Radiation Thickness, Collisional Thickness, and Most Probable Collisional Energy Loss for E97110: The GDH Sum Rule, the Spin Structure of $^3$He and the Neutron using Nearly Real Photons

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Abstract

Catalog of radiation lengths and materials for E97110. Also a catalog of collisional thickness and most probable energy loss. Still needs a list of uncertainties and a more complete list of references. Landau40 and Tsa71 for example. Better equation references. More discussion about the “1/18” term. Maybe explicitly write out bremsstrahlung spectrum? better table placement. reference/verify for GE180 composition. make comment about $\beta \approx 1$ and about the energy independence of $B$ for our beam energies and momenta. references for densities of oxides. $\delta$ subscripts on stuff.

1 General Formulas for Calculating Radiation Lengths

Bremsstrahlung is radiation emitted when an electron is accelerated in the Coulombic fields of both the nucleus [1] and electrons [2, 3] of an atom. The amount of energy an electron loses to bremsstrahlung at a given frequency is given by:

$$d^2E = -h\nu \ [N] \frac{d\sigma(\nu, E)}{d\nu} \ dx \ d\nu$$

where $h\nu$ is the energy of the radiated photons, $[N]$ is the number density of the atoms, $d\sigma(\nu, E)/d\nu$ is differential cross section for the production of a photon, $dx$ is the thickness of the material, and $d\nu$ is width of frequency range. To calculate the amount of energy lost per unit depth over all frequencies, one must integrate over the entire bremsstrahlung frequency spectrum:

$$\int \frac{d^2E}{dx} = \frac{dE}{dx} = -[N] \int_0^E h\nu \frac{d\sigma(\nu, E)}{d\nu} \ d\nu$$

It is convenient to change the integration variable to $u \equiv h\nu/E$, which is the fraction of the electron energy carried away by the radiated photons:

$$\frac{dE}{dx} = -[N]E \int_0^1 u \frac{d\sigma(u, E)}{du} \ du$$

For electron energies over 50 MeV, the integrand is nearly independent of the electron energy $E$ [4]:

$$\int_0^1 u \frac{d\sigma(u, E)}{du} \ du \approx \int_0^1 u \frac{d\sigma(u)}{du} \ du \equiv \sigma_{rad}$$
Consequently $\sigma_{\text{rad}}$, to a very good approximation, depends only on the charge of the nucleus $Z$ and is given by [5, 6, 7]:

$$\sigma_{\text{rad}}(Z) = 4\alpha r_e^2 \left[ Z^2 (L_{\text{rad}}(Z) - f(Z\alpha)) + ZL'_{\text{rad}}(Z) \right]$$

$$4\alpha r_e^2 = 2.31787 \text{ millibarns}$$

where $\alpha$ is the fine structure constant and $r_e$ is the classical electron radius. The term proportional to $Z^2$ is due to the field of the nucleus and the term proportional to $Z$ is due to the field of the atomic electrons. When evaluated in the complete screening limit, the radiation integrals $L_{\text{rad}}$ & $L'_{\text{rad}}$ are given by:

$$L_{\text{rad}}(Z) = \begin{cases} 
5.31 & Z = 1 \\
4.79 & Z = 2 \\
4.74 & Z = 3 \\
4.71 & Z = 4 \\
\log (184.15Z^{-1/3}) & Z \geq 5 
\end{cases}$$

$$L'_{\text{rad}}(Z) = \begin{cases} 
6.144 & Z = 1 \\
5.621 & Z = 2 \\
5.805 & Z = 3 \\
5.924 & Z = 4 \\
\log (1194Z^{-2/3}) & Z \geq 5 
\end{cases}$$

where $f(z)$ is known as the “Coulomb correction” [8]:

$$f(z) = z^2 \sum_{\nu=1}^{\infty} \frac{1}{\nu (\nu^2 + z^2)} \approx z^2 \left[ \frac{1}{1 + z^2} + 0.20206 - 0.0369z^2 + 0.0083z^4 - 0.002z^6 \right]$$

and we have dropped the $Z^2$ “1/18” term altogether [4]. When the Coulomb correction is neglected, $L_{\text{rad}}$ is approximated by $\log (183Z^{-1/3})$. If only the lowest terms in the Coulomb correction are kept, then $L_{\text{rad}}$ is approximated by $\log (191Z^{-1/3})$. Note that the original calculation of $L'_{\text{rad}}$ [2] contains an error [3] that results in this commonly used but incorrect form of $L'_{\text{rad}} = \log (1440Z^{-2/3})$.

Returning to the approximation that $\sigma_{\text{rad}}$ is independent of the electron energy, the energy of the electron decays exponentially as it penetrates the material:

$$\frac{dE}{E} = -[N]\sigma_{\text{rad}}dx \to E(x) = E(0) \exp \left(-[N]\sigma_{\text{rad}}(Z)x\right)$$

If the material is a uniform mixture of different atoms and one makes the reasonable assumption that the electron interacts with only one atom at a time, then the cumulative energy loss is given by:

$$E(x) = E(0)\Pi_k \exp \left(-[N]_k\sigma_{\text{rad}}(Z_k)x\right) = E(0) \exp \left(-\sum_k [N]_k\sigma_{\text{rad}}(Z_k)x\right)$$

where $k$ labels the atomic isotope. It is convenient to characterize a material by the thickness required for an electron to lose $1 - 1/e$ of its energy. This parameter is called the radiation length and is given by:

$$X_0 = \left[ \sum_k [N]_k\sigma_{\text{rad}}(Z_k) \right]^{-1}$$

In practice, it’s more convenient to work with mass densities $\rho$ then with number densities $[N]$:

$$X_0 = \frac{1}{[N]\sigma_{\text{rad}}(Z)} = \frac{A}{\rho N_A\sigma_{\text{rad}}(Z)} = \frac{X_0}{\rho}$$

where $A$ is the molecular weight of the isotope and $N_A$ is the Avogadro constant. Note that even though $X_0$ and $X_0$ have different units, they are interchangeably called the radiation length in the literature [7, 9].
We will distinguish the two by a bar and their units. For a single isotope, the radiation length in mass per unit “area” is given by:

\[ X_0 = \frac{A}{N_A \sigma_{\text{rad}}(Z)} \]  

(14)

For a composite material, the radiation length in mass per unit “area” is given by the sum:

\[ X_0^{-1} = \sum_k w_k \frac{N_A \sigma_{\text{rad}}(Z_k)}{A_k} = \sum_k \frac{w_k}{X_0^k} \]  

(15)

where \( w_k \) is the fraction by mass for the \( k \)-th isotope. Finally the unitless radiation thickness is given by:

\[ t = \sum_j \rho_j \ell_j X_0^j \]  

(16)

where the sum over \( j \) runs over each composite material with physical thickness \( \ell_j \) and mass density \( \rho_j \).

