The nucleus

The atomic nucleus consists of protons and neutrons

Protons and Neutrons are therefore called nucleons

A nucleus is characterized by:
• A: Mass Number = number of nucleons
• Z: Charge Number = number of protons
• N: Neutron Number

Of course A=Z+N

Usual notation:

\[ ^{12}\text{C} \]

Element symbol – defined by charge number
C is Carbon and Z=6

So this nucleus is made of 6 protons and 6 neutrons
Abundance of a nucleus
How can we describe the relative abundances of nuclei of different species and their evolution in a given sample (say, a star, or the Universe)?

1) Number density

We could use the number density $n_i = \text{number of nuclei of species } i \text{ per } \text{cm}^3$

Disadvantage: tracks not only nuclear processes that create or destroy nuclei, but also density changes, for example due to compression or expansion of the material.

→ not useful as characterization of composition
2) Mass fraction

Mass fraction \( X_i \) is fraction of total mass of sample that is made up by nucleus of species i

\[
n_i = \frac{X_i \rho}{m_i}
\]

\( \rho \) : mass density (g/cm\(^3\))

\( m_i \) mass of atom of species i

Or in terms of moles:

\[
n_i = \frac{X_i \rho N_A}{m_{i\text{ mole}}}
\]

\( m_{i\text{ mole}} \) (atomic) mole mass of species i

Of course:

\[
\sum_i X_i = 1
\]

Disadvantage: depends on mass of nucleus (for equal numbers of particles a heavier nucleus has a larger mass fraction)
3) Abundance

reactions mostly affect particle numbers, therefore sometimes one wants a quantity that is a measure for the number of particles (if two species have the same number density one wants the “abundance” to be the same regardless of mass)

So call this abundance \( Y_i \)

\[
n_i = \frac{X_i}{m_{i\text{mole}}} \rho N_A
\]

so

\[
n_i = Y_i \rho N_A
\]

The abundance \( Y \) is proportional to number density but changes only if the nuclear species gets destroyed or produced. Changes in density are factored out.

Unit: in principle mole/g

So abundance \( Y \) is the number of moles of a species per gram matter (sometimes called mole fraction BUT does not sum to 1 !!!)
Common approximation: \[ m_{\text{mole}} = A \] With nuclear mass number A

- Neglects nuclear binding energy
- Neglects electron masses (and binding energies)
- ONLY VALID IN CGS UNITS

With that:

And typically Y is considered unit less

\[ Y = \frac{X}{A} \]

\[ n_i = Y_i \rho N_A \] Still works out unit wise as \( N_A = 1/m_u \) in CGS units
Some useful quantities and relations

of course $\sum_i X_i = 1$ but, as $Y=X/A < X$ $\sum_i Y_i < 1$

- Mean molecular weight $\mu_i$
  
  $\mu_i = \frac{\sum_i A_i Y_i}{\sum_i Y_i} = \frac{1}{\sum_i Y_i}$ or
  
  $\mu_i = \frac{1}{\sum_i Y_i}$

- Electron Abundance $Y_e$

  As matter is electrically neutral, for each nucleus with charge number $Z$ there are $Z$ electrons:

  $Y_e = \sum_i Z_i Y_i$ and as with nuclei, electron density $n_e = \rho N_A Y_e$

  can also write: $Y_e = \frac{\sum_i Z_i Y_i}{\sum_i A_i Y_i}$ prop. to number of protons

  prop. to number of nucleons

  So $Y_e$ is ratio of protons to nucleons in sample
  (counting all protons including the ones contained in nuclei
  - not just free protons as described by the “proton abundance”)

Abundance is not a fraction!
some special cases:

For 100% hydrogen: $Y_e=1$
For equal number of protons and neutrons ($N=Z$ nuclei): $Y_e=0.5$
For pure neutron gas: $Y_e=0$