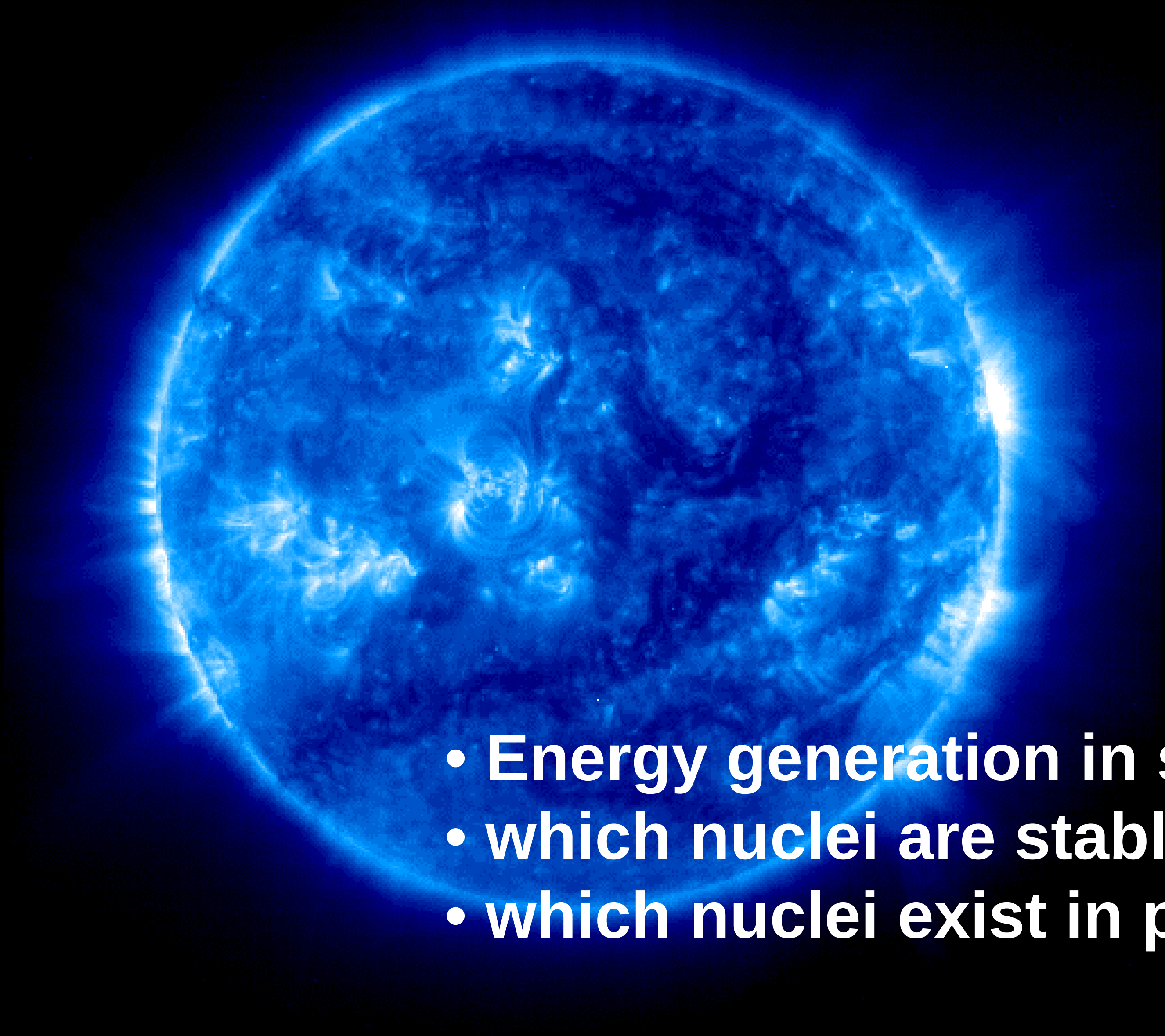
The mass of a nucleus



- **Energy generation in stars**
- **which nuclei are stable**
- **which nuclei exist in principle**

SOHO, 171A Fe emission line

Nucleons

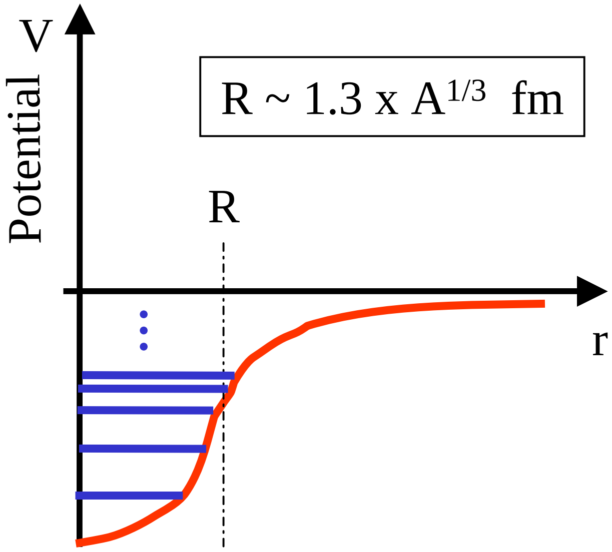
	Mass	Spin	Charge
Proton	938.272 MeV/c^2	1/2	+1e
Neutron	939.565 MeV/c^2	1/2	0

size: ~1 fm

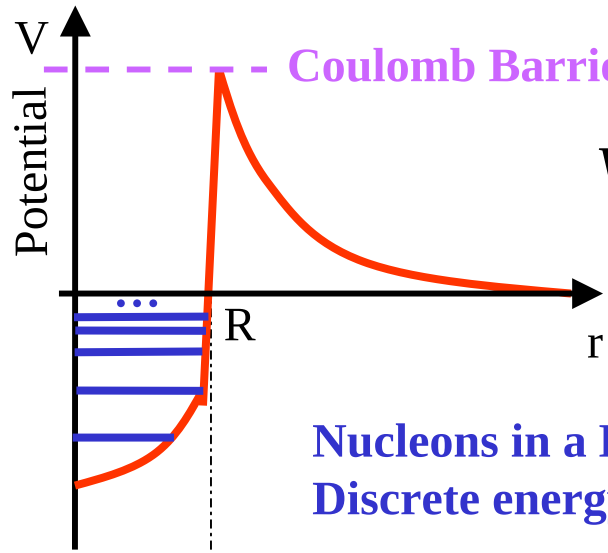
Nuclei

nucleons attract each other via the strong force (range ~ 1 fm)
a bunch of nucleons bound together create a potential for an additional :

neutron



proton
(or any other charged particle)



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

Nucleons in a Box:
Discrete energy levels in nucleus

→ Nucleons are bound by attractive force. Therefore, mass of nucleus is smaller than the total mass of the nucleons by the binding energy dm=B/c^2

Nuclear Masses and Binding Energy

Energy that is released when a nucleus is assembled from neutrons and protons

$$m(Z,N) = Zm_p + Nm_n - B/c^2$$

m_p = proton mass, m_n = neutron mass, m(Z,N) = mass of nucleus with Z,N

- B>0
- With B the mass of the nucleus is determined.
- B is very roughly ~A

Masses are usually tabulated as **atomic masses**

$$m = m_{nuc} + Z\,m_e - B_e$$

Nuclear Mass
~ 1 GeV/A

Electron Mass
511 keV/Z

Electron Binding Energy
13.6 eV (H)
to 116 keV (K-shell U) / Z

Most tables give atomic mass excess Δ in MeV: $m = Am_u + \Delta / c^2$

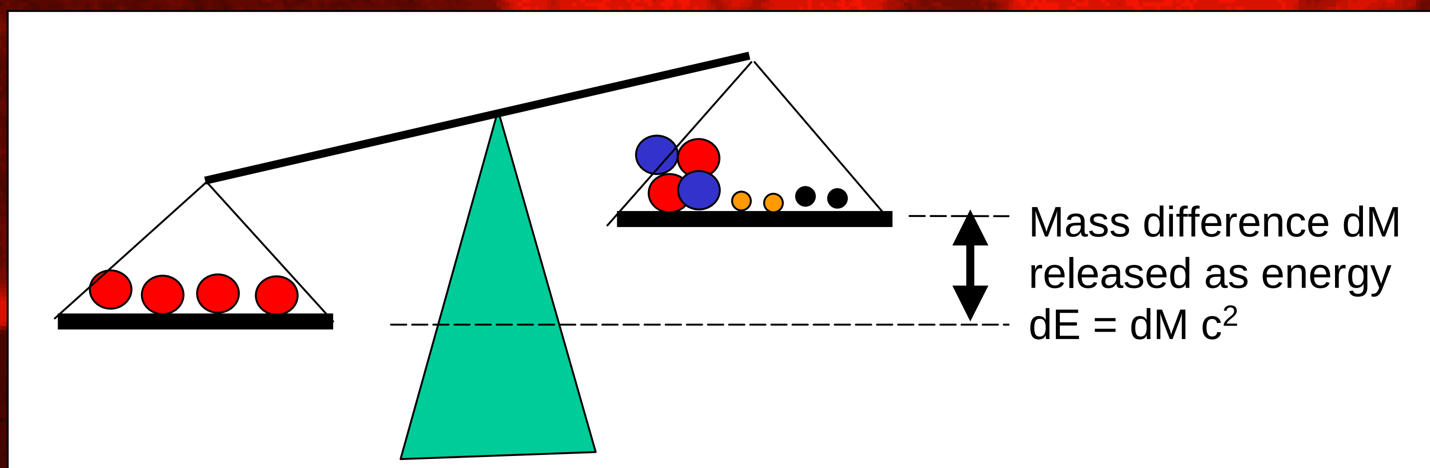
(so for ^{12}C: Δ=0) (see nuclear wallet cards for a table)

Q-value

Energy released in a nuclear reaction (>0 if energy is released, <0 if energy is use

Example: The sun is powered by the fusion of hydrogen into helium:

$$4p \rightarrow {}^{4}He + 2\,e^{+} + 2\nu_e$$



$$Q/c^2 = 4m_{nuc\ p} - m_{nuc\ 4He} - 2m_e - 2m_\nu$$

(using nuclear masses !)

