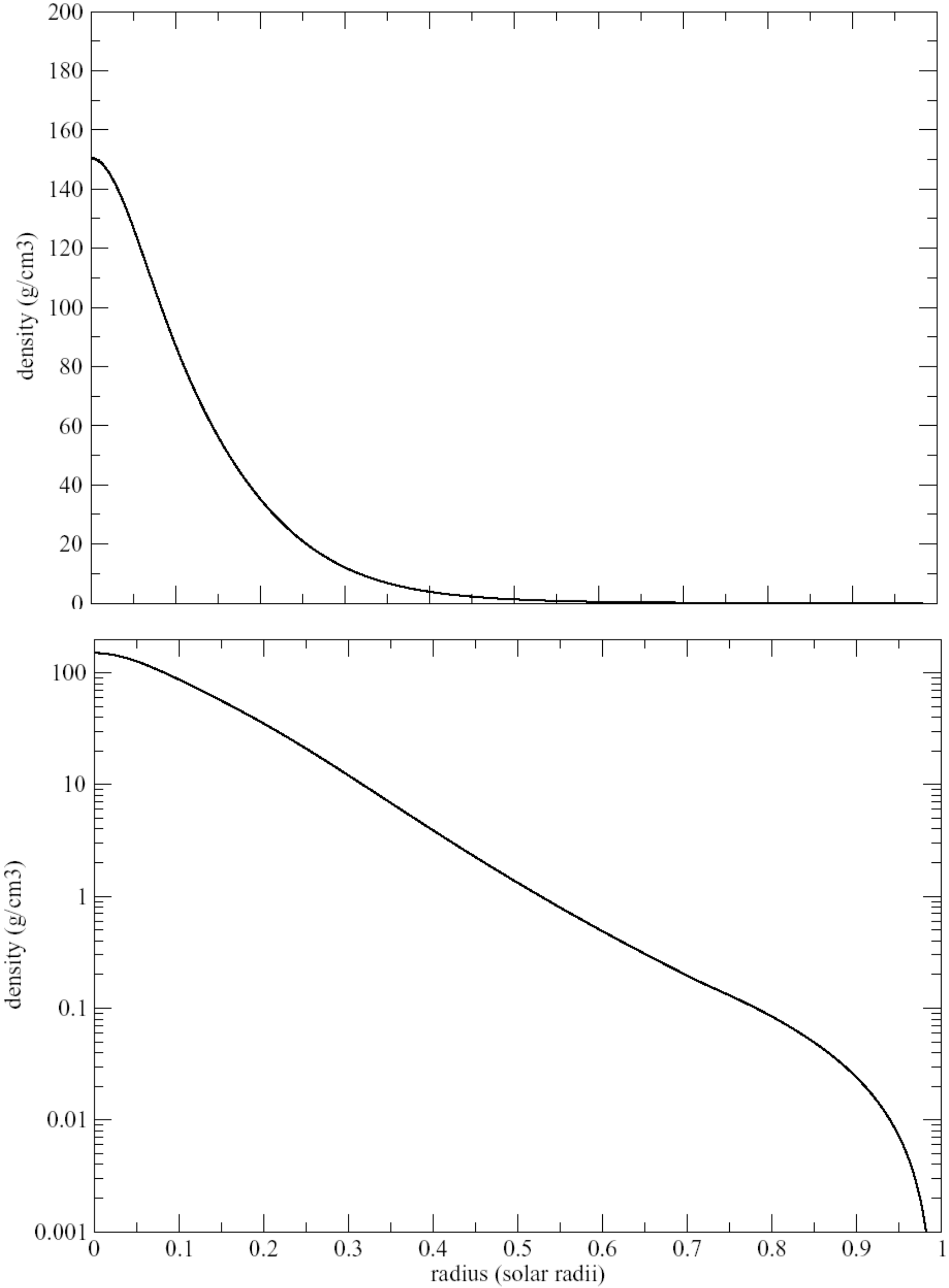
oscillations with periods of 1-20 minutes