2 General Formulas for Calculating Collisional Energy Loss

Electrons can undergo elastic collisions with atomic electrons within the materials along the beam path. Very often these collisions result in the ionization of the struck atom. Consequently the process is interchangeably called “ionization loss,” “collisional loss,” and “loss to ionizing collisions.” We will also use these terms interchangeably. The mean energy loss is given by the celebrated Bethe-Bloch equation, while the most probable energy loss was first calculated by Landau. Both equations have the form of energy lost to collisions per unit mass density per unit length [9]:

\[ \frac{\Delta \rho x}{\rho x} = \left[ \frac{\xi}{\rho x} \right] 2 \log \left( \frac{pc}{I} \right) - \delta(X) + g \]  

(17)

\[ \frac{\xi}{\rho x} = \frac{Za}{A \beta^2} \]  

(18)

\[ a = 2\pi N_A r_e^2 m_e c^2 = 0.15353747 \text{ MeV} \cdot \text{cm}^2/\text{mol} \]  

(19)

where \( Z \) & \( A \) are the effective atomic number and weight (in g/mol) of the material, \( p \) & \( E \) are the electron’s momentum and energy, \( \rho \) & \( x \) are the material’s mass density and thickness, \( I \) is the mean excitation potential of the material, \( \delta(X) \) is the density correction [10], and we’ll call \( \xi \) the “collisional” thickness as used in TSAI71. The specific form of \( g \) depends on which energy loss is desired:

\[ \bar{g} = \log(\gamma - 1) - F(\gamma) \quad \text{mean energy loss (Bethe – Bloch)} \]  

(20)

\[ g_{mp} = \log \left[ \frac{25}{m_e c^2 \gamma^2} \right] - \beta^2 + 0.198 \quad \text{most probable (Landau)} \]  

(21)

\[ F(\gamma) = \left[ 1 + \frac{2}{\gamma} - \frac{1}{\gamma^2} \right] \log(2) - \frac{1}{8} \left[ 1 - \frac{1}{\gamma} \right]^2 - \frac{1}{\gamma^2} \approx \log(2) - \frac{1}{8} = 0.568 \quad \text{for } \gamma \gg 1 \]  

(22)

\[ \beta = \frac{v}{c} = \frac{pc}{E} \]  

(23)

\[ \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m_e c^2} \]  

(24)

Note that constant term in \( g_{mp} \) was originally calculated by Landau to be 0.373 and subsequently recalculated more accurately [11] to be 0.198. The energy lost per unit length can be written alternatively as:

\[ \frac{dE}{dx} = -(m_e c^2)[N] \sigma_{\text{coll}} \]  

(25)

\[ \sigma_{\text{coll}} = 2\pi r_e^2 \left( \frac{Z}{\beta^2} \right) B = \frac{A}{N_A m_e c^2} \left[ \frac{\xi}{\rho x} \right] B \]  

(26)

\[ B = \left[ 2 \log \left( \frac{pc}{I} \right) - \delta(X) + g \right] \]  

(27)
where $\sigma_{\text{coll}}$ is the collisional cross section and $B$ is called the stopping number when $g = g$.

The density correction $\delta$ is given by [12]:

$$
\delta(X) = \begin{cases} 
\delta(X_0^\delta) \times 10^{2(X - X_0^\delta)} & X \leq X_0^\delta \\
2 \log(10) (X - X_0^\delta) + a_\delta (X_0^\delta - X)^{m_\delta} & X_0^\delta < X < X_1^\delta \\
2 \log(10) (X - X_1^\delta) & X \geq X_1^\delta 
\end{cases}
$$

(28)

$$
X = \log_{10} \left( \frac{p}{m_e c} \right)
$$

(29)

$$
X_\alpha^\delta = -\frac{C_\delta}{2 \log(10)}
$$

(30)

$$
C_\delta = 2 \log \left( \frac{\hbar \omega_p}{I} \right) - 1
$$

(31)

$$
a_\delta = \frac{\delta(X_0^\delta) + 2 \log(10) (X_\alpha^\delta - X_0^\delta)}{(X_1^\delta - X_0^\delta)^{m_\delta}}
$$

(32)

$$
\delta(X_0^\delta) = \begin{cases} 
0.0 & \text{insulators} \\
0.06, 0.08, 0.10, 0.12, \text{ or } 0.14 & \text{conductors} 
\end{cases}
$$

(33)

where $\omega_p, X_0^\delta, X_1^\delta, m_\delta$, and $\delta(X_0^\delta)$ depend on the material. The density correction below $X_0^\delta$ depends on whether the material is an insulator or conductor [13]. The plasma frequency [14] is a function of the electron number density in the material, $[N_e]$:

$$
\omega_p = \sqrt{4 \pi [N_e] r_e c^2}
$$

(34)

$$
[N_e] = \langle Z \rangle [N] = \frac{\langle Z \rangle N_A \rho}{\langle A \rangle} = N_A \rho \left( \frac{Z}{A} \right)
$$

(35)

Note that above $X \geq X_1^\delta$, the mean energy loss becomes depends only logrithamically on energy and the most probable loss becomes independant of energy:

$$
\frac{p}{m_e c} \geq \frac{p_1}{m_e c} = 10 X_1^\delta
$$

(36)

$$
\delta(p > p_1) = 2 \log(10) (X - X_\alpha^\delta) = 2 \log \left( \frac{p}{p_a} \right)
$$

(37)

$$
\frac{p_a}{m_e c} = 10 X_\alpha^\delta = \exp \left( -\frac{C_\delta}{2} \right) = \left( \frac{I}{\hbar \omega_p} \right)^2 \sqrt{e}
$$

(38)

$$
\left[ \frac{\Delta}{\rho x} \right] = \left[ \frac{\xi}{\rho x} \right] \left[ 2 \log \left( \frac{p_a c}{T} \right) + g \right]
$$

(39)

We use the density correction parameters from [12]. If the density of a material is different than that listed in [12], then the following substitutions are made [14]:

$$
X_\alpha^{\delta'} = X_\alpha^\delta - \frac{1}{2} \log_{10} \left( \frac{[N]}{[N_0]} \right)
$$

(40)

$$
X_0^{\delta'} = X_0^\delta - \frac{1}{2} \log_{10} \left( \frac{[N]}{[N_0]} \right)
$$

(41)

$$
X_1^{\delta'} = X_1^\delta - \frac{1}{2} \log_{10} \left( \frac{[N]}{[N_0]} \right)
$$

(42)

where $[N_0]$ is the number density listed in table and $[N]$ is the desired number density. This is particularly useful for gases because table blah uses the density for 20°C and 1 atm.