In practice one often uses mass excess Δ and atomic masses.

Q-value with mass excess Δ

As A is always conserved in nuclear reactions the mass excess Δ can always be used instead of the masses (the Am_u term cancels)

(as nucleon masses cancel on both sides, its really the binding energies that entirely determine the Q-values !)

Q-value with atomic masses:

If Z is conserved (no weak interaction) atomic masses can be used instead of nuclear masses (Zme and most of the electron binding energy cancels)

Otherwise: For each positron emitted subtract $2m_e/c^2$= 1.022 MeV from the Q-value

Example: $4p \rightarrow {}^4He + 2\,e^+ + 2\nu_e$ Z changes an 2 positrons are emitted

$$Q/c^2 = 4m_H - m_{He} - 4m_e$$ With atomic masses

$$Q/c^2 = 4\Delta_H - \Delta_{He} - 4m_e$$ With atomic mass excess

The liquid drop mass model for the binding energy: (Weizaecker Formula)
(assumes incompressible fluid (volume ~ A) and sharp surface)

$$B(Z,A) = a_V A$$

Volume Term (each nucleon gets bound by about same energy)

$$- a_s A^{2/3}$$

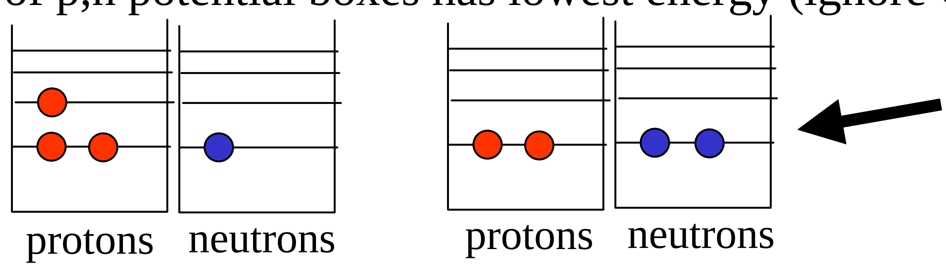
Surface Term ~ surface area (Surface nucleons less bound)

$$- a_C \frac{Z^2}{A^{1/3}}$$

Coulomb term. Coulomb repulsion leads to reduction
uniformly charged sphere has E=3/5 Q^2/R

$$- a_A \frac{(Z - A/2)^2}{A}$$

Asymmetry term: Pauli principle to protons: symmetric filling of p,n potential boxes has lowest energy (ignore Coulomb)



lower total energy = more bound

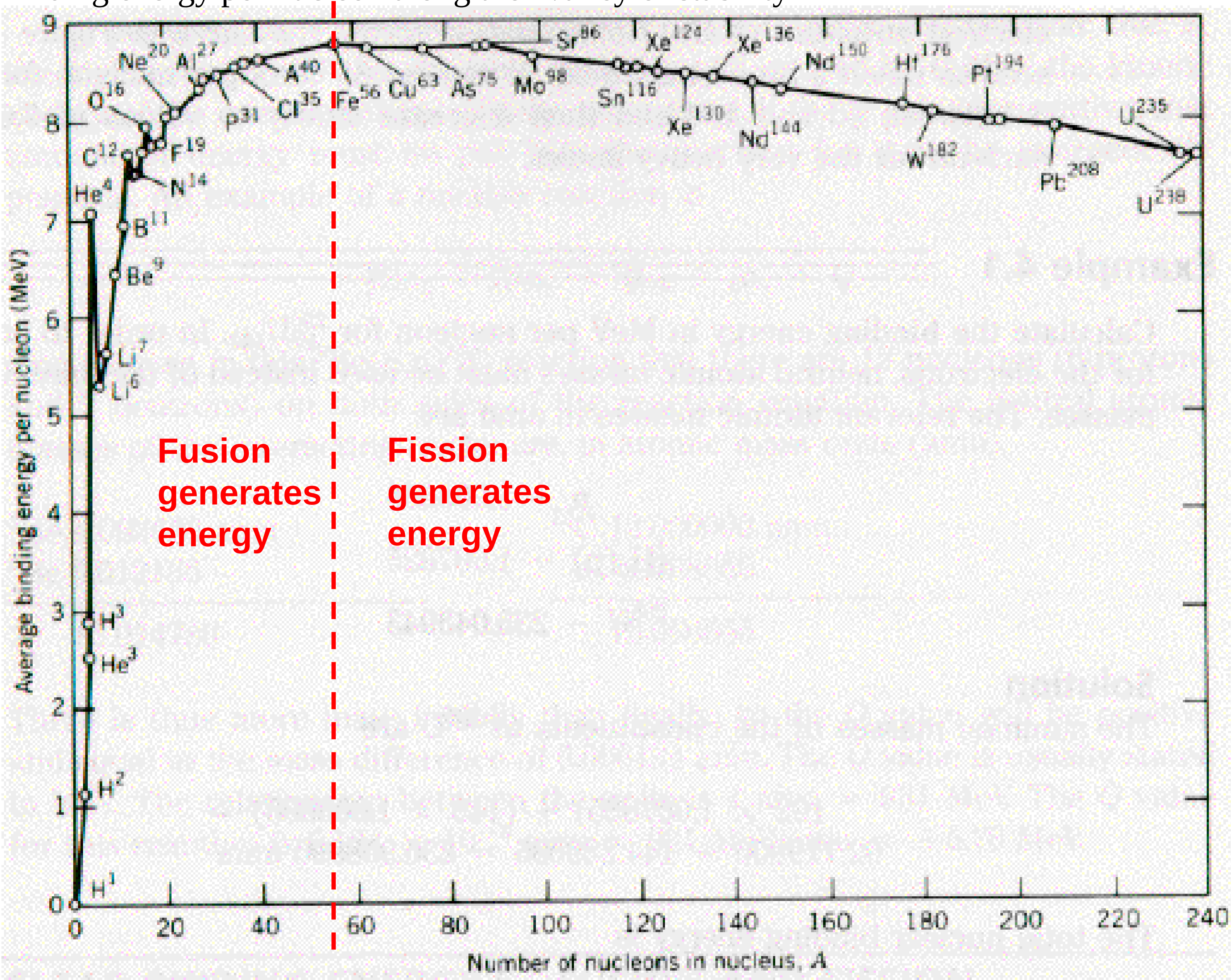
and in addition: p-n more bound than p-p or n-n (S=1,T=0 more bound than S=0,T=1)

$$+ a_p A^{-1/2} \begin{cases} \text{x } 1 & \text{ee} \\ \text{x } 0 & \text{oe/eo} \\ \text{x } (-1) & \text{oo} \end{cases}$$

Pairing term: even number of like nucleons favoured

(e=even, o=odd referring to Z, N respectively)

Binding energy per nucleon along the “valley of stability”

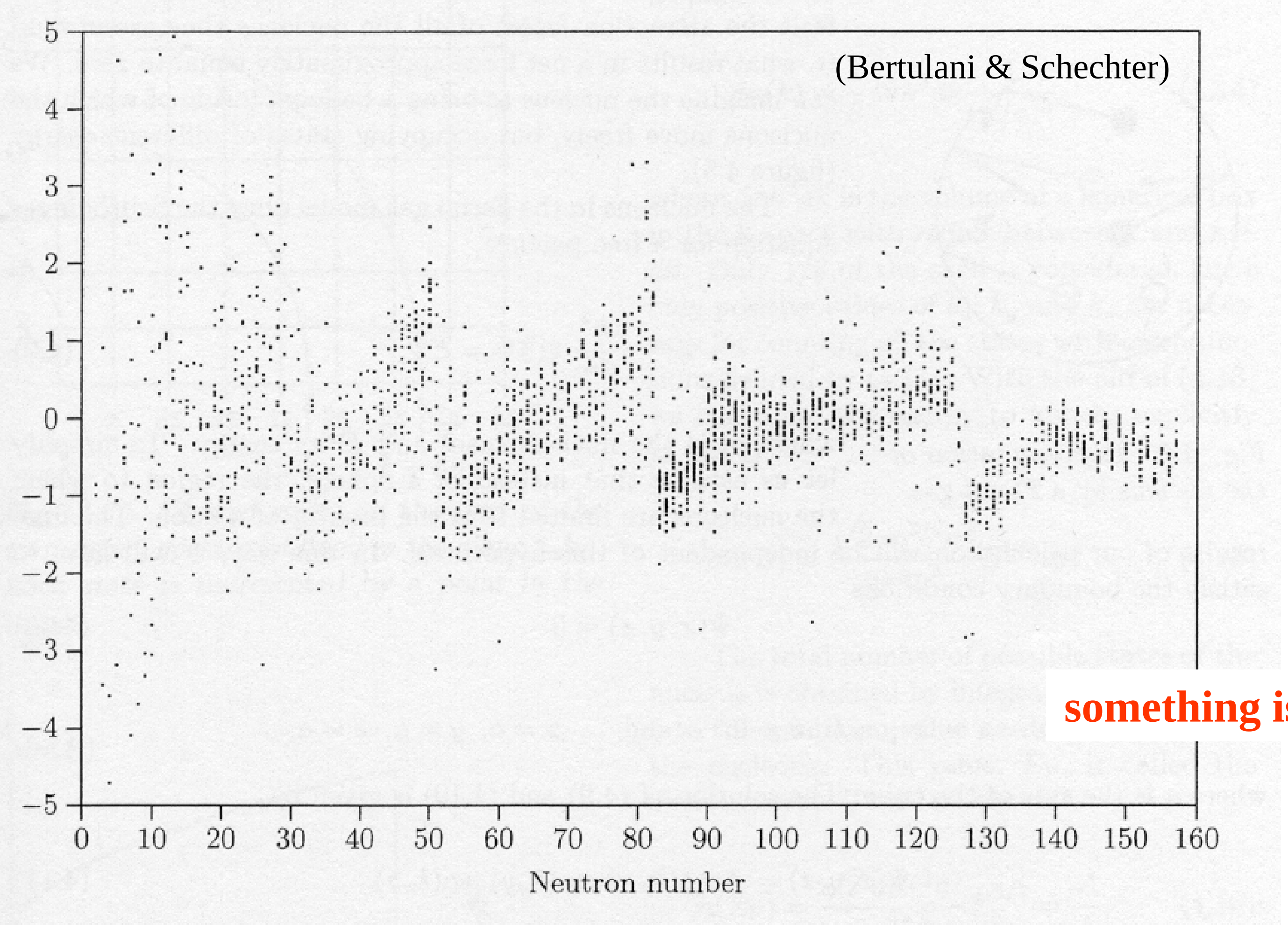


Best fit values (from A.H. Wapstra, Handbuch der Physik 38 (1958) 1)