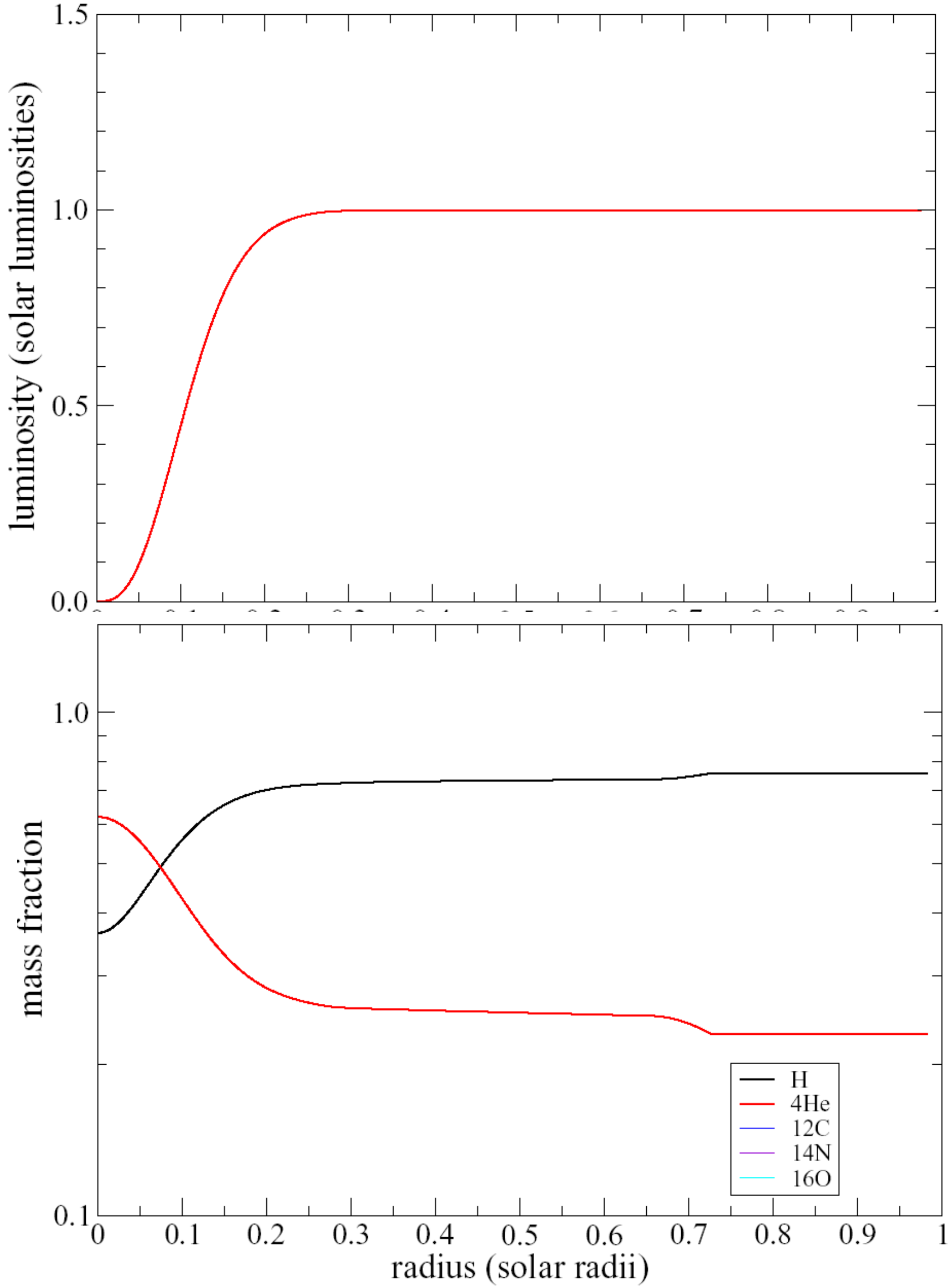
max 0.1 m/s

Conditions in the sun
(J. Bahcall, BS05
standard solar model)

