EXPLORATIONS ALONG THE NEUTRON DRIPLINE

M. THOENNESSEN
Department of Physics & Astronomy and
National Superconducting Cyclotron Laboratory
Michigan State University, East Lansing, MI 48824, USA

Abstract. Reaching the limits of nuclear stability offers unique opportunities to understand basic nuclear properties. Current and future techniques to produce and study very neutron-rich nuclei will be discussed. A few examples of light nuclei which are currently in reach at and even beyond the neutron dripline will be presented.

1. Introduction

The study of nuclei under extreme conditions has always been a goal in order to understand the nuclear forces [1]. Nuclei at high densities and temperatures seize to exist as nuclei and dissolve into the quark-gluon plasma [2]. Very fast rotating nuclei can change their shape to be extremely deformed [3]. The nuclear binding in nuclei with an extreme excess of neutrons or protons may not be sufficient to bind the last neutron or proton [4]. These limits are called the driplines. How to reach the neutron dripline with current methods as well as future opportunities will be discussed in the present paper. The location of the neutron dripline is presently only known up to oxygen. However, recent experiments are reevaluating the location of the neutron dripline even for the very lightest nuclei. These new searches and the study of nuclei beyond the neutron dripline will also be presented in the following sections.

2. Bound and Unbound Nuclei

There are many combinations of neutrons and protons that can make up a nucleus of a given mass. For each mass there are only a few isotopes which are stable. They form the “valley of stability”. Changing protons into neutrons or changing neutrons into protons increases the energy and the isotopes become
radioactive or unstable with respect to $\beta$-decay. Figure 1 shows the mass (top) and the single proton and neutron separation energies (bottom) of the $A=21$ isotopes. If the difference between neutrons and protons becomes too large, the separation energies can become negative and the last neutron or proton is not bound to the remaining nucleus anymore. These are the isotopes beyond the dripline. They are particle unbound or just unbound nuclei. In Figure 1, $^{21}$C and $^{21}$Al are neutron and proton unbound, respectively. Along the proton dripline these nuclei -- although unbound -- can have fairly long lifetimes because of the Coulomb barrier. In contrast, the absence of the Coulomb barrier for neutron rich nuclei makes nuclei beyond the neutron dripline extremely short lived. Sometimes they are not referred to as nuclei at all, but rather only identified as resonances.

The question of what constitutes a nucleus is not well defined. A possible limit could be set by the definition of radioactivity. Cerny and Hardy stated that “…lifetimes longer than $10^{12}$ s, a possible lower limit for the process to be called radioactivity” [5].

The International Union of Pure and Applied Chemistry (IUPAC) has published guidelines for the discovery of a chemical element [6]. In addition to other criteria they state that the “Discovery of a chemical element is the experimental
demonstration, beyond reasonable doubt, of the existence of a nuclide with an atomic number \( Z \) not identified before, existing for at least \( 10^{-14} \) s.” The justification for this limit is also given: “This lifetime is chosen as a reasonable estimate of the time it takes a nucleus to acquire its outer electrons. It is not considered self-evident that talking about an ‘element’ makes sense if no outer electrons, bearers of the chemical properties, are present.”

The above definition of an element does not necessarily have to be the same for a nucleus which might be less restrictive. Figure 2 shows the range of lifetimes for different nuclear decay modes. As a comparison elementary particles or resonances are also listed. The boundary between particles and resonances is not well defined. The discussion about what constitutes a nucleus is more or less philosophical.

Two question arise for the very short lifetimes: (i) how does one measure these times and (ii) are the lifetimes sufficient to identify them as separate entities. Traditional or direct methods are limited to \( \sim 10^{-9} \) s for particle emission (Gamma decay has been measured down to \( \sim 10^{-15} \) s). For shorter lifetimes it is possible to use the uncertainty principle relating the lifetime to the decay width as indicated in Figure 2. Detector resolutions of the order of \( > \) keV, however, limit this method to times shorter than \( \sim 10^{-19} \) s. Thus there is a wide range of lifetimes that is currently not accessible (\( 10^{-10} \) s – \( 10^{-19} \) s).