in MeV/c^2

a_V	a_S	a_C	a_A	a_P
15.85	18.34	0.71	92.86	11.46

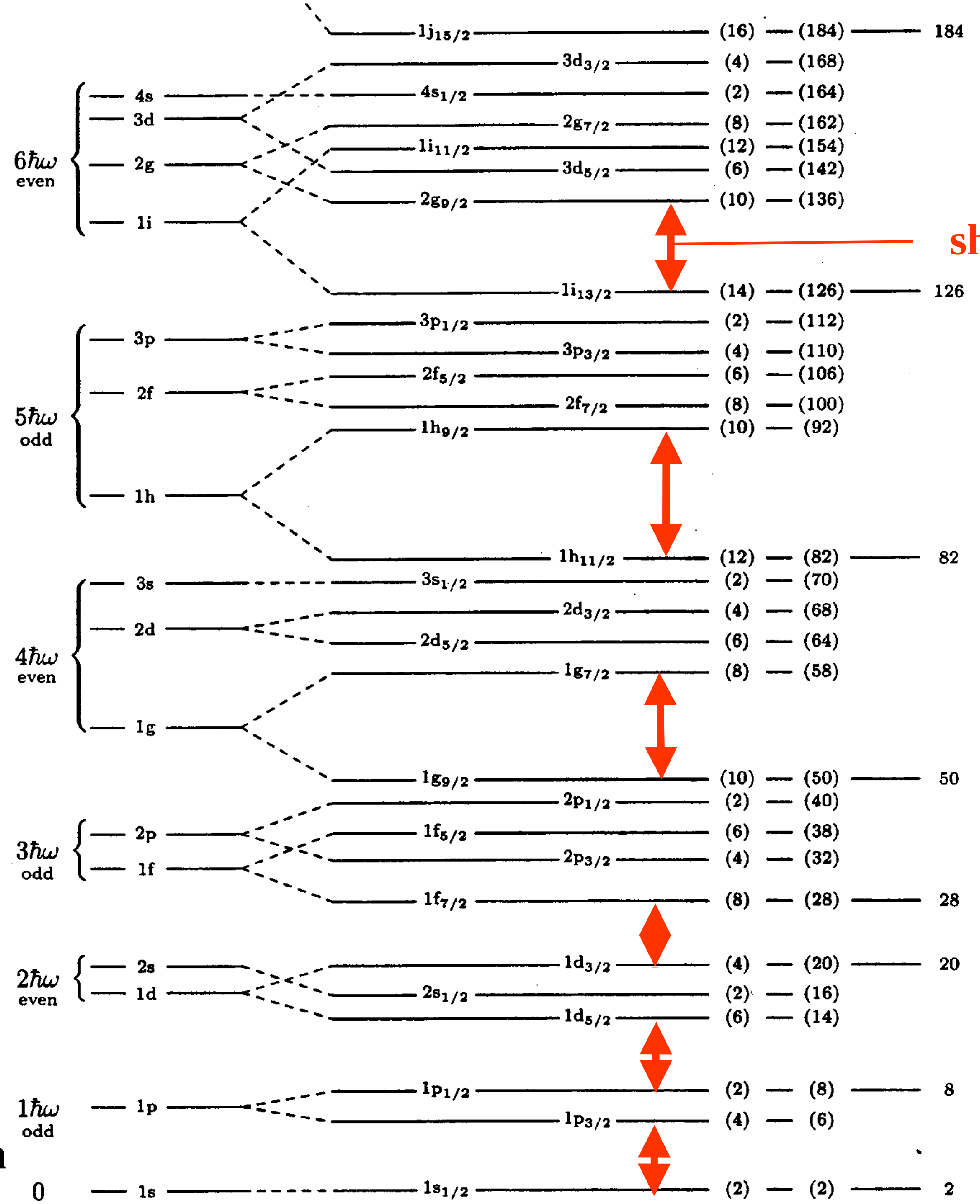
Deviation (in MeV) to experimental masses:



something is missing !

Shell model:
(single nucleon
energy levels)

are not evenly spaced



shell gaps

less bound than average

more bound than average

need to add shell correction term S(Z,N)

Magic numbers

Modern mass models

Global mass models – 2 basic philosophies:

1) Microscopic – Macroscopic mass models

Macroscopic part: liquid drop, droplet, or refinements thereof)

Microscopic part: shell correction, pairing correction, refinement of surface term accounting for finite range of nuclear force …

2) Microscopic mass models

based on some (parametrized) nucleon-nucleon interaction

Problem: not very accurate due to limitations of current microscopic theories

Solution: Fit parameters of interaction specifically to masses to obtain a mass model

Local mass “models”:

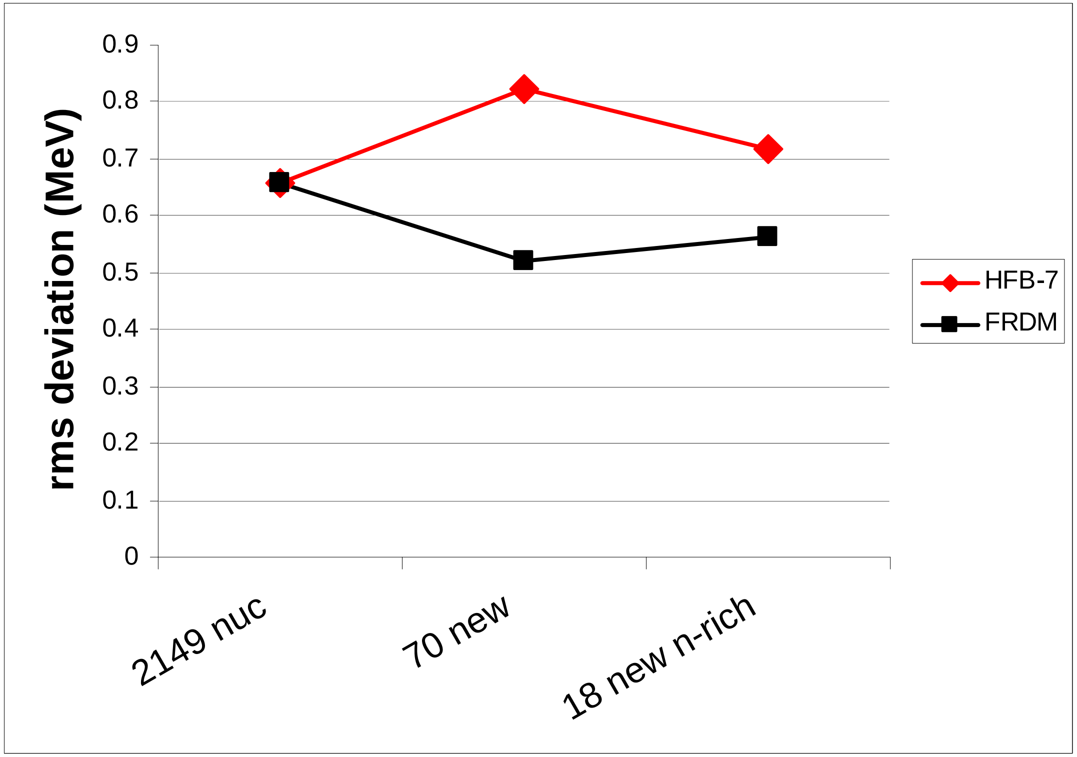
- extrapolations based on neighboring masses (Atomic Mass Evaluation)
- mirror symmetry: Coulomb shifts, IMME
- Garvey-Kelson …

Mass measurements have sufficiently progressed so that global mass models are only needed for very neutron rich nuclei (r-process, neutron star crusts)

Modern mass models – how well are they doing?

Example: mic model: HFB series (Goriely, Pearson) currently at HFB-15 (2008)
mic-mac : Finite Range Droplet Model FRDM (Moller et al.) unchanged since 1993