If the material is not listed, then one must calculate effective values for $I \& Z/A$ using Bragg’s additivity law. If the constituents are elements, the mean ionization potential is underestimated because electrons are
more tightly bound in molecules than in the free atoms of the constituent elements. Therefore we’ll use the “13 percent rule” to compensate [15]. If the constituents are molecules such as various types of glass, we do not increase the ionization potentials. This is because the binding effect was accounted for when the molecular ionization potential were calculated from their constituent elements. Consequently the effective \( Z/A \) values are given by [15]:

\[
\langle \frac{Z}{A} \rangle = \frac{\sum_i n_i Z_i}{\sum_i n_i A_i} = \sum_i w_i \langle \frac{Z}{A} \rangle_i \quad (43)
\]

\[
\log \langle I' \rangle = \sum_i \frac{n_i Z_i}{Z} \log (I'_i) = \langle \frac{A}{Z} \rangle \sum_i w_i \langle \frac{Z}{A} \rangle_i \log (I_i) \quad (44)
\]

where \( n_i \) are the number of atoms of the \( i \)-th type in the molecule, \( I' \) is the modified elemental ionization potential, \( w_i \) is the fraction of the \( i \)-th molecule by weight in the mixture, and \( I \) is the molecular ionization potential. Once the effective plasma frequency, ionization potential, and \( Z/A \) for the material have been calculated, then one uses Tab. (1) to select appropriate value of \( X^4_{a,0,1} \). Finally the density correction \( \delta \), collisional thickness \( \xi \), and energy loss are calculated only after first deriving effective density correction parameters for a given material.

### 3 Materials in the Path of the Beam

The electron beam exits the beam pipe and enters the target region through a 5 mil beryllium window. The target region is enclosed in a helium “bag.” For now, we will assume that the helium gas displaces all of the air and that its temperature & pressure are 20°C & and 1 atm. The electron beam then enters the target chamber portion of the cell which is made of C1720 glass. The end “windows” are much thinner than the side “walls.” About 10 amagats of polarized \(^3\)He and 0.1 amag of unpolarized \( \text{N}_2 \) fill the target chamber. For now, we’ll also make the traditional approximation that the scattering occurs at the center of the target. This effects the thickness of polarized \(^3\)He, unpolarized \( \text{N}_2 \), and glass wall traversed by the scattered electron.

After scattering, the electron passes through more polarized \(^3\)He, unpolarized \( \text{N}_2 \), and the side wall of the cell. The distance travelled through these two materials after scattering is inversely proportional to

<table>
<thead>
<tr>
<th>Phase of Material and Conditions</th>
<th>( X^4_{0} )</th>
<th>( X^4_{1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I &lt; 100 \text{ eV} )</td>
<td>( C_\delta &gt; -3.681 )</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>( C_\delta \leq -3.681 )</td>
<td>-0.326( C_\delta )</td>
</tr>
<tr>
<td>( I \geq 100 \text{ eV} )</td>
<td>( C_\delta &gt; -5.215 )</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>( C_\delta \leq -5.215 )</td>
<td>-0.326( C_\delta )</td>
</tr>
<tr>
<td>Gases, 20°C at 1 atm</td>
<td>(-10.000 &lt; C_\delta )</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>(-10.500 &lt; C_\delta \leq -10.000 )</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(-11.000 &lt; C_\delta \leq -10.500 )</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>(-11.500 &lt; C_\delta \leq -11.000 )</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>(-12.250 &lt; C_\delta \leq -11.500 )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>(-13.804 &lt; C_\delta \leq -12.250 )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>(-13.804 \leq C_\delta )</td>
<td>-0.326( C_\delta )</td>
</tr>
</tbody>
</table>

Table 1: General Parameterization of the Density Correction [14, 15]. \( C_\delta \) is calculated from the plasma frequency. For all cases \( m_\delta = 3.0 \) and \( X^4_{a} \& a_\delta \) are calculated using equations blah.
the sine of the central scattering angle. After exiting the cell, the electrons travel out of the helium “bag” and through the sieve slit box. At this point the electrons enter the bore of the septum magnet. Both the entrance and exit apertures of the septum bore were covered by 2 inch thick pieces of polystyrene “foam.” The density of this foam is discussed in detail in the next section. The sieve slit box is filled with helium gas. However there is some ambiguity regarding whether the septum bore and the small amount of space between the septum and the HRS were filled with air or helium gas.

The spacing of the sieve slit box and septum magnet were almost certainly changed during the six to nine degree switch. In addition, the path of the central ray within the septum bore is different between six and nine degrees. The only consequence of these two differences is that the amount of helium gas traversed by the scattered electron is different. Since this makes up only a tiny fraction of the radiation thickness, we will assume the geometry for nine degrees is identical to the geometry for six degrees. Finally, according to E. Folts, the electrons enter the HRS through a 10 mil Kapton window.

4 Density of the Polystyrene Foam

Solid polystyrene, used in scintillators, has a density of $1.032 \text{ g/cm}^3$ [7]. The density of the polystyrene foam that we used was measured independently by V. Sulkosky and A. Deur:

$$\rho = \left( \frac{0.03151 + 0.03220}{2} \right) \text{g/cm}^3 = 0.03186 \text{ g/cm}^3$$  \hspace{1cm} (45)

Clearly a large fraction of the polystyrene foam is occupied by some unknown gas. In principle, this gas affects the radiation length of the foam:

$$\frac{1}{X_0} = \frac{w_p}{X_p} + \frac{w_g}{X_g}$$  \hspace{1cm} (46)

where $w$ is the fraction by mass and the subscripts $p$ & $g$ refer to the polystyrene & the gas. Taking advantage of $w_p + w_g = 1$ and $w_g = \rho_g/\rho$, we find:

$$\frac{X_p}{X_0} - 1 = \frac{\rho_g}{\rho} \left[ \frac{X_p}{X_g} - 1 \right]$$  \hspace{1cm} (47)

If the gas is at room temperature and 1 atm, then the gas density is on order of $10^{-3} \text{ g/cm}^3$ and consequently the true radiation length of the foam differs from the radiation length of polystyrene at the level of a fraction of a percent. Since the difference in the two measurements for the density of the foam is larger than the estimated correction due to the trapped gas, we neglect it’s effect on the radiation length:

$$\left[ \frac{\rho}{X_0} \right]_{\text{foam}} \approx \frac{\rho_{\text{foam}}}{X_{\text{polystyrene}}}$$  \hspace{1cm} (48)
Figure 1: Scaled Geometry of Target Region.
$\alpha^{-1} = 137.0359991$ cm/s
$c = 29979245800$ cm/s
$r_e = 2.817940325 \times 10^{-13}$ cm
$m_e c^2 = 0.510998918$ MeV
$h = 6.58211915 \times 10^{-16}$ eV·s

$R = 8.314472$ J/mol/K
$N_A = 6.0221415 \times 10^{23}$ \#/mol
1 ang = $2.6867773 \times 10^{19}$ \#/cm$^3$
1 ang = $4.4614981 \times 10^{-5}$ mol/cm$^3$
1 mil = 0.001 inch