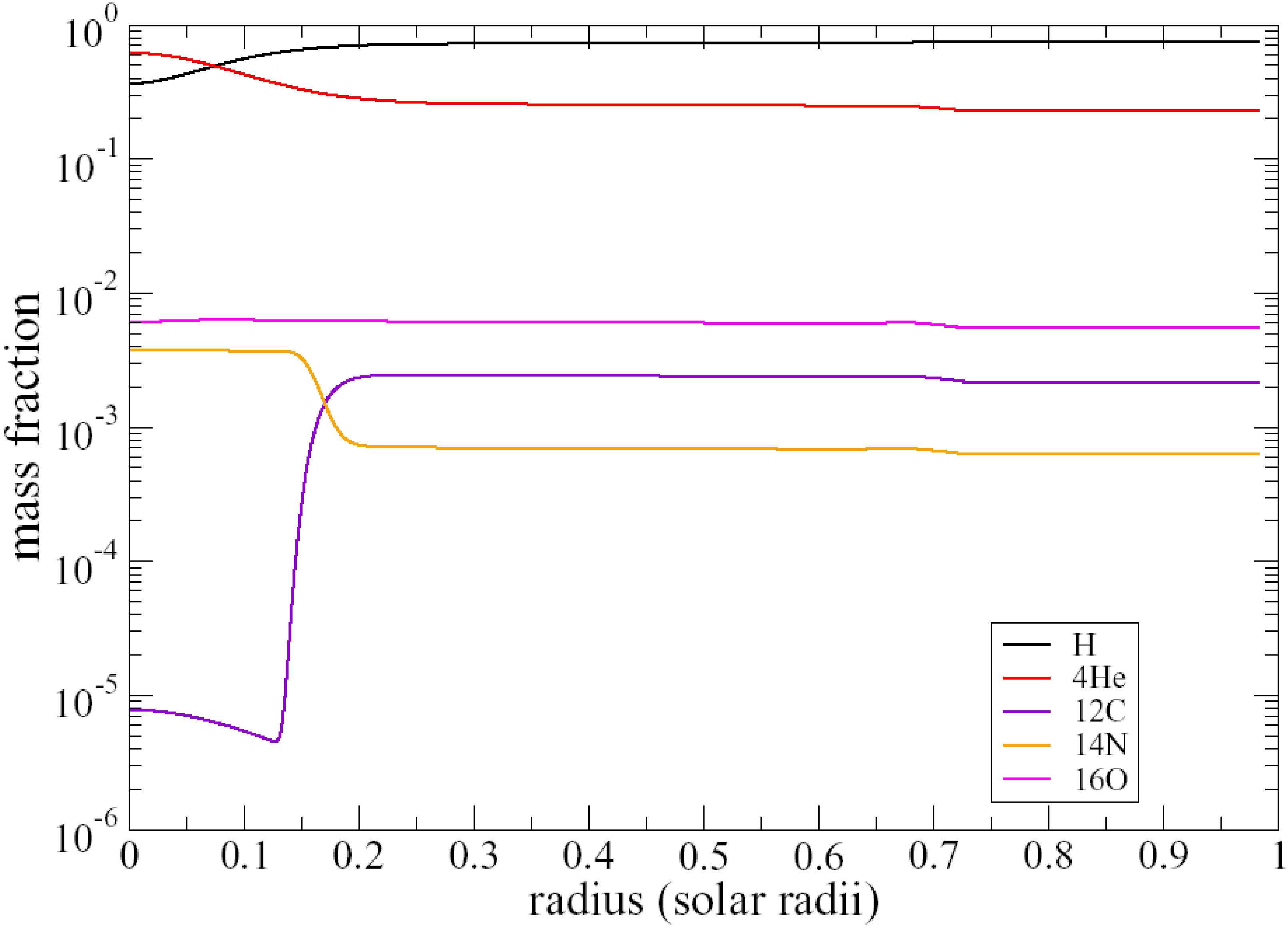





density (g/cm3)
radius (solar radii)
0
20
40
60
80
100
120
140
160
180
200
0.001
0.01
0.1
1
10
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