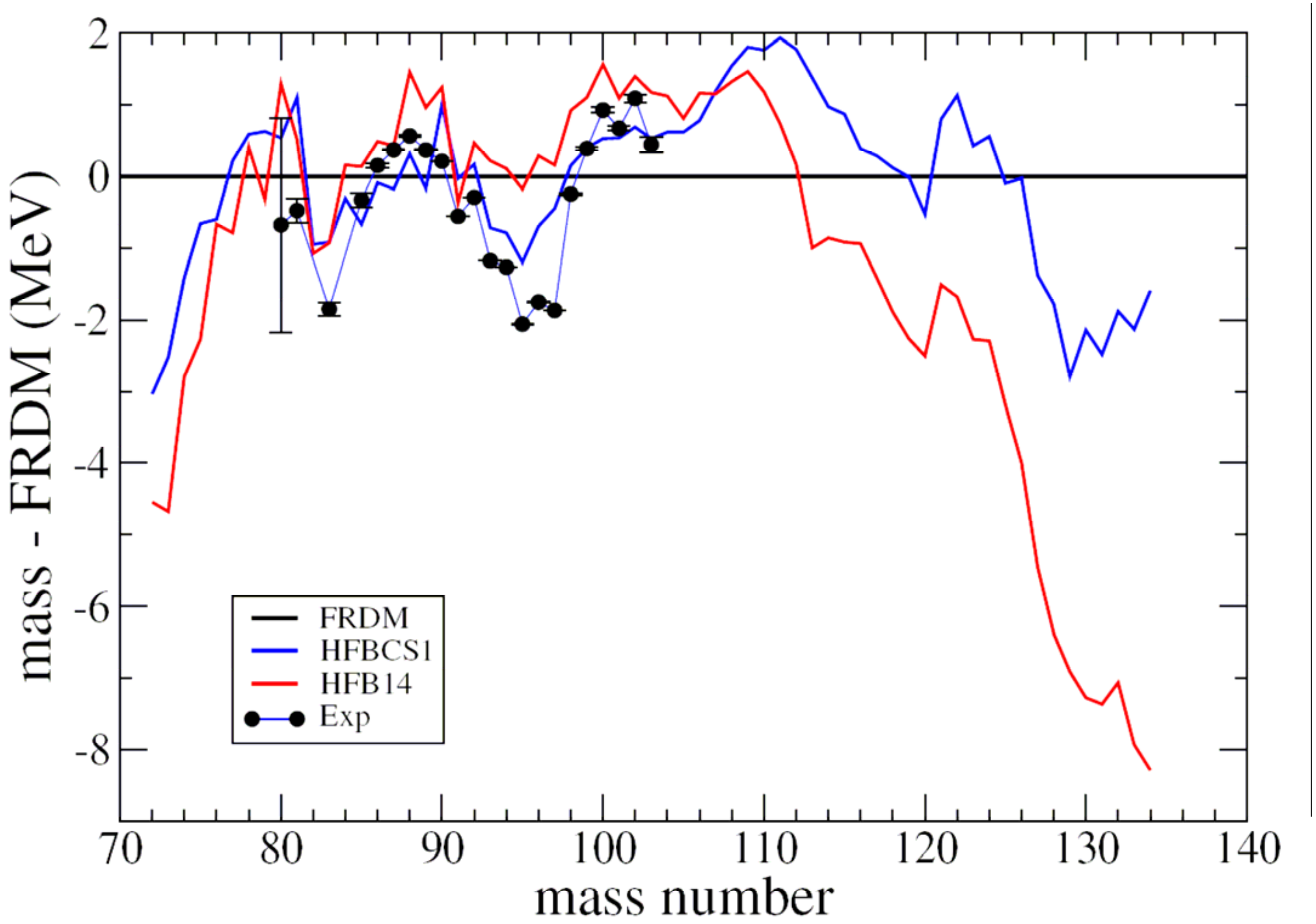
Compare rms deviations to experiment:



Important is not how well the model fits known masses, but how well it predicts unknown masses !

Modern mass models – how well are they doing?

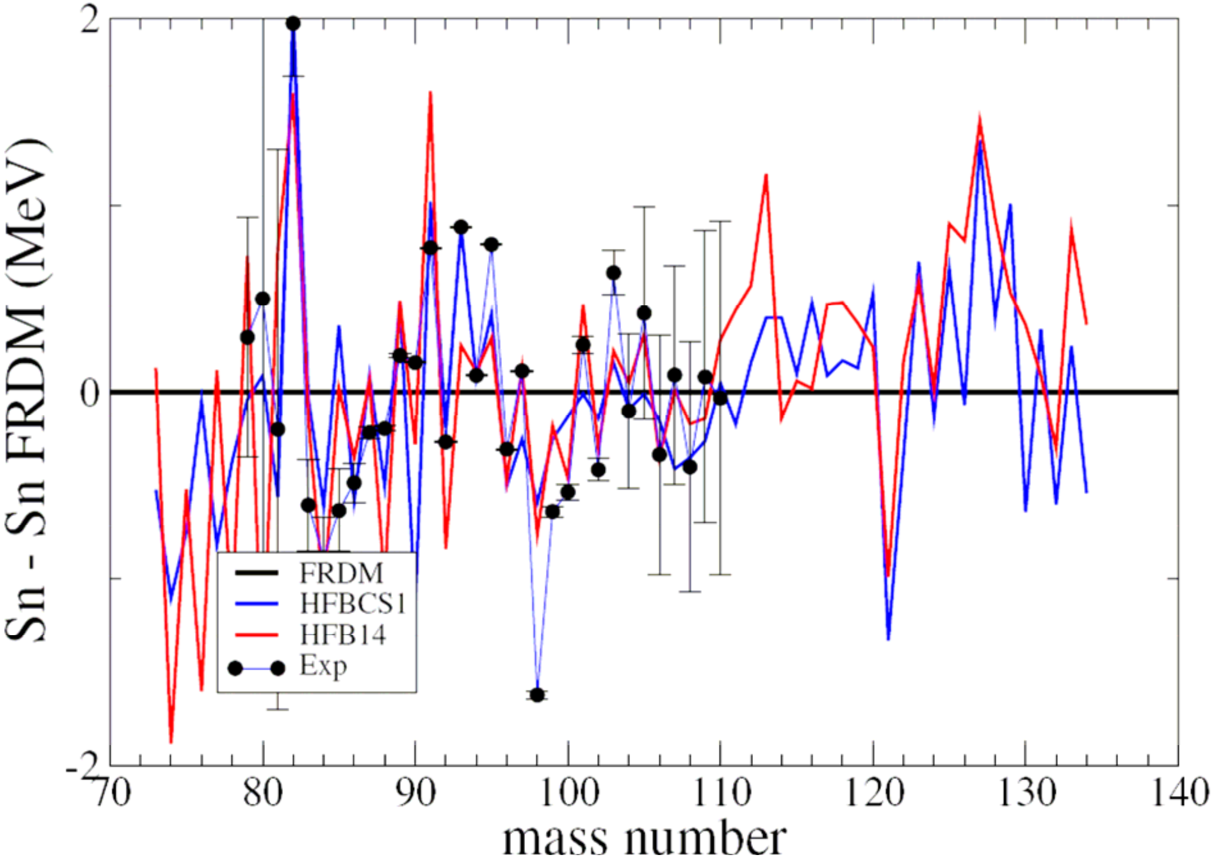
Example: predicted masses for Zr isotopes



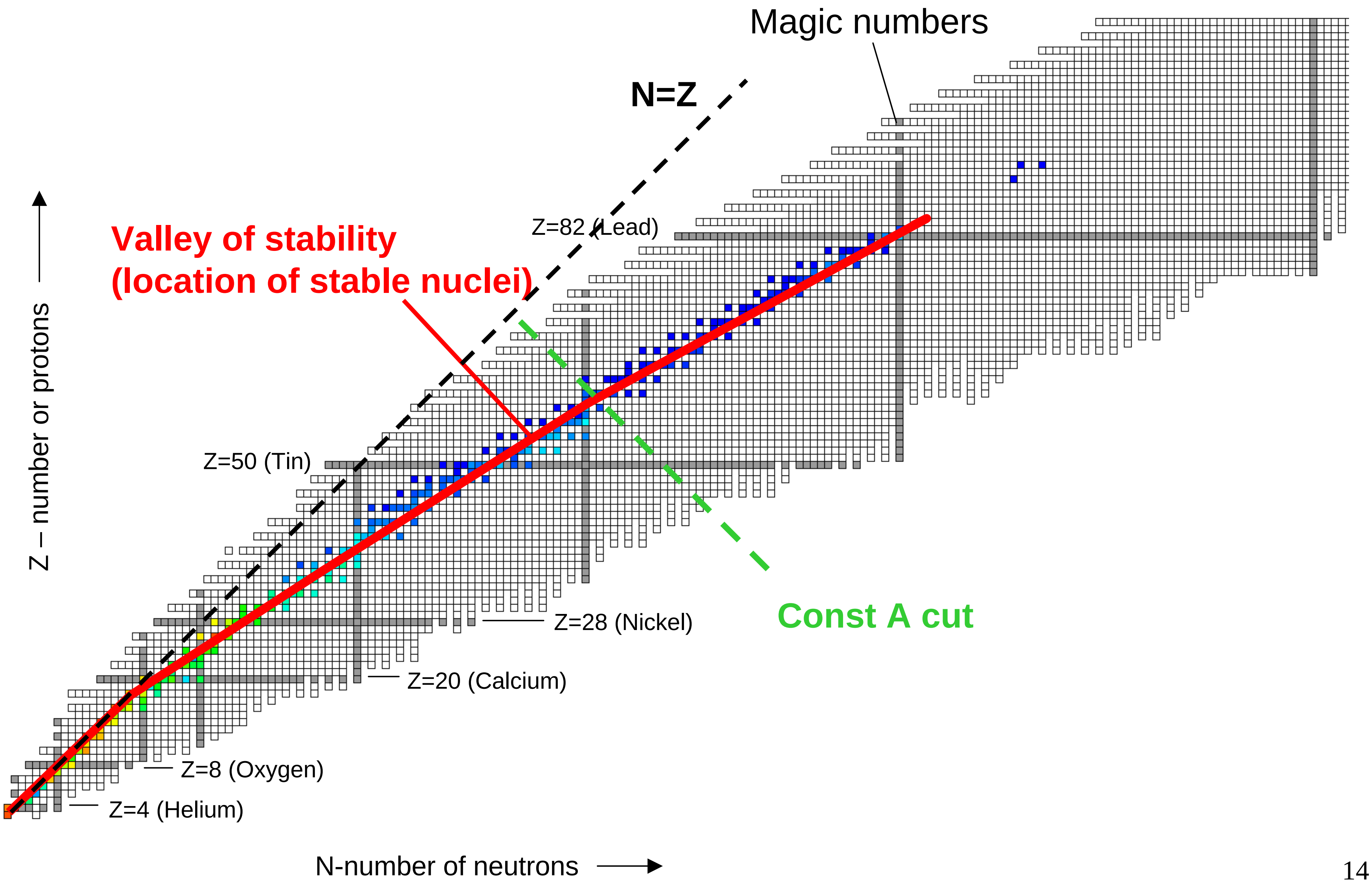
Modern mass models – how well are they doing?

What about mass differences?