Table 2: Physical Constants and Conversions. [16]

<table>
<thead>
<tr>
<th>$Z$</th>
<th>atom</th>
<th>$A$ (amu)</th>
<th>$\sigma_{\text{rad}}$ (barns)</th>
<th>$X_0$ (g/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>1.00794</td>
<td>0.02655</td>
<td>63.0435</td>
</tr>
<tr>
<td>2</td>
<td>$^3$He</td>
<td>3.01603</td>
<td>0.07047</td>
<td>71.0736</td>
</tr>
<tr>
<td>2</td>
<td>He</td>
<td>4.00260</td>
<td>0.07047</td>
<td>94.3224</td>
</tr>
<tr>
<td>4</td>
<td>Be</td>
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<td>0.22956</td>
<td>65.1900</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>10.81100</td>
<td>0.34073</td>
<td>52.6868</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>12.01070</td>
<td>0.46711</td>
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</tr>
<tr>
<td>7</td>
<td>N</td>
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<td>0.61226</td>
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<td>Mg</td>
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<td>1.61235</td>
<td>25.0315</td>
</tr>
<tr>
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<td>Al</td>
<td>26.98154</td>
<td>1.86596</td>
<td>24.0112</td>
</tr>
<tr>
<td>14</td>
<td>Si</td>
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<td>2.13706</td>
<td>21.8231</td>
</tr>
<tr>
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<td>K</td>
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</tr>
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</tr>
<tr>
<td>33</td>
<td>As</td>
<td>74.92160</td>
<td>10.41949</td>
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</tr>
<tr>
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<td>Sr</td>
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<td>13.51897</td>
<td>10.7624</td>
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<tr>
<td>56</td>
<td>Ba</td>
<td>137.32700</td>
<td>27.45233</td>
<td>8.3066</td>
</tr>
</tbody>
</table>

Table 3: Radiation Length by Atomic Species.
Table 4: Density Corrections Parameters for Elemental Materials. \( I' \) are the ionization potentials to be used when calculating effective molecular ionization potentials. The densities are given for the natural form of the element (gas, diatomic gas, liquid, solid) at 1 atm and 20°C.

<table>
<thead>
<tr>
<th>( Z/A )</th>
<th>( I ) (eV)</th>
<th>( I' ) (g) (eV)</th>
<th>( I' ) (l/s) (eV)</th>
<th>( \rho ) (g/cm(^3))</th>
<th>(-C)</th>
<th>( X_0 )</th>
<th>( X_1 )</th>
<th>( a )</th>
<th>( m )</th>
<th>( \delta(X_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>He 0.9922</td>
<td>19.2</td>
<td>19.2</td>
<td>19.2</td>
<td>8.3748E-05</td>
<td>9.5835</td>
<td>1.8639</td>
<td>3.2718</td>
<td>0.14095</td>
<td>5.7273</td>
<td>0.00</td>
</tr>
<tr>
<td>He 0.4997</td>
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<td>41.8</td>
<td>47.2</td>
<td>1.6632E-04</td>
<td>11.1393</td>
<td>2.2017</td>
<td>3.6122</td>
<td>0.13443</td>
<td>5.8347</td>
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</tr>
<tr>
<td>Be 0.6631</td>
<td>63.7</td>
<td>63.7</td>
<td>72.0</td>
<td>1.8480E+00</td>
<td>2.7847</td>
<td>0.0592</td>
<td>1.6922</td>
<td>0.76146</td>
<td>2.4339</td>
<td>0.14</td>
</tr>
<tr>
<td>B 0.4625</td>
<td>76.0</td>
<td>76.0</td>
<td>85.9</td>
<td>2.3700E+00</td>
<td>2.8477</td>
<td>0.0305</td>
<td>1.9688</td>
<td>0.56221</td>
<td>2.4512</td>
<td>0.14</td>
</tr>
<tr>
<td>C 0.4995</td>
<td>78.0</td>
<td>70.0</td>
<td>81.0</td>
<td>2.0000E+00</td>
<td>2.9925</td>
<td>-0.0351</td>
<td>2.4860</td>
<td>0.20489</td>
<td>3.0036</td>
<td>0.10</td>
</tr>
<tr>
<td>N 0.4998</td>
<td>82.0</td>
<td>82.0</td>
<td>82.0</td>
<td>1.1653E-03</td>
<td>10.5400</td>
<td>1.7378</td>
<td>4.1323</td>
<td>0.15955</td>
<td>3.2125</td>
<td>0.00</td>
</tr>
<tr>
<td>O 0.5000</td>
<td>95.0</td>
<td>97.0</td>
<td>106.0</td>
<td>1.3315E-03</td>
<td>10.7004</td>
<td>1.7541</td>
<td>4.3213</td>
<td>0.11778</td>
<td>3.2913</td>
<td>0.00</td>
</tr>
<tr>
<td>Na 0.4785</td>
<td>149.0</td>
<td>149.0</td>
<td>168.4</td>
<td>9.7100E-01</td>
<td>5.0526</td>
<td>0.2880</td>
<td>3.1962</td>
<td>0.07608</td>
<td>3.6452</td>
<td>0.08</td>
</tr>
<tr>
<td>Mg 0.4937</td>
<td>156.0</td>
<td>156.0</td>
<td>176.3</td>
<td>1.7400E+00</td>
<td>4.5297</td>
<td>0.1499</td>
<td>3.0668</td>
<td>0.08162</td>
<td>3.6166</td>
<td>0.08</td>
</tr>
<tr>
<td>Al 0.4818</td>
<td>166.0</td>
<td>166.0</td>
<td>187.6</td>
<td>2.6989E+00</td>
<td>4.2395</td>
<td>0.1708</td>
<td>3.0127</td>
<td>0.07934</td>
<td>3.6345</td>
<td>0.12</td>
</tr>
<tr>
<td>Si 0.4985</td>
<td>173.0</td>
<td>173.0</td>
<td>195.5</td>
<td>2.3300E+00</td>
<td>4.4351</td>
<td>0.2014</td>
<td>2.8175</td>
<td>0.14840</td>
<td>3.2546</td>
<td>0.14</td>
</tr>
<tr>
<td>K 0.4860</td>
<td>190.0</td>
<td>190.0</td>
<td>214.7</td>
<td>8.6200E-01</td>
<td>5.6423</td>
<td>0.3851</td>
<td>3.1724</td>
<td>0.20027</td>
<td>2.9233</td>
<td>0.10</td>
</tr>
<tr>
<td>Ca 0.4990</td>
<td>191.0</td>
<td>191.0</td>
<td>215.8</td>
<td>1.5500E+00</td>
<td>5.0396</td>
<td>0.3228</td>
<td>3.1191</td>
<td>0.15475</td>
<td>3.0745</td>
<td>0.14</td>
</tr>
<tr>
<td>As 0.4405</td>
<td>347.0</td>
<td>347.0</td>
<td>392.1</td>
<td>5.7300E+00</td>
<td>5.0510</td>
<td>0.1767</td>
<td>3.5702</td>
<td>0.06725</td>
<td>3.4176</td>
<td>0.08</td>
</tr>
<tr>
<td>Sr 0.4337</td>
<td>366.0</td>
<td>366.0</td>
<td>413.6</td>
<td>2.5400E+00</td>
<td>5.9867</td>
<td>0.4585</td>
<td>3.6778</td>
<td>0.07058</td>
<td>3.4435</td>
<td>0.14</td>
</tr>
<tr>
<td>Ba 0.4078</td>
<td>491.0</td>
<td>491.0</td>
<td>554.8</td>
<td>3.5000E+00</td>
<td>6.3153</td>
<td>0.4190</td>
<td>3.4547</td>
<td>0.18267</td>
<td>2.8906</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table 5: Radiation Length of Polymers. [7, 17]