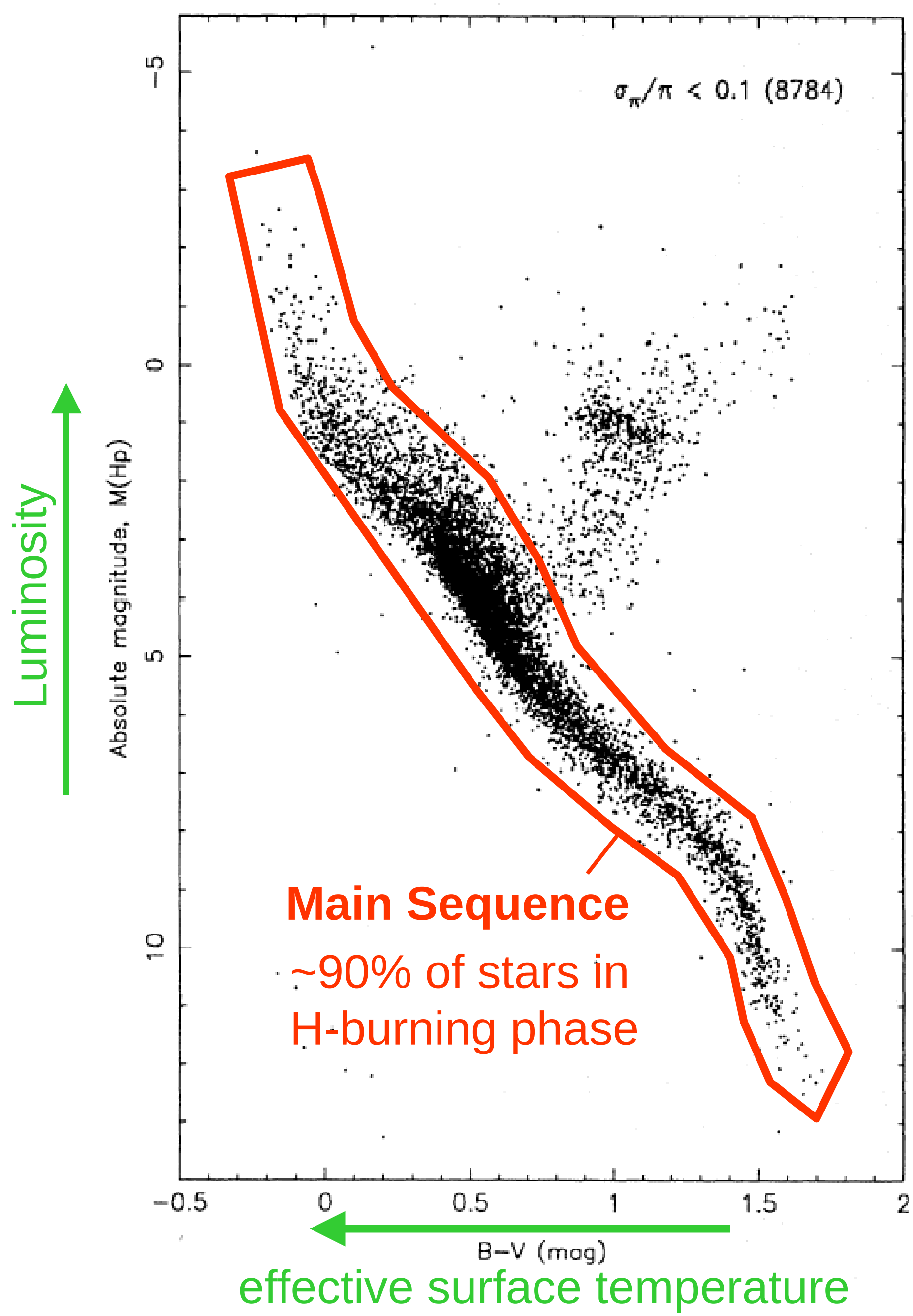


luminosity (solar luminosities)
1.5
1.0
0.5
0.0
mass fraction
1.0
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
radius (solar radii)
H
4He
12C
14N
16O



mass fraction
radius (solar radii)
H
4He
12C
14N
16O

Hertzsprung-Russell diagram



Perryman et al. A&A 304 (1995) 69

HIPPARCOS distance measurements

Magnitude:

Measure of stars brightness

Def: difference in magnitudes m from ratio of brightnesses b:

$$m_2 - m_1 = 2.5 \log \frac{b_1}{b_2}$$

(star that is x100 **brighter** has by 5 **lower** magnitude)

absolute scale for apparent magnitude historically defined
(Sirius: -1.5, Sun: -26.72
naked eye easy: <0, limit: <4)

absolute magnitude is measure of luminosity = magnitude that star would have at 10 pc distance

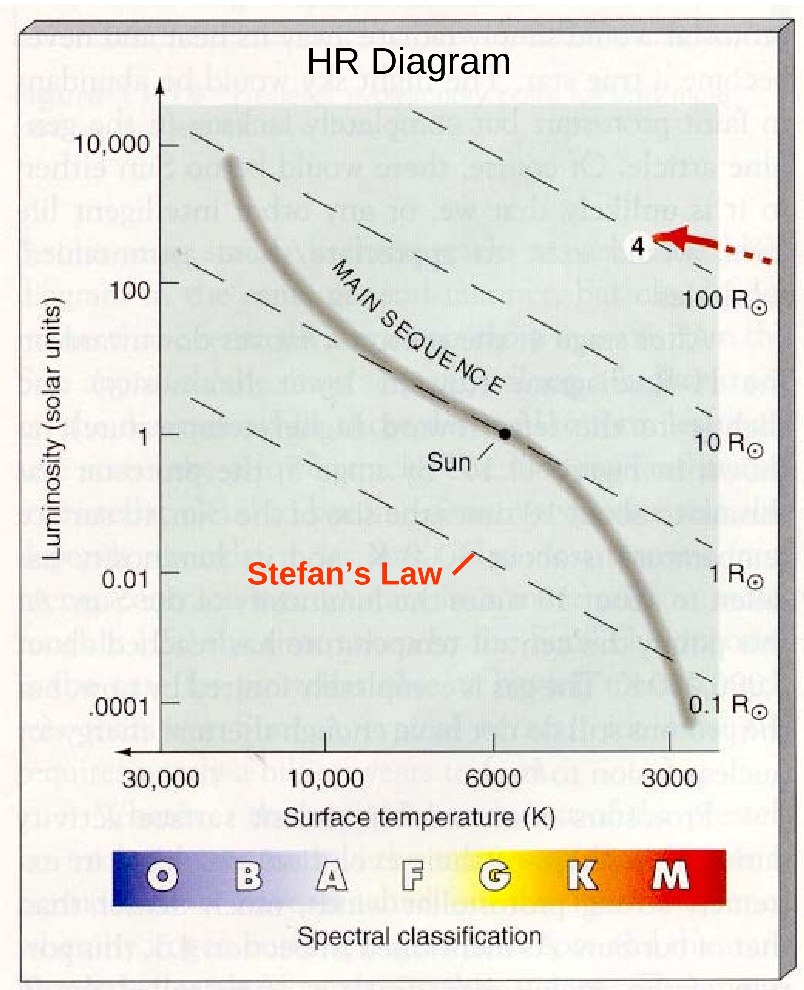
Sun: + 4.83 (visual magnitude)
+ 4.75 (bolometric magnitude)