Neutron capture Q-values for Zr isotopes
(neutron separation energy Sn)



The valley of stability



Const. A cut:

Binding energy per nucleon along const A due to asymmetry term in mass formula

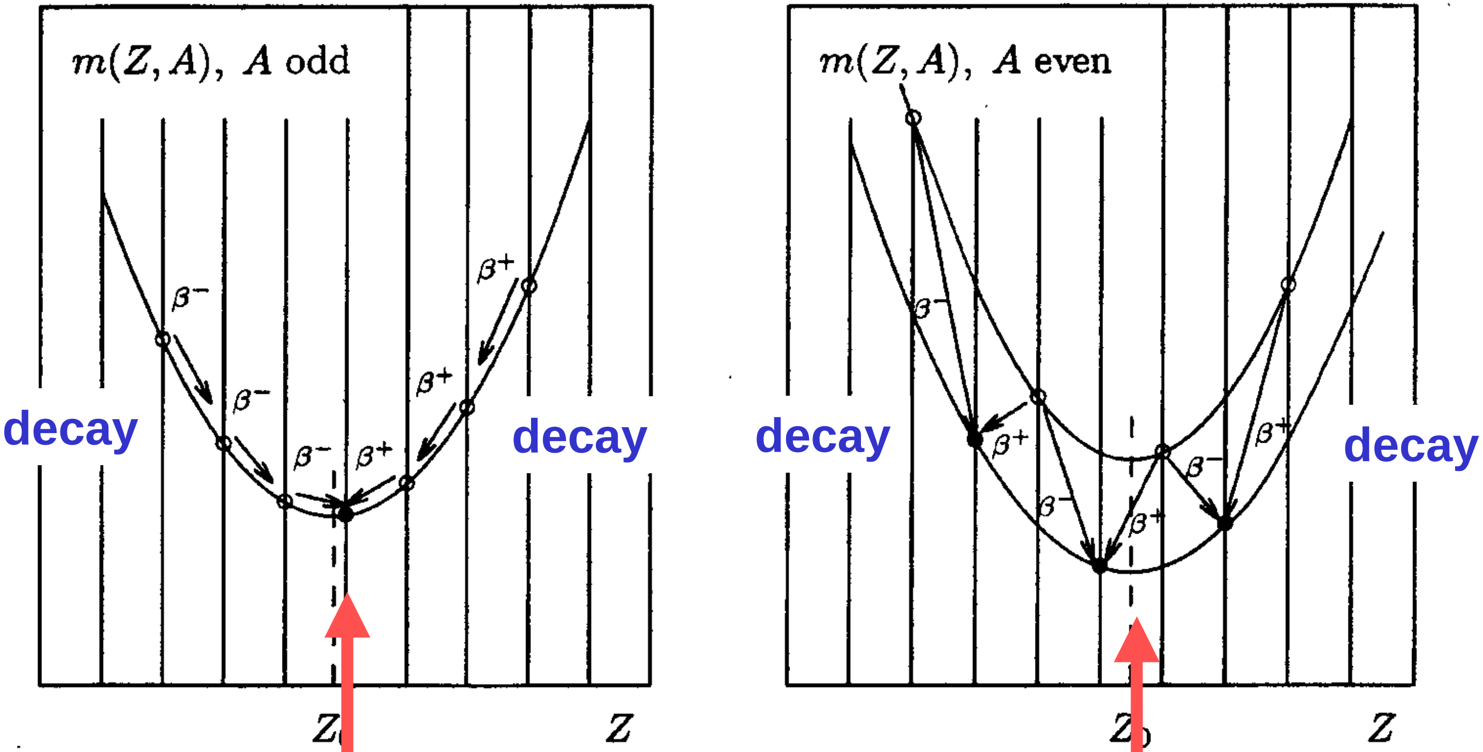


Fig. 4.2 - Mass of nuclei with a fixed A. The stable nuclei are represented by full circles.

(Bertulani & Schechter)

valley of stability

What happens when a nucleus outside the valley of stability is created ? (for example in a nuclear reaction inside a star ?)

Decay - energetics and decay law

Decay of A in B and C is possible if reaction A $\longrightarrow$ B+C has positive Q-value
(again – masses are critcical !)

BUT: there might be a barrier that prolongs the lifetime

Decay is described by quantum mechanics and is a pure random process, with a constant probability for the decay of a single nucleus to happen in a given time interval.

N: Number of nuclei A (Parent)
λ : decay rate (decays per second and parent nucleus)

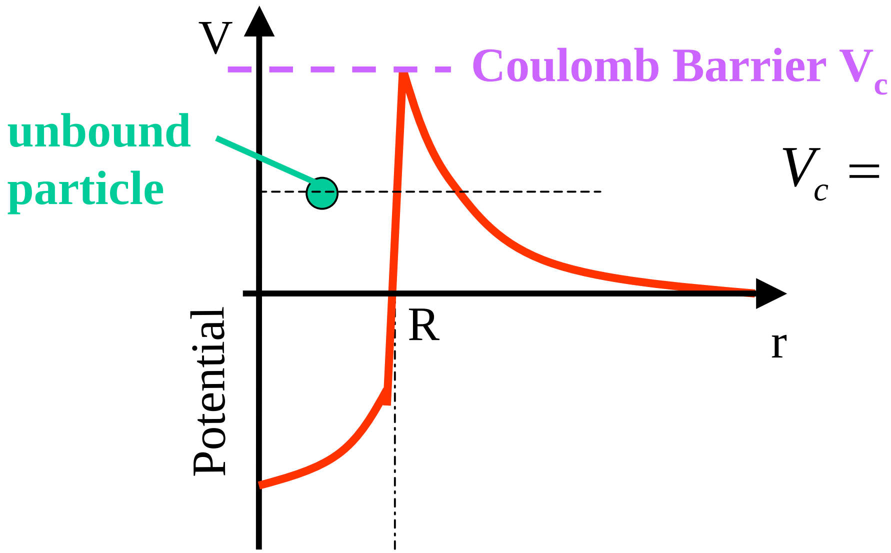
$$dN = -\lambda N dt \qquad \text{therefore} \qquad N(t) = N(t=0)\,\mathrm{e}^{-\lambda t}$$

lifetime $\tau = 1/\lambda$

half-life $T_{1/2} = \tau \ln 2 = \ln 2/\lambda$ is time for half of the nuclei present to decay

Decay modes

for anything other than a neutron (or a neutrino) emitted from the nucleus there is a Coulomb barrier



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

If that barrier delays the decay beyond the lifetime of the universe (~ 14 Gyr) we consider the nucleus as being stable.

Example: for $^{197}Au \rightarrow {}^{58}Fe + {}^{139}I$ has Q ~ 100 MeV !

yet, gold is stable.

➔ **not all decays that are energetically possible happen**

most common:
- β decay
- n decay
- p decay
- α decay
- fission

β decay

p $\leftrightarrow$ n   conversion within a nucleus via weak interaction

Modes (for a proton/neutron in a nucleus):

β^+ decay	$p \rightarrow n + e^+ + \nu_e$	Favourable for n-deficient nuclei
electron capture	$e^- + p \rightarrow n + \nu_e$	Favourable for n-deficient nuclei
β^- decay	$n \rightarrow p + e^- + \bar{\nu}_e$	Favourable for n-rich nuclei

Electron capture (or EC) of atomic electrons or, in astrophysics, of electrons in the surrounding plasma

Q-values for decay of nucleus (Z,N)

with nuclear masses   **with atomic masses**

$$Q_{\beta+} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z-1,N+1) - m_e \ = m(Z,N) - m(Z-1,N+1) - 2m_e$$

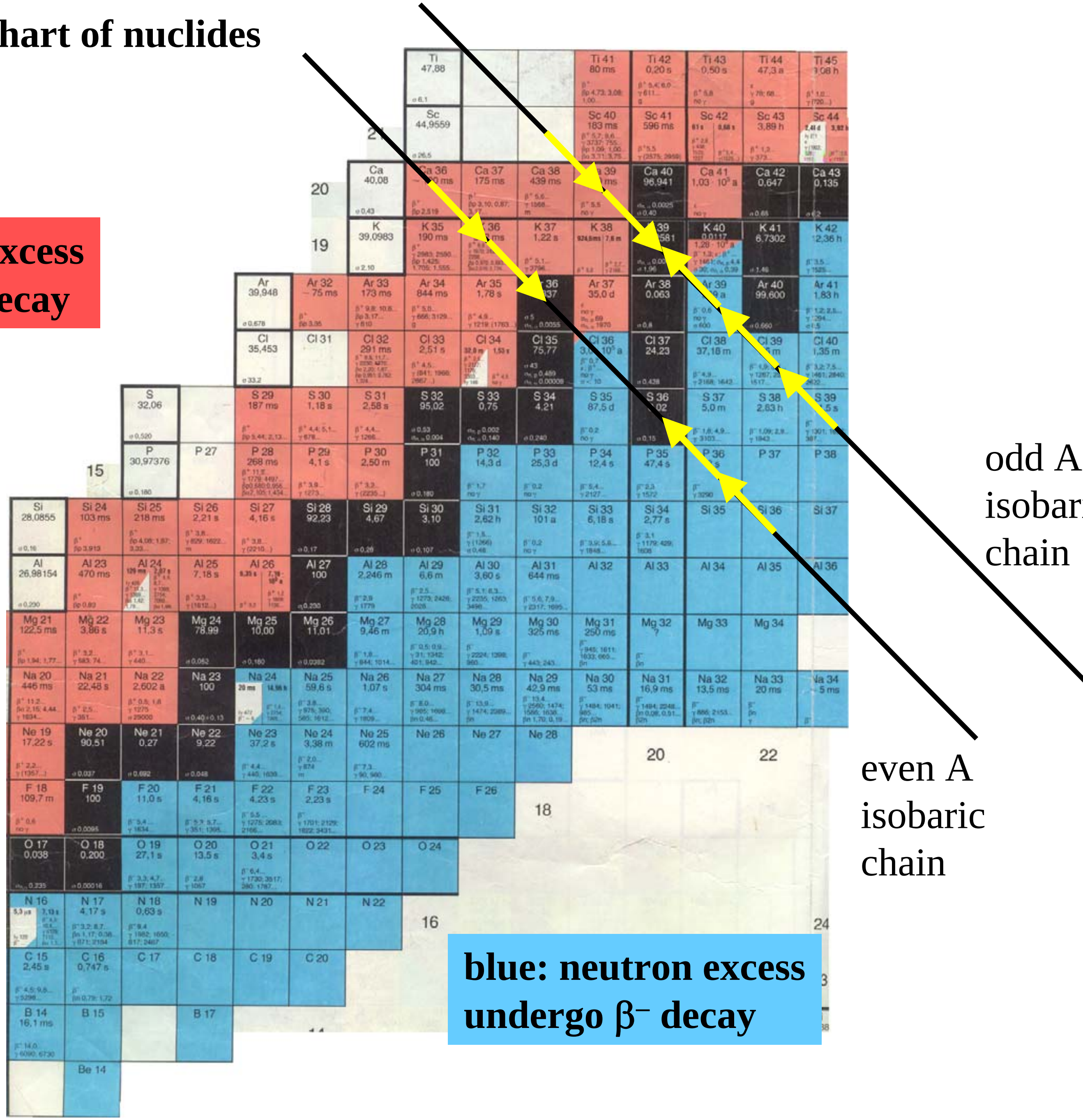
$$Q_{EC} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z-1,N+1) + m_e \ = m(Z,N) - m(Z-1,N+1)$$

$$Q_{\beta-} / c^2 = m_{nuc}(Z,N) - m_{nuc}(Z+1,N-1) - m_e \ = m(Z,N) - m(Z+1,N-1)$$

