Today

• Announcements:
  – HW#4 is due by 8:00 am Wednesday February 14th.

• General Relativity

• Comments on COSMOS by Carl Sagan
  – Watch part of episode 8:
    http://www.youtube.com/watch?v=SIfrZhttNos
  – I have copies to loan if you want to watch the episode
Another consequence of special relativity

- $F = ma$.
- What happens as we accelerate to near the speed of light? We can’t continue to accelerate. If we did we would exceed the speed of light and that is not allowed.
- Newton’s law must be valid, so it must be that mass increases near the speed of light. Mass is relative!
- We define the rest mass as the mass of an object at rest.
This lead Einstein to his famous equation…

- Energy is the ability to do work. It comes in two main types
  - Kinetic energy: the energy of motion
  - Potential energy: the energy of position
- Work = force x distance (it’s a scalar measured in Joules, J)
- Einstein’s Energy-mass relation:
  \[ E = m \ c^2 \]
The following is a picture of a chemical reaction:

Start with some initial mass (kg) \rightarrow Something happens \rightarrow End up with some final mass (kg)

Some fraction, \( f \), is converted to energy

\( f \)- see the table on the next page

\[ \text{ENERGY (Joules, J)} \]

The amount of energy is \( E = m_{\text{converted}} c^2 \)

\( m_{\text{converted}} = \text{(Mass to start)} \times \text{fraction} \)
Fraction of Energy Converted

- In a chemical reaction not all the mass can be converted to energy. Actually only a very small fraction (the exact value of the fraction depends on the chemical reaction) about $1 \times 10^{-10}$ of all the mass is converted to energy.

- Some other fractions:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Fraction</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matter-Antimatter Annihilation</td>
<td>1</td>
<td>No common example</td>
</tr>
<tr>
<td>Fusion</td>
<td>0.007</td>
<td>Power source of the Sun</td>
</tr>
<tr>
<td>Fission</td>
<td>0.001</td>
<td>Nuclear power plant</td>
</tr>
<tr>
<td>Chemical</td>
<td>$1 \times 10^{-10}$</td>
<td>Burning coal</td>
</tr>
<tr>
<td>Mechanical</td>
<td>$1 \times 10^{-15}$</td>
<td>Compressing a spring</td>
</tr>
</tbody>
</table>
Some Samples

- A power plant generates 500 MW of electrical power and 700 MW of waste heat (plants always make more waste heat than electrical power). How many Joules of energy does the plant generate in 1 day? Data: 1 Watt = 1 Joule/s

Electrical Energy (1 day) = (500 MW) × seconds in a day

= 1200 × 10^6 J \frac{60s}{s} \times \frac{60m}{m} \times \frac{24hr}{hr} \times 1d

Electrical Energy (produced in 1 day) = 4.32E13 J
More on the power plant

Assume the power plant in the previous problem burns 2.2 kg of oxygen and 1 kg of carbon from coal to make 33 MJ of energy. How many kg of carbon and oxygen will the plant use in a day?

\[
\text{mass (kg)} = \frac{\text{total energy produced}}{\text{energy generated/mass}} = \frac{\text{electrical + waste energy}}{\text{energy generated/mass}}
\]

\[
\text{mass (kg)} = \frac{4.32 \times 10^{13} \text{ J} + 6.048 \times 10^{13} \text{ J}}{33 \times 10^6 \text{ J}/(2.2 \text{ kg} + 1.0 \text{ kg})} = 1.005 \times 10^7 \text{ kg}
\]
How much of that mass was converted to energy?

\[ E = m_{\text{converted}} c^2 \Rightarrow m_{\text{converted}} = \frac{E}{c^2} \]

\[ m_{\text{converted}} = \frac{1.04 \times 10^{14} \text{ J}}{(3 \times 10^8 \text{ m/s})^2} = 1.16 \times 10^{-3} \text{ kg} \]

But we used more than \(10^7\) kg (10,000 metric tons), where did it all go?

Hint: The main byproduct of burning coal is \(\text{CO}_2\).
How long will the Sun burn?

The sun generates $3.82 \times 10^{24}$ W of power by fusion of hydrogen into helium. The fraction of mass converted for fusion is 0.007. How many kg of protons and electrons does the Sun use every second?

$$m_{\text{burned each s}} = \frac{m_{\text{converted}}}{f} = \frac{E}{f} = \frac{3.82 \times 10^{24} J}{\left(3 \times 10^8 \frac{m}{s}\right)^2} \times \frac{1}{0.007} = 6.06 \times 10^9 \text{ kg}$$

Years Sun will last = (Total mass of the core/mass used per second) x (years/s) 1 year = $3.156 \times 10^7$ s
General Relativity

• Why is gravitational mass (mg) the same as inertial mass (ma)?

• This is one of the questions that inspired Einstein. His answer is General Relativity.

• Recall – Special Relativity was for non-accelerating frames of reference. General Relativity covers both accelerating and non-accelerating.
General Relativity continued

• Main Postulate: Acceleration in one direction is like gravity in the other direction. It is not possible distinguish the two. This is called the principle of equivalence.

• Mass warps space

• Space and times are combined into a 4-dimensional space-time
Gravity is actually the result of warped space. What we perceive as acceleration (and hence say is due to a force) is really just stretched space.
Gravitational Time Dilation

• Mass stretches space, but since space and time are connected (space-time) it also affects time.

• Near a mass, time runs more slowly. On the surface of the Earth this affect is only $10^{-9}$ s, but near a black hole it could be infinite.

• Why? As you travel through space you travel through time. Where space is stretched, time is compacted.

• Metric equation: 

\[ (\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 \]

The (-) is part of what is called the metric of space time.
• Bending of star light – the gravitational field of the Sun bends star light by 1.75 arcseconds. This was observed by A. Eddington in 1919 during an eclipse.
Gravitational Lensing: Routinely observed and used to measure the mass of distant clusters of galaxies.
Real picture from the Hubble Telescope

Abel galaxy cluster
Wormholes

This could be the basis for a time machine.
Paradoxes in Time Travel

• If time is a dimension like the other three, can we move back and forth in time?
• If we can travel back in time, it would be possible for use to influence things so that we are not born.
• Three theories to resolve the paradox
  – Travel back in time is not possible
  – There are a very large number of parallel universes
  – Something about nature prevents us from influencing the past
Einstein Equation

\[ R_{ij} - \frac{1}{2} Rg_{ij} - \lambda g_{ij} = \frac{8\pi G}{c^4} T_{ij} \]

- A tensor equation that describes how space-time is influenced by mass.
- The details of what the symbols mean does not matter. Approximately, the left side is the curvature and motion of space and the right side is the location and motion of mass.
- \( R_{ij} \) is the Ricci tensor, \( g \) is the metric of space, \( G \) is the gravitational constant, etc.