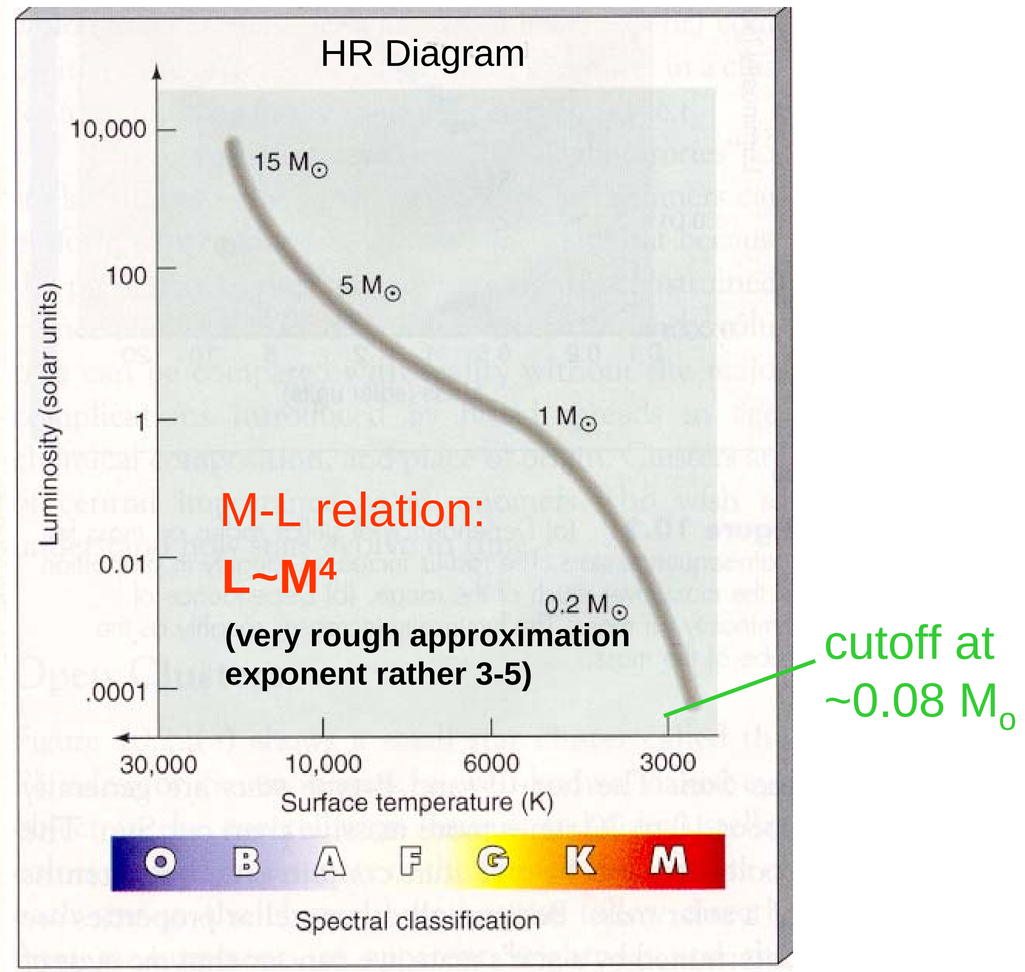
Temperature,Luminosity, Mass relation during core H-burning:

It turns out that as a function of mass there is a rather unique relationship between

- surface temperature (can be measured from contineous spectrum)
- luminosity (can be measured if distance is known)

(recall Stefan's Law $L \sim R^2\, T^4$)





(from Chaisson McMillan)

Mass – Radius relation:

In solar units: $R \sim M^{0.8}$ (10 M_sol: 6.3 x R_sol, 100 M_sol: 40 x R_sol)
(really exponent is ~0.8 for M<M_sol, 0.57 for M>M_sol)