Note: $Q_{EC} > Q_{\beta+}$ by 1.022 MeV

Typical part of the chart of nuclides

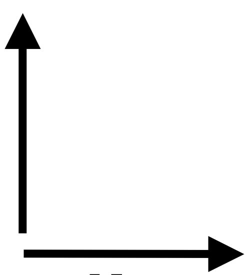
red: proton excess undergo β⁺ decay



odd A isobaric chain

even A isobaric chain

blue: neutron excess undergo β⁻ decay



Z

N

<u>Typical β decay half-lives:</u>

very near “stability” : occasionally Mio’s of years or longer

more common within a few nuclei of stability: minutes - days

most exotic nuclei that can be formed: ~ milliseconds

Proton or neutron decay:

Usually, the protons and neutrons in a nucleus are bound
→ Q-value for proton or neutron decay is negative

For extreme asymmetries in proton and neutron number nuclei become proton or neutron unbound

→ Proton or neutron decay is then possible

A nucleus that is proton unbound (Q-value for p-decay > 0) is beyond the "**proton drip line**"

A nucleus that is neutron unbound (Q-value for n-decay >0) is beyond the "**neutron drip line**"

NOTE: nuclei can exist beyond the proton and neutron drip line:
- for very short time
- for a "long" time beyond p-drip if Q-value for p-decay is small (Coulomb barrier !)
- for a long time beyond n-drip at extreme densities inside neutron stars

6.4. α decay

emission of an α particle (= ^{4}He nucleus)

Coulomb barrier twice as high as for p emission, but exceptionally strong bound, so larger Q-value

emission of other nuclei does not play a role (but see fission !) because of

- increased Coulomb barrier
- reduced cluster probability

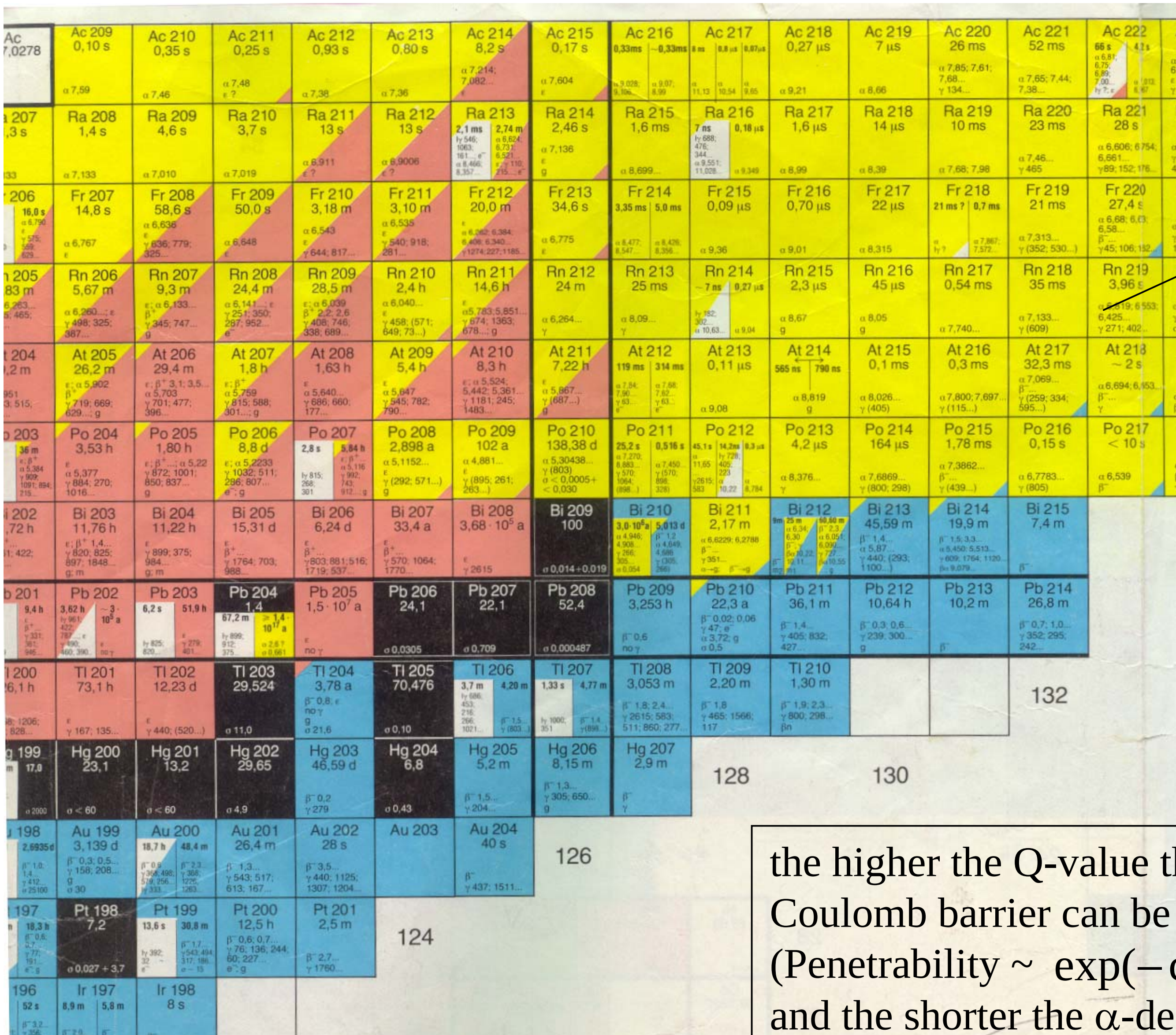
Q-value for α decay:

$$
\begin{aligned}
Q_\alpha &= m(Z,A) - m(Z-2,A-4) - m_\alpha \\
&= \underbrace{-B(Z,A) + B(Z-2,A-4)}_{<0\text{, but closer to 0 with larger A,Z}} + m_\alpha
\end{aligned}
$$

→ large A therefore favored

lightest α emitter: ^{144}Nd (Z=60) (Qα=1.9 MeV but still $T_{1/2}$=2.3 x 10^{15} yr)

beyond Bi α emission ends the valley of stability !



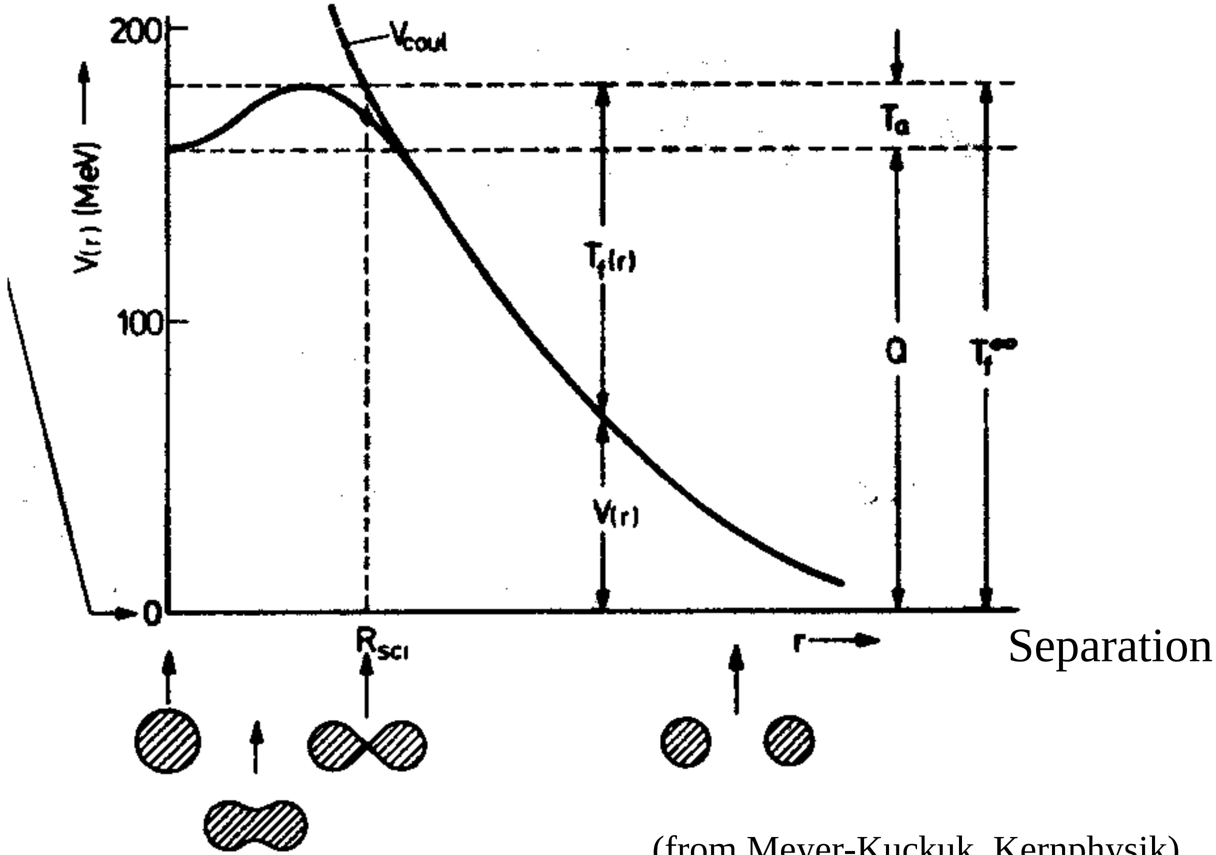
yellow are α emitter

the higher the Q-value the easier the Coulomb barrier can be overcome (Penetrability ~ $\exp(-\mathrm{const}\cdot E^{-1/2})$) and the shorter the α-decay half-lives

6.5. Fission

Very heavy nuclei can fission into two parts (Q>0 if heavier than ~iron already)

For large nuclei surface energy less important - large deformations less prohibitive. Then, with a small amount of additional energy (Fission barrier) nucleus can be deformed sufficiently so that coulomb repulsion wins over nucleon-nucleon attraction and nucleus fissions.