<table>
<thead>
<tr>
<th>material</th>
<th>by wt.</th>
<th>( X_0 ) (g/cm(^2))</th>
<th>by ( X_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C by wt.</td>
<td>0.922582</td>
<td>0.691133</td>
<td></td>
</tr>
<tr>
<td>O by wt.</td>
<td>-</td>
<td>0.209235</td>
<td></td>
</tr>
<tr>
<td>N by wt.</td>
<td>-</td>
<td>0.073270</td>
<td></td>
</tr>
<tr>
<td>H by wt.</td>
<td>0.077418</td>
<td>0.026362</td>
<td></td>
</tr>
<tr>
<td>( X_0 ) (g/cm(^2))</td>
<td>43.7911</td>
<td>40.5761</td>
<td></td>
</tr>
</tbody>
</table>

\[
X_0 = 26.8379 \text{ g/cm}^2 \\
\rho = 2.53 \text{ g/cm}^3 \\
\bar{X}_0 = 10.608 \text{ cm}
\]

Table 6: Radiation Length of Corning 1729 (C1720). [18]

<table>
<thead>
<tr>
<th>material</th>
<th>by wt.</th>
<th>( X_0 ) (g/cm(^2))</th>
<th>by ( X_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO(_2)</td>
<td>0.599</td>
<td>27.0460</td>
<td>0.594</td>
</tr>
<tr>
<td>Al(_2)O(_3)</td>
<td>0.182</td>
<td>27.9399</td>
<td>0.175</td>
</tr>
<tr>
<td>CaO</td>
<td>0.074</td>
<td>19.0098</td>
<td>0.104</td>
</tr>
<tr>
<td>MgO</td>
<td>0.088</td>
<td>28.0227</td>
<td>0.084</td>
</tr>
<tr>
<td>B(_2)O(_3)</td>
<td>0.047</td>
<td>38.4158</td>
<td>0.033</td>
</tr>
<tr>
<td>Na(_2)O</td>
<td>0.010</td>
<td>29.1660</td>
<td>0.009</td>
</tr>
<tr>
<td>K(_2)O</td>
<td>0.000</td>
<td>18.9019</td>
<td>negligible</td>
</tr>
<tr>
<td>As(_2)O(_3)</td>
<td>0.000</td>
<td>14.1807</td>
<td>negligible</td>
</tr>
</tbody>
</table>

\[
X_0 = 19.4246 \text{ g/cm}^2 \\
\rho = 2.76 \text{ g/cm}^3 \\
\bar{X}_0 = 7.038 \text{ cm}
\]

Table 7: Radiation Length of GE180.
<table>
<thead>
<tr>
<th>Z/A</th>
<th>I [eV]</th>
<th>ρ [g/cm³]</th>
<th>−C</th>
<th>X₀</th>
<th>X₁</th>
<th>a</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al₂O₃</td>
<td>0.4904</td>
<td>145.2</td>
<td>3.9700E+00</td>
<td>3.5682</td>
<td>0.0402</td>
<td>2.8665</td>
<td>0.08499</td>
</tr>
<tr>
<td>As₂O₃</td>
<td>0.4549</td>
<td>276.6</td>
<td>3.8600E+00</td>
<td>4.9606</td>
<td>0.2000</td>
<td>3.0000</td>
<td>0.18402</td>
</tr>
<tr>
<td>BaO</td>
<td>0.4174</td>
<td>451.1</td>
<td>5.7000E+00</td>
<td>5.6349</td>
<td>0.3370</td>
<td>3.0000</td>
<td>0.21620</td>
</tr>
<tr>
<td>B₂O₃</td>
<td>0.4884</td>
<td>99.6</td>
<td>1.8120E+00</td>
<td>3.6027</td>
<td>0.1843</td>
<td>2.7379</td>
<td>0.11547</td>
</tr>
<tr>
<td>CaO</td>
<td>0.4993</td>
<td>176.1</td>
<td>3.3000E+00</td>
<td>4.1209</td>
<td>-0.0172</td>
<td>3.0171</td>
<td>0.12127</td>
</tr>
<tr>
<td>MgO</td>
<td>0.4962</td>
<td>143.8</td>
<td>3.5800E+00</td>
<td>3.6404</td>
<td>0.0575</td>
<td>2.8580</td>
<td>0.08313</td>
</tr>
<tr>
<td>K₂O</td>
<td>0.4883</td>
<td>189.9</td>
<td>2.3200E+00</td>
<td>4.6463</td>
<td>0.0480</td>
<td>3.0110</td>
<td>0.16789</td>
</tr>
<tr>
<td>SiO₂</td>
<td>0.4993</td>
<td>139.2</td>
<td>2.3200E+00</td>
<td>4.0029</td>
<td>0.1385</td>
<td>3.0025</td>
<td>0.08407</td>
</tr>
<tr>
<td>Na₂O</td>
<td>0.4840</td>
<td>148.8</td>
<td>2.2700E+00</td>
<td>4.1892</td>
<td>0.1652</td>
<td>2.9793</td>
<td>0.07501</td>
</tr>
<tr>
<td>SrO</td>
<td>0.4439</td>
<td>326.4</td>
<td>5.7000E+00</td>
<td>4.9259</td>
<td>0.2000</td>
<td>3.0000</td>
<td>0.18244</td>
</tr>
<tr>
<td>C1720</td>
<td>0.4967</td>
<td>141.0</td>
<td>2.5300E+00</td>
<td>3.9477</td>
<td>0.2000</td>
<td>3.0000</td>
<td>0.13788</td>
</tr>
<tr>
<td>GE180</td>
<td>0.4829</td>
<td>171.8</td>
<td>2.7600E+00</td>
<td>4.2834</td>
<td>0.2000</td>
<td>3.0000</td>
<td>0.15317</td>
</tr>
<tr>
<td>kapton</td>
<td>0.5126</td>
<td>79.6</td>
<td>1.4200E+00</td>
<td>3.3497</td>
<td>0.1509</td>
<td>2.5631</td>
<td>0.15971</td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.5377</td>
<td>68.7</td>
<td>1.0600E+00</td>
<td>3.2999</td>
<td>0.1647</td>
<td>2.5031</td>
<td>0.16454</td>
</tr>
</tbody>
</table>

Table 8: Density Correction Parameters for Composite Materials. For all these materials δ(X₀) = 0.