Main Sequence evolution:

Main sequence lifetime:

H Fuel reservoir F~M
Luminosity $L \sim M^4$ $\longrightarrow$ lifetime $\tau_{\mathrm{MS}} = \frac{F}{L} \propto M^{-3}$

Recall from Homework: H-burning lifetime of sun ~ 10^{10} years

$$\boxed{\tau_{\mathrm{MS}} \approx \left(\frac{M}{M_{\oplus}}\right)^{-3} 10^{10} \text{ years}}$$

note: very approximate
exponent is really
between 2 and 3

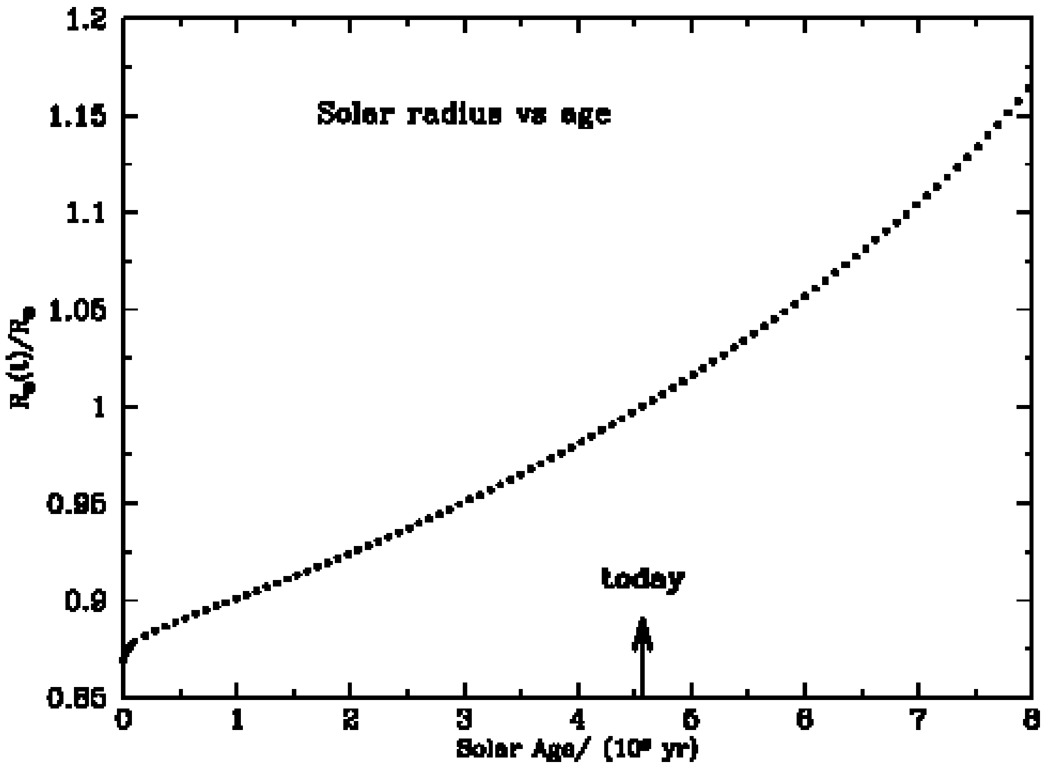
so a 10 solar mass star lives only for 10-100 Mio years
a 100 solar mass star only for 10-100 thousand years !