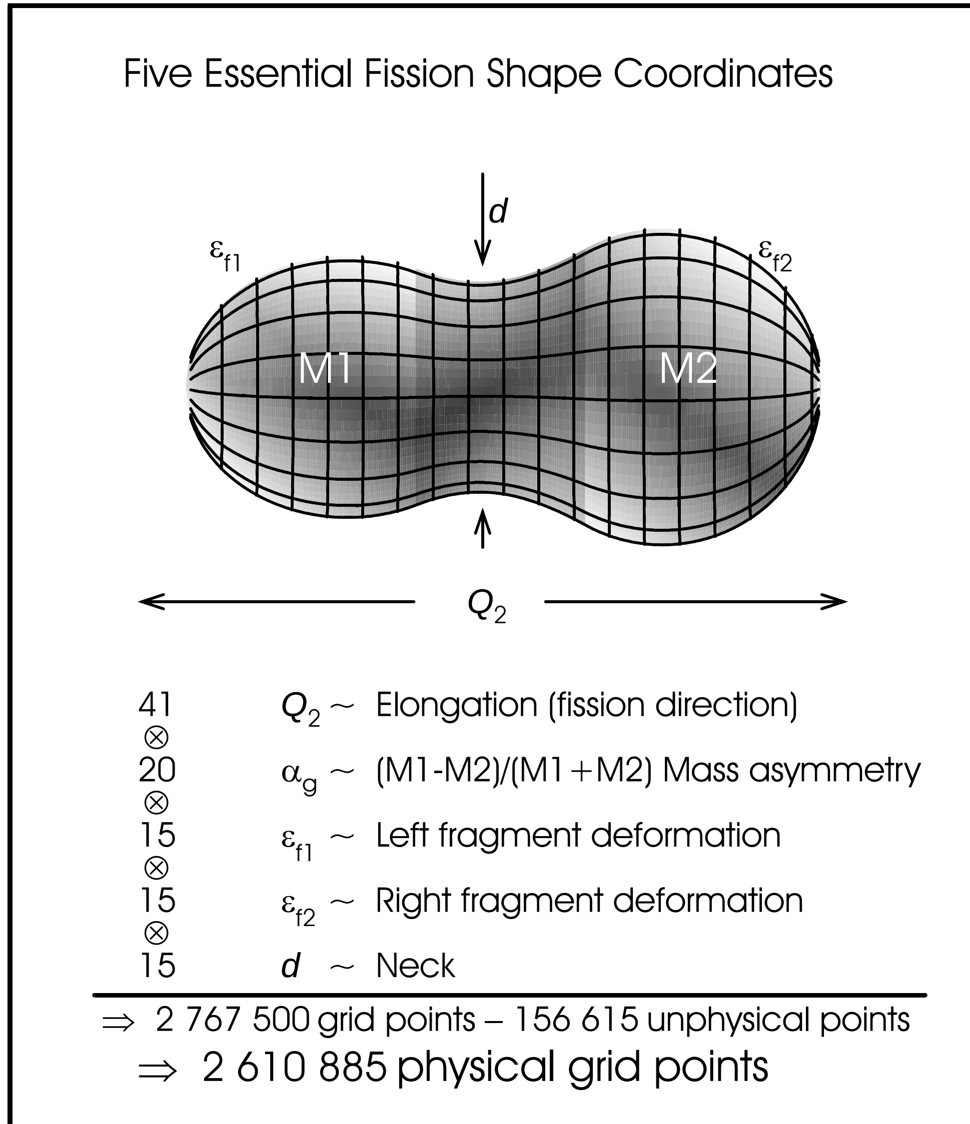


(from Meyer-Kuckuk, Kernphysik)

Real fission barriers:

Fission barrier depends on how shape is changed (obviously, for example. it is favourable to form a neck).

Real theories have many more shape parameters - the fission barrier is then a landscape with mountains and valleys in this parameter space. The minimum energy needed for fission along the optimum valley is “the fission barrier”



Example for parametrization in Moller et al. Nature 409 (2001) 485

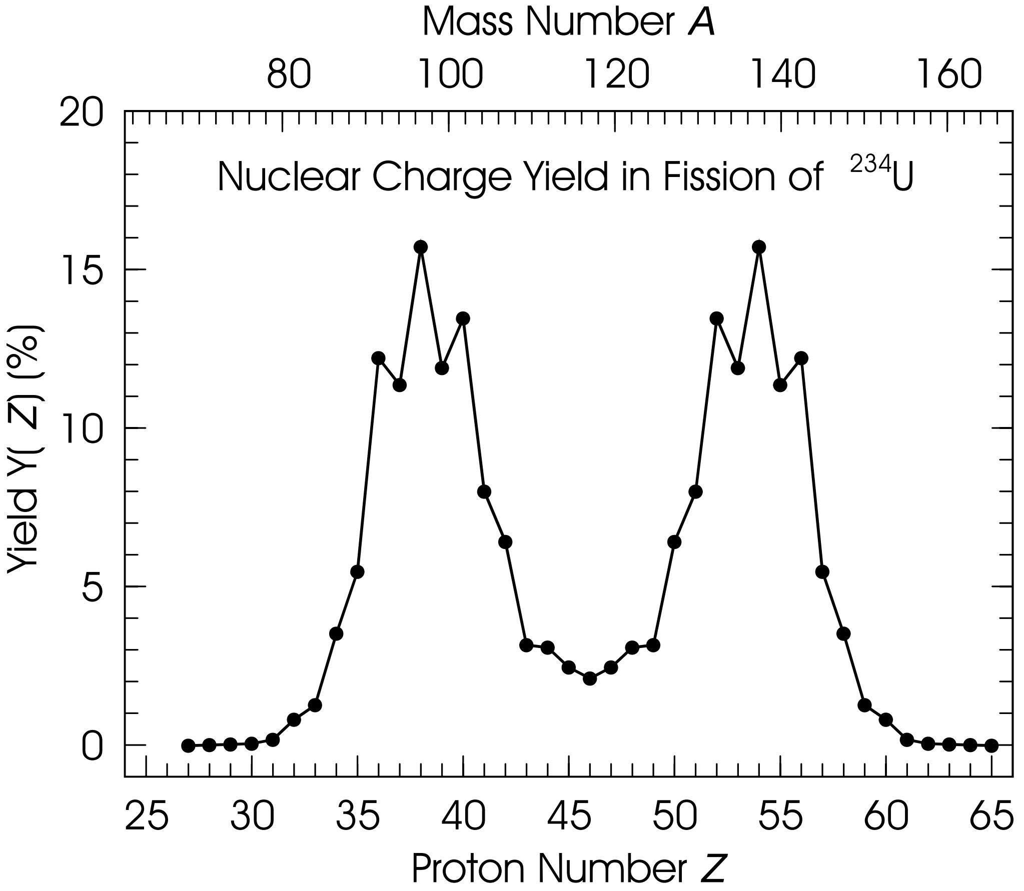
Fission fragments:

Naively splitting in half favourable (symmetric fission)

There is a asymmetric fission mode due to shell effects

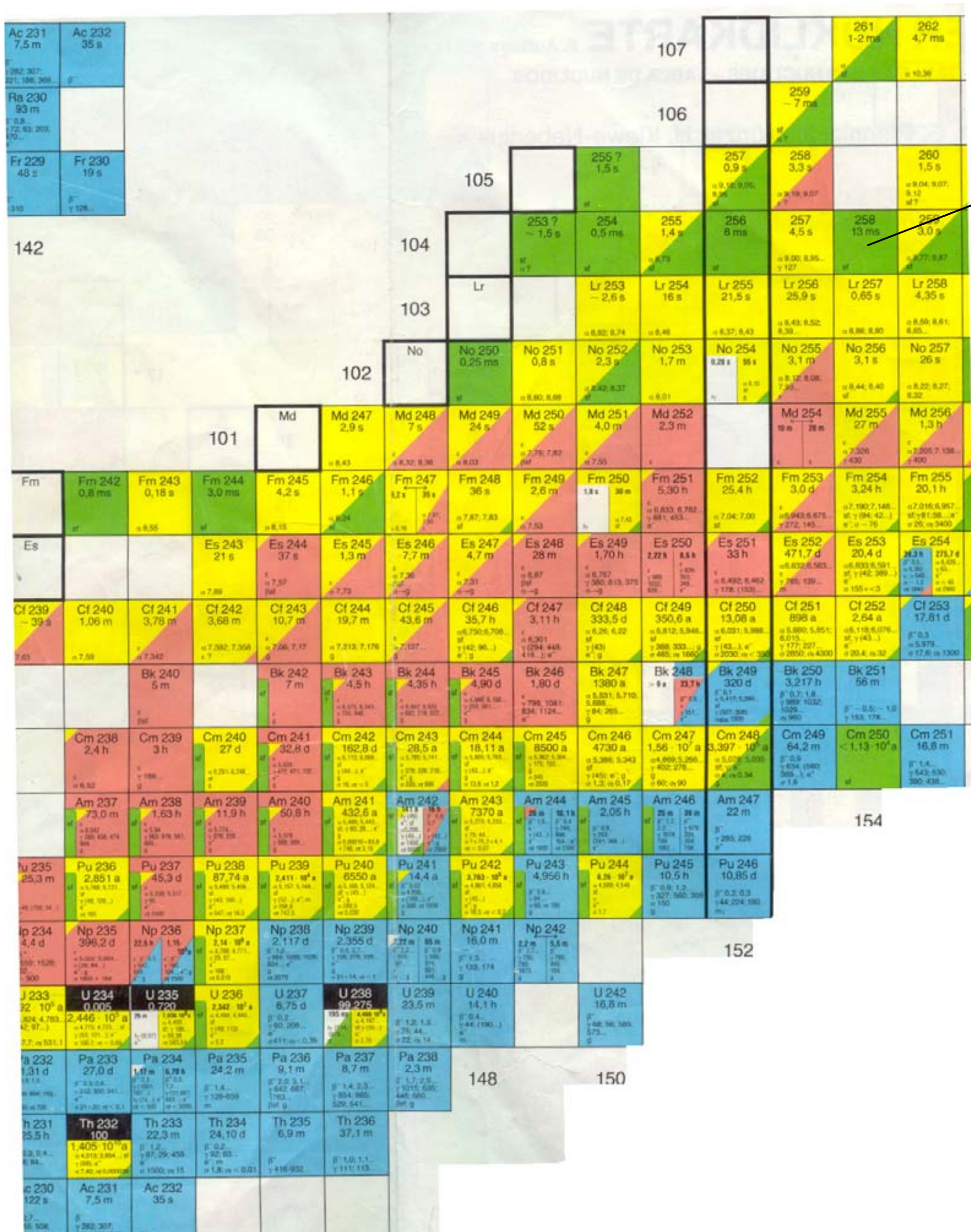
(somewhat larger or smaller fragment than exact half might be favoured if more bound due to magic neutron or proton number)