<table>
<thead>
<tr>
<th>Z/A</th>
<th>I [eV]</th>
<th>ρ [g/cm³]</th>
<th>−C</th>
<th>X₀</th>
<th>X₁</th>
<th>a</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>³He</td>
<td>0.6631</td>
<td>41.8</td>
<td>1.4330E-03</td>
<td>1.6726</td>
<td>3.0831</td>
<td>0.13443</td>
<td>5.8347</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.4080E-03</td>
<td>1.6764</td>
<td>3.0869</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>1.6758</td>
<td>3.0863</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.3370E-04</td>
<td>2.2080</td>
<td>4.6025</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.3870E-04</td>
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<td>4.5945</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.4000E-04</td>
<td>2.1980</td>
<td>4.5925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N₂</td>
<td>0.4998</td>
<td>82.0</td>
<td>1.05400</td>
<td>1.2342</td>
<td>3.6287</td>
<td>0.15350</td>
<td>3.2125</td>
</tr>
<tr>
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<td>1.1850E-02</td>
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<td>3.6283</td>
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<td>1.2366</td>
<td>3.6311</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.1720E-02</td>
<td>1.2366</td>
<td>3.6311</td>
<td></td>
<td></td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.5377</td>
<td>68.7</td>
<td>3.1860E-02</td>
<td>3.2999</td>
<td>0.9257</td>
<td>3.2641</td>
<td>0.16454</td>
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</tbody>
</table>

Table 9: Density Correction Parameters for Materials at Different Mass Densities.
<table>
<thead>
<tr>
<th>cell</th>
<th>Penelope</th>
<th>Ref. Cell 1</th>
<th>Priapus</th>
<th>Ref. Cell 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>window, upstream (μm)</td>
<td>132 ± 2</td>
<td>127 ± 2</td>
<td>128 ± 2</td>
<td>131 ± 2</td>
</tr>
<tr>
<td></td>
<td>138 ± 2</td>
<td>120 ± 2</td>
<td>142 ± 2</td>
<td>150 ± 2</td>
</tr>
<tr>
<td>window, downstream (μm)</td>
<td>622 ± 10</td>
<td>638 ± 10</td>
<td>600 ± 10</td>
<td>610 ± 10</td>
</tr>
<tr>
<td></td>
<td>694 ± 10</td>
<td>693 ± 10</td>
<td>760 ± 10</td>
<td>711 ± 10</td>
</tr>
<tr>
<td>wall, beam right (μm)</td>
<td>1.9249</td>
<td>1.920</td>
<td>1.9160</td>
<td>1.920</td>
</tr>
<tr>
<td>wall, beam left (μm)</td>
<td>10.65</td>
<td>-</td>
<td>10.46/10.49</td>
<td>-</td>
</tr>
<tr>
<td>outer diameter (cm)</td>
<td>0.107</td>
<td>9.48</td>
<td>0.111/0.112</td>
<td>9.50/9.38</td>
</tr>
<tr>
<td>[3He] (amg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[N2] (amg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Cell Target Chamber Parameters. Densities for Priapus refer to 6/9 degrees.
<table>
<thead>
<tr>
<th>material</th>
<th>$X_0$</th>
<th>$\rho$</th>
<th>$\bar{X}_0$</th>
<th>$\ell$</th>
<th>$t$</th>
<th>fraction of tot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm²</td>
<td>g/cm³</td>
<td>cm</td>
<td>cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C1720 Window</td>
<td>26.8379</td>
<td>2.530E+00</td>
<td>1.0608E+01</td>
<td>1.320E-02</td>
<td>1.244E-03</td>
<td>0.5879</td>
</tr>
<tr>
<td>Polarized $^3$He</td>
<td>71.0736</td>
<td>1.433E-03</td>
<td>4.9596E+04</td>
<td>1.983E+01</td>
<td>3.998E-04</td>
<td>0.1889</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>65.1900</td>
<td>1.848E+00</td>
<td>3.5276E+01</td>
<td>1.270E-02</td>
<td>3.600E-04</td>
<td>0.1701</td>
</tr>
<tr>
<td>Unpolarized N₂</td>
<td>37.9879</td>
<td>1.337E-04</td>
<td>2.8406E+05</td>
<td>1.983E+01</td>
<td>6.981E-05</td>
<td>0.0330</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>94.3224</td>
<td>1.664E-04</td>
<td>5.6687E+05</td>
<td>2.409E+01</td>
<td>4.250E-05</td>
<td>0.0201</td>
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<tr>
<td><strong>Total Before</strong></td>
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<td></td>
<td></td>
<td></td>
<td><strong>2.116E-03</strong></td>
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<tr>
<td>C1720 Wall</td>
<td>26.8379</td>
<td>2.530E+00</td>
<td>1.0608E+01</td>
<td>5.951E-01</td>
<td>5.610E-02</td>
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<tr>
<td>Polystyrene Foam</td>
<td>43.7911</td>
<td>3.186E-02</td>
<td>1.3745E+03</td>
<td>1.016E+01</td>
<td>7.392E-03</td>
<td>0.1139</td>
</tr>
<tr>
<td>Kapton Window</td>
<td>40.5761</td>
<td>1.420E+00</td>
<td>2.8575E+01</td>
<td>2.540E-02</td>
<td>8.889E-04</td>
<td>0.0137</td>
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<tr>
<td>Helium Gas</td>
<td>94.3224</td>
<td>1.664E-04</td>
<td>5.6687E+05</td>
<td>1.912E+02</td>
<td>3.373E-04</td>
<td>0.0052</td>
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<tr>
<td>Polarized $^3$He</td>
<td>71.0736</td>
<td>1.433E-03</td>
<td>4.9596E+04</td>
<td>8.612E+00</td>
<td>1.737E-04</td>
<td>0.0027</td>
</tr>
<tr>
<td>Unpolarized N₂</td>
<td>37.9879</td>
<td>1.337E-04</td>
<td>2.8406E+05</td>
<td>8.612E+00</td>
<td>3.032E-05</td>
<td>0.0005</td>
</tr>
<tr>
<td><strong>Total After</strong></td>
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<td></td>
<td></td>
<td></td>
<td><strong>6.492E-02</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>material</th>
<th>$X_0$</th>
<th>$\rho$</th>
<th>$\bar{X}_0$</th>
<th>$\ell$</th>
<th>$t$</th>
<th>fraction of tot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm²</td>
<td>g/cm³</td>
<td>cm</td>
<td>cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>3.2062E+03</td>
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Table 11: Radiation Thicknesses Before & After Scattering from Penelope at 6 deg.