Changes during Main Sequence evolution:

With the growing He abundance in the center of the star slight changes occur (star gets somewhat cooler and bigger) and the stars moves in the HR diagram slightly

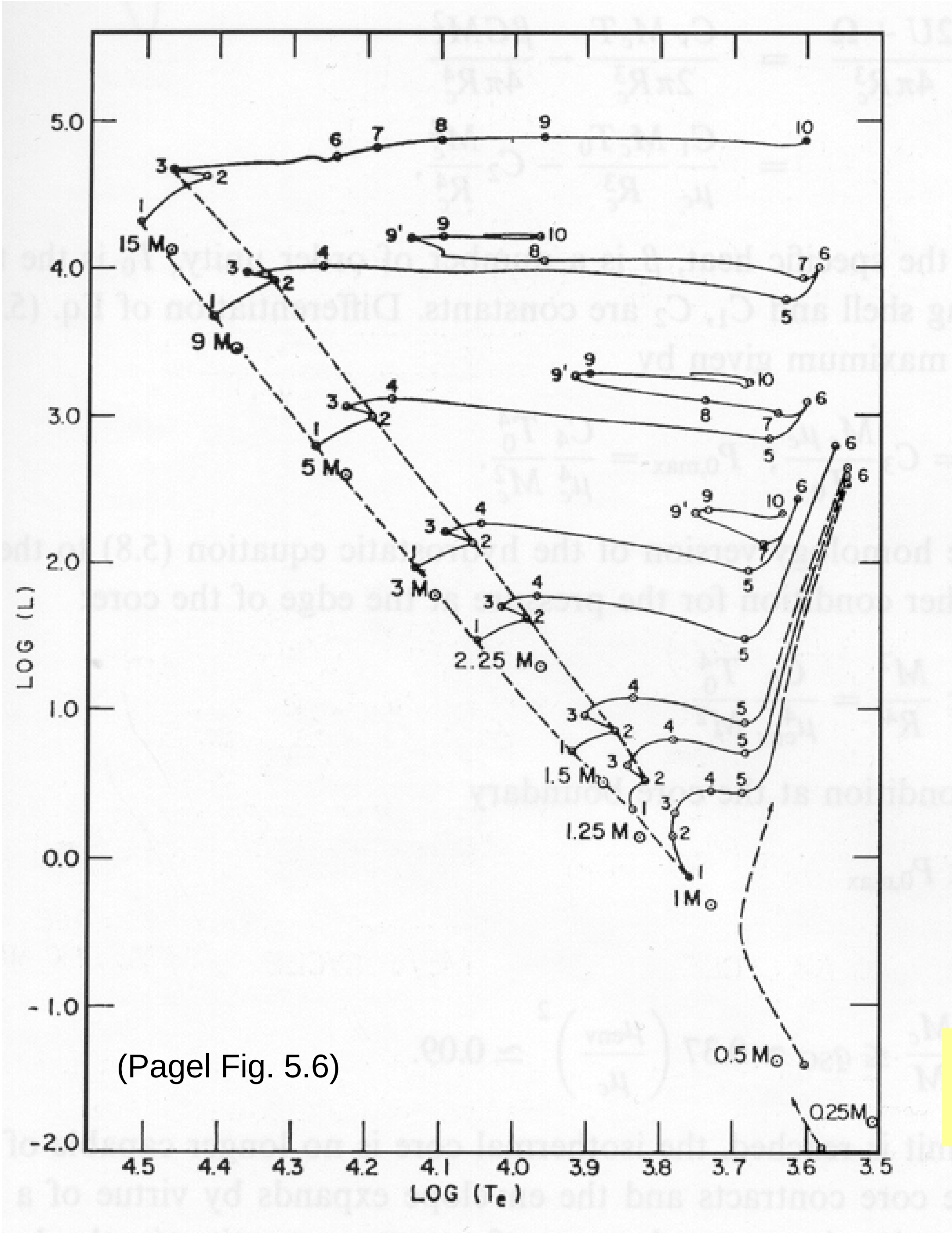
→ main sequence is a band with a certain width

For example, predicted radius change of the sun according to Bahcall et al. ApJ555(2001)990



Zero Age Main Sequence (ZAMS): “1”

End of Main Sequence: “2”



(Pagel Fig. 5.6)

Stellar masses are usually given in ZAMS mass !