Both modes occur



Example from
Moller et al. Nature 409 (2001) 485

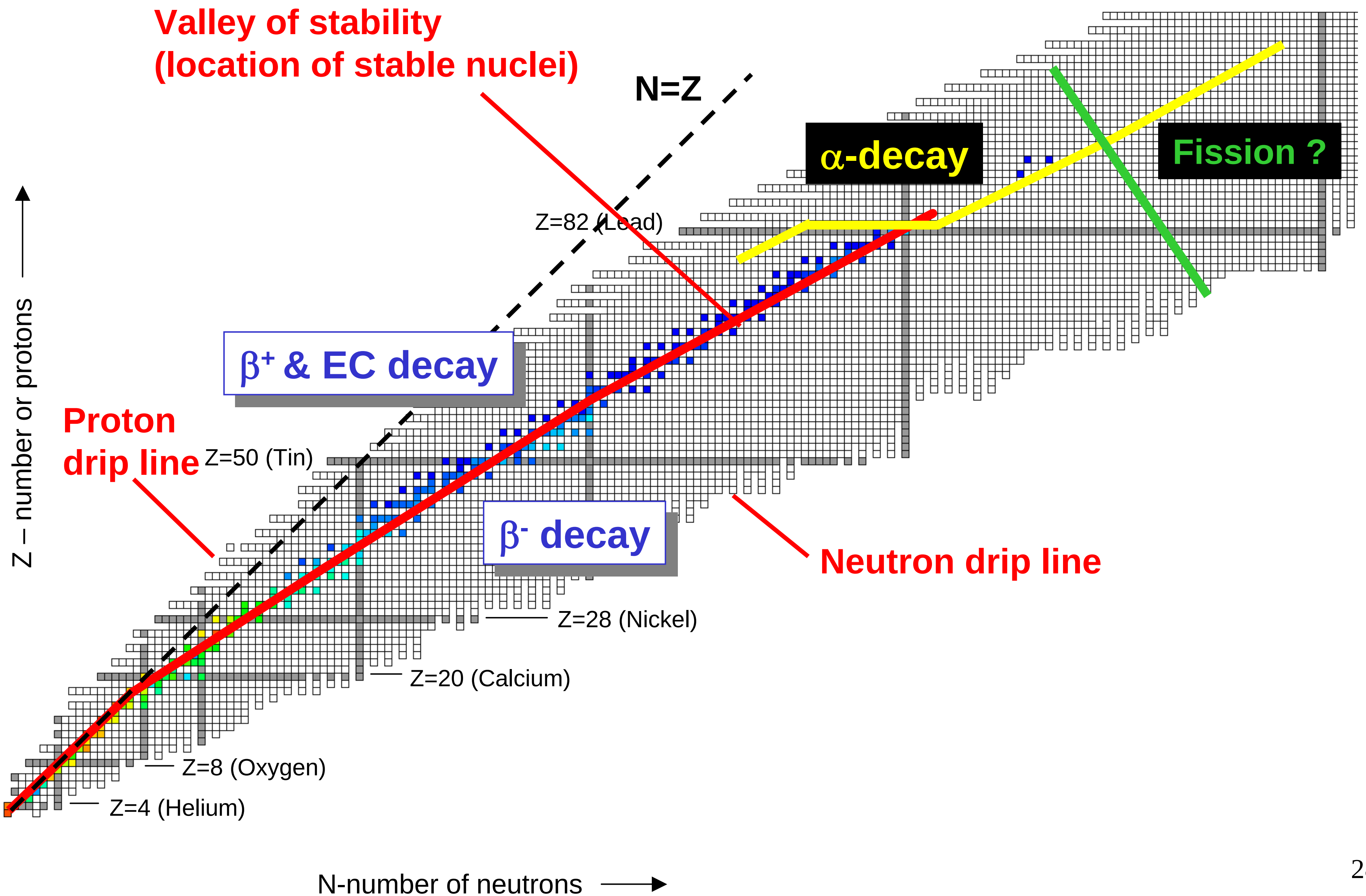
If fission barrier is low enough spontaneous fission can occur as a decay mode



green = spontaneous fission

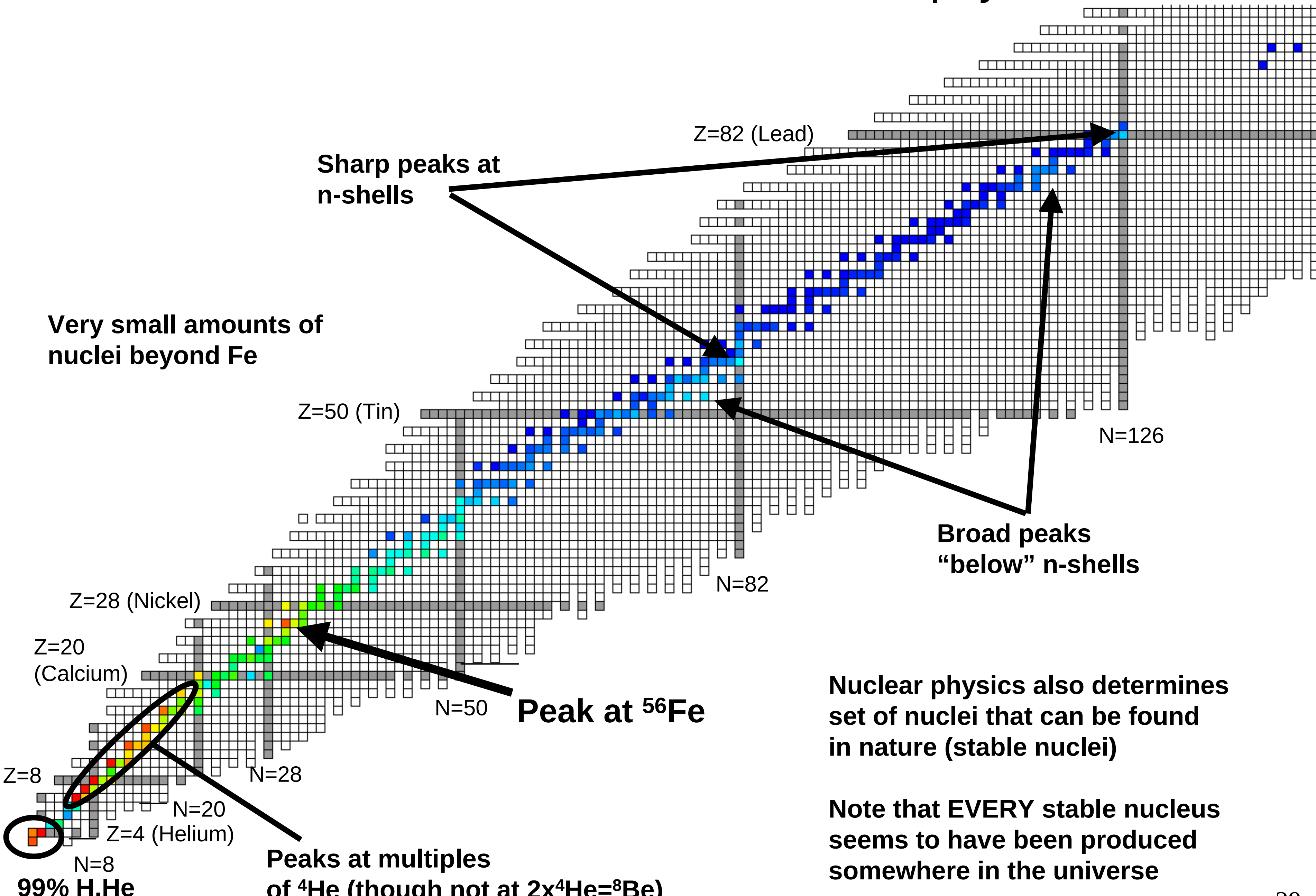
spontaneous fission is the limit of existence for heavy nuclei

Summary



Valley of stability
(location of stable nuclei)
N=Z
α-decay
Fission ?
Z=82 (Lead)
β+ & EC decay
Proton
drip line
Z=50 (Tin)
β- decay
Neutron drip line
Z=28 (Nickel)
Z=20 (Calcium)
Z=8 (Oxygen)
Z=4 (Helium)
Z – number or protons
N-number of neutrons

Solar abundances and nuclear physics



Nuclear physics also determines set of nuclei that can be found in nature (stable nuclei)

Note that EVERY stable nucleus seems to have been produced somewhere in the universe