Table 12: Radiation Thicknesses Before & After Scattering from Ref. Cell 1 at 6 deg.
<table>
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<tr>
<th>material</th>
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<th>$\bar{X}_0$</th>
<th>$\ell$</th>
<th>$t$</th>
<th>fraction of tot</th>
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<td>26.8379</td>
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<td>1.0608E+01</td>
<td>1.280E-02</td>
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<td>1.408E-03</td>
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<td>1.967E+01</td>
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<td>3.5276E+01</td>
<td>1.270E-02</td>
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Table 13: Radiation Thicknesses Before & After Scattering from Priapus at 6 deg.

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<th>$X_0$</th>
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<th>$\bar{X}_0$</th>
<th>$\ell$</th>
<th>$t$</th>
<th>fraction of tot</th>
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<tbody>
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<td>1.0608E+01</td>
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<td>1.848E+00</td>
<td>3.5276E+01</td>
<td>1.270E-02</td>
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Table 14: Radiation Thicknesses Before & After Scattering from Ref. Cell 2 at 6 deg.
### Table 15: Radiation Thicknesses Before & After Scattering from Priapus at 9 deg.

<table>
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<th>$\ell$</th>
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<td>3.600E−04</td>
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<td>2.7138E+05</td>
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<td>5.6687E+05</td>
<td>2.425E+01</td>
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<td>1.0608E+01</td>
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### Table 16: Radiation Thicknesses Before & After Scattering from Ref. Cell 2 at 9 deg.

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<th>fraction of tot</th>
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<td>g/cm²</td>
<td>g/cm³</td>
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<td>cm</td>
<td>-</td>
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<td>1.0608E+01</td>
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Table 17: Before ($p = 2134.3$ MeV) & After ($p = 1806.4$ MeV) Scattering from Penelope at 6 deg.

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<td>g/cm$^3$</td>
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<td>MeV</td>
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</tr>
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<td>C1720 Window</td>
<td>2.530E+00</td>
<td>1.270E-02</td>
<td>2.450E-03</td>
<td>0.036</td>
<td>0.069</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>1.848E+00</td>
<td>1.270E-02</td>
<td>1.599E-03</td>
<td>0.024</td>
<td>0.046</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>2.417E+01</td>
<td>3.086E-04</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td><strong>Total Before</strong></td>
<td></td>
<td></td>
<td></td>
<td>2.232E-02</td>
<td>0.466</td>
</tr>
<tr>
<td>C1720 Wall</td>
<td>2.530E+00</td>
<td>6.104E-01</td>
<td>1.178E-01</td>
<td>2.209</td>
<td>3.290</td>
</tr>
<tr>
<td>Polystyrene Foam</td>
<td>3.186E-02</td>
<td>1.016E+01</td>
<td>2.672E-02</td>
<td>0.577</td>
<td>0.861</td>
</tr>
<tr>
<td>Unpolarized N$_2$</td>
<td>1.185E-02</td>
<td>8.574E+00</td>
<td>7.795E-03</td>
<td>0.167</td>
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</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>1.912E+02</td>
<td>2.442E-03</td>
<td>0.060</td>
<td>0.092</td>
</tr>
<tr>
<td>Kapton Window</td>
<td>1.420E+00</td>
<td>2.540E-02</td>
<td>2.839E-03</td>
<td>0.044</td>
<td>0.081</td>
</tr>
<tr>
<td><strong>Total After</strong></td>
<td></td>
<td></td>
<td></td>
<td>1.576E-01</td>
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</tr>
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</table>

Table 18: Before ($p = 2134.3$ MeV) & After ($p = 1806.4$ MeV) Scattering from Ref. Cell 1 at 6 deg.
<table>
<thead>
<tr>
<th>material</th>
<th>$\rho$</th>
<th>$\ell$</th>
<th>$\xi$</th>
<th>mp</th>
<th>$dE$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm$^3$</td>
<td>cm</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
</tr>
<tr>
<td>Polarized $^3$He</td>
<td>1.408E-03</td>
<td>1.967E+01</td>
<td>2.819E-03</td>
<td>0.063</td>
<td>0.101</td>
</tr>
<tr>
<td>C1720 Window</td>
<td>2.530E+00</td>
<td>1.280E-02</td>
<td>2.470E-03</td>
<td>0.037</td>
<td>0.070</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>1.848E+00</td>
<td>1.270E-02</td>
<td>1.599E-03</td>
<td>0.024</td>
<td>0.046</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>2.425E+01</td>
<td>3.096E-04</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>Unpolarized N$_2$</td>
<td>1.387E-04</td>
<td>1.967E+01</td>
<td>2.094E-04</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td>Total Before</td>
<td></td>
<td></td>
<td>7.407E-03</td>
<td>0.136</td>
<td>0.237</td>
</tr>
</tbody>
</table>

| C1720 Wall       | 2.530E+00  | 5.740E-01 | 1.108E-01 | 2.071      | 3.102        |
| Polystyrene Foam | 3.186E-02  | 1.016E+01 | 2.672E-02 | 0.577      | 0.863        |
| Helium Gas       | 1.664E-04  | 1.913E+02 | 2.442E-03 | 0.060      | 0.092        |
| Kapton Window    | 1.420E+00  | 2.540E-02 | 2.839E-03 | 0.044      | 0.081        |
| Polarized $^3$He | 1.408E-03  | 8.591E+00 | 1.231E-03 | 0.026      | 0.043        |
| Unpolarized N$_2$| 1.387E-04  | 8.591E+00 | 9.146E-05 | 0.002      | 0.003        |
| Total After      |            |           | 1.441E-01 | 2.780      | 4.184        |

Table 19: Before ($p = 3145.3$ MeV) & After ($p = 1941.5$ MeV) Scattering from Priapus at 6 deg.

<table>
<thead>
<tr>
<th>material</th>
<th>$\rho$</th>
<th>$\ell$</th>
<th>$\xi$</th>
<th>mp</th>
<th>$dE$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm$^3$</td>
<td>cm</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
</tr>
<tr>
<td>Unpolarized N$_2$</td>
<td>1.187E-02</td>
<td>1.975E+01</td>
<td>1.800E-02</td>
<td>0.400</td>
<td>0.609</td>
</tr>
<tr>
<td>C1720 Window</td>
<td>2.530E+00</td>
<td>1.310E-02</td>
<td>2.528E-03</td>
<td>0.038</td>
<td>0.072</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>1.848E+00</td>
<td>1.270E-02</td>
<td>1.599E-03</td>
<td>0.024</td>
<td>0.046</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>2.417E+01</td>
<td>3.086E-04</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>Total Before</td>
<td></td>
<td></td>
<td>2.244E-02</td>
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<td>0.739</td>
</tr>
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</table>

| C1720 Wall       | 2.530E+00  | 5.836E-01 | 1.126E-01 | 2.107      | 3.154        |
| Polystyrene Foam | 3.186E-02  | 1.016E+01 | 2.672E-02 | 0.577      | 0.863        |
| Unpolarized N$_2$| 1.187E-02  | 8.601E+00 | 7.836E-03 | 0.168      | 0.261        |
| Helium Gas       | 1.664E-04  | 1.912E+02 | 2.442E-03 | 0.060      | 0.092        |
| Kapton Window    | 1.420E+00  | 2.540E-02 | 2.839E-03 | 0.044      | 0.081        |
| Total After      |            |           | 1.524E-01 | 2.956      | 4.451        |

Table 20: Before ($p = 3145.3$ MeV) & After ($p = 1941.5$ MeV) Scattering from Ref. Cell 2 at 6 deg.
<table>
<thead>
<tr>
<th>material</th>
<th>$\rho$</th>
<th>$\ell$</th>
<th>$\xi$</th>
<th>$mp$</th>
<th>$dE$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>g/cm³</td>
<td>cm</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
</tr>
<tr>
<td>Polarized $^3$He</td>
<td>1.412E-03</td>
<td>1.967E+01</td>
<td>2.827E-03</td>
<td>0.063</td>
<td>0.101</td>
</tr>
<tr>
<td>C1720 Window</td>
<td>2.530E+00</td>
<td>1.280E-02</td>
<td>2.470E-03</td>
<td>0.037</td>
<td>0.070</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>1.848E+00</td>
<td>1.270E-02</td>
<td>1.599E-03</td>
<td>0.024</td>
<td>0.046</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>2.425E+01</td>
<td>3.096E-04</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>Unpolarized N₂</td>
<td>1.400E-04</td>
<td>1.967E+01</td>
<td>2.113E-04</td>
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<td>0.008</td>
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<tr>
<td><strong>Total Before</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.417E-03</td>
<td>0.136</td>
<td>0.237</td>
<td></td>
<td></td>
</tr>
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<td>3.835E-01</td>
<td>7.400E-02</td>
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<td>2.075</td>
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<tr>
<td>Polystyrene Foam</td>
<td>3.186E-02</td>
<td>1.016E+01</td>
<td>2.672E-02</td>
<td>0.577</td>
<td>0.864</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>1.943E+02</td>
<td>2.481E-03</td>
<td>0.061</td>
<td>0.093</td>
</tr>
<tr>
<td>Kapton Window</td>
<td>1.420E+00</td>
<td>2.540E-02</td>
<td>2.839E-03</td>
<td>0.044</td>
<td>0.081</td>
</tr>
<tr>
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<td>1.412E-03</td>
<td>5.740E+00</td>
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<td>0.029</td>
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<tr>
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<td>1.400E-04</td>
<td>5.740E+00</td>
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<td>0.002</td>
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<td></td>
</tr>
<tr>
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<td>1.069E-01</td>
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</table>

Table 21: Before ($p = 3219.9$ MeV) & After ($p = 2007.0$ MeV) Scattering from Priapus at 9 deg.

<table>
<thead>
<tr>
<th>material</th>
<th>$\rho$</th>
<th>$\ell$</th>
<th>$\xi$</th>
<th>$mp$</th>
<th>$dE$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cm³</td>
<td>cm</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
</tr>
<tr>
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<td>1.172E-02</td>
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<td>1.777E-02</td>
<td>0.395</td>
<td>0.602</td>
</tr>
<tr>
<td>C1720 Window</td>
<td>2.530E+00</td>
<td>1.310E-02</td>
<td>2.528E-03</td>
<td>0.038</td>
<td>0.072</td>
</tr>
<tr>
<td>Beryllium Window</td>
<td>1.848E+00</td>
<td>1.270E-02</td>
<td>1.599E-03</td>
<td>0.024</td>
<td>0.046</td>
</tr>
<tr>
<td>Helium Gas</td>
<td>1.664E-04</td>
<td>2.417E+01</td>
<td>3.086E-04</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td><strong>Total Before</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.221E-02</td>
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<td>0.732</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>3.899E-01</td>
<td>7.524E-02</td>
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<td>2.110</td>
</tr>
<tr>
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<td>3.186E-02</td>
<td>1.016E+01</td>
<td>2.672E-02</td>
<td>0.577</td>
<td>0.864</td>
</tr>
<tr>
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<td>1.943E+02</td>
<td>2.480E-03</td>
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<td>0.093</td>
</tr>
<tr>
<td>Kapton Window</td>
<td>1.420E+00</td>
<td>2.540E-02</td>
<td>2.839E-03</td>
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<td>0.081</td>
</tr>
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<td><strong>Total After</strong></td>
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<td></td>
</tr>
<tr>
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<td>1.124E-01</td>
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Table 22: Before ($p = 3219.9$ MeV) & After ($p = 2007.0$ MeV) Scattering from Ref. Cell 2 at 9 deg.
<table>
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<th>angle</th>
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<th>before</th>
<th>after</th>
<th>total</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>deg.</td>
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<td>mp</td>
<td>dE</td>
<td>mp</td>
</tr>
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</tr>
<tr>
<td>penelope</td>
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<td>2134.3</td>
<td>0.138</td>
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</tr>
<tr>
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<td>400.0</td>
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<td>0.222</td>
<td>2.780</td>
</tr>
<tr>
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<td>2134.9</td>
<td>0.135</td>
<td>0.234</td>
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<td>0.135</td>
<td>0.236</td>
<td>2.780</td>
</tr>
<tr>
<td>priapus</td>
<td>6</td>
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<td>0.135</td>
<td>0.239</td>
<td>2.780</td>
</tr>
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<td>0.135</td>
<td>0.222</td>
<td>2.054</td>
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<tr>
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<td>0.235</td>
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<tr>
<td>priapus</td>
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<td>0.135</td>
<td>0.238</td>
<td>2.054</td>
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<tr>
<td>priapus</td>
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<td>0.135</td>
<td>0.239</td>
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<tr>
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<td>4404.2</td>
<td>0.135</td>
<td>0.240</td>
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<td>6</td>
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</tr>
<tr>
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</tr>
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<td>3775.5</td>
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<td>9</td>
<td>4404.2</td>
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<td>0.739</td>
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</tbody>
</table>

Table 23: Collisional Energy Loss for Penelope, Priapus, and the Ref. Cells for Different Electron Momenta. Note that the energy loss is insensitive to the electron momentum. Because of this insensitivity in our momentum range, we use average momenta for each cell and angle in the calculation of the collisional thickness $\xi$, the most probable energy loss $m_p$, and the mean energy loss $dE$. 

References