The second question is related to the production of these nuclei. As will be discussed in the following section a very effective way to reach very neutron
rich nuclei is fragmentation. The nuclei at and beyond the dripline are produced by fragmenting a heavier nucleus into smaller fragments. This process is very fast and the fragments leave the reaction zone within ~10^{-22}s; before they decay and so they can be identified as a separated entity. Thus these reactions are fast enough to study nuclei with lifetimes that are longer than ~10^{-22}s.

As an example, Figure 3 shows the production of {\textsuperscript{10}}Li from the fragmentation of {\textsuperscript{18}}O. After the fragmentation which occurs within ~10^{-22}s the {\textsuperscript{10}}Li has left the target nucleus (\textsuperscript{9}Be) with approximately the initial beam velocity of ~40% of the speed of light (80 MeV/nucleon). Assuming a decay width of 1 MeV corresponding to a lifetime of ~10^{-21}s the {\textsuperscript{10}}Li travels about 80 fm away from the target before it decays into \textsuperscript{9}Li and a neutron.

3. Production of Nuclei Along the Neutron Dripline

There are several methods to produce extremely neutron rich nuclei. In the light mass region it is possible to use multi-particle transfer reaction to reach the dripline because it is still fairly close to the valley of stability. For example the reaction \textsuperscript{16}C(\textsuperscript{12}C,\textsuperscript{12}N){\textsuperscript{10}}Li has been used to explore \textsuperscript{10}Li [7]. The mass and the excited states are then deduced from the energy-loss spectrum of the \textsuperscript{12}C. It is not possible to measure the spectrum of the \textsuperscript{10}Li directly because it will decay into \textsuperscript{9}Li and a neutron. This method is clearly limited to the very lightest nuclei.
Another obvious reaction to populate neutron rich nuclei is fission [8]. However, fission populates primarily medium mass nuclei where the neutron dripline is extremely far away from the valley of stability. Although fission (and especially fission of fast beams) is an extremely useful tool to explore very neutron-rich nuclei, its reach is not sufficient to reach the neutron dripline. Presently, the optimum method is fragmentation. Although target fragmentation has been used initially to map the dripline for very light nuclei [9], the method of choice especially for very short-lived nuclei is projectile fragmentation. In the following a few specific examples for the production of rare isotopes using projectile fragmentation will be discussed. Fragmentation can be viewed as a statistical process where depending on the impact parameter of the reaction a certain number of neutrons and protons are stripped from the projectile. For a given initial projectile a wide variety of fragments will be produced. The availability of initial energy and separation capability will determine the rates of available exotic nuclei. Figure 4 shows the distribution of fragments following the fragmentation of 80 MeV/nucleon $^{18}$O on a $^9$Be target. The initial $^{18}$O intensity is $\sim 10^{12}$ particles per second. The figure demonstrates that fragmentation is an extremely low probability event because even the most intense fragments are only produced at a rate of at most $\sim 10^6$ particles per second. The most exotic nuclei are of course produced at even much smaller quantities. For example, the intensity of $^{11}$Li is only $\sim 100$ per
second. As a rule of thumb, for each additional nucleon removed the intensity drops by a factor of ~10. The gap in the Li isotopes (darker columns) corresponds to \(^{10}\text{Li}\) which is particle unbound. Although it will be produced in the reaction as shown earlier it will have decayed before it can be used as a secondary beam.

In addition to the production, it is essential to be able to separate the isotope of interest from the significantly more intense (at least a factor of \(\sim 10^6\)) primary beam and the other produced fragments. A magnetic fragment separator immediately following the production target is thus necessary to select the isotope of interest.

As an example, the functionality of a fragment separator is demonstrated for the newly commissioned A1900 fragment separator at the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University [10,11]. The A1900 label refers to the magnetic rigidity similar to the terminology to classify cyclotrons. The NSCL has two superconducting cyclotrons, the K500 and the K1200. The K values refer to the maximum bending power of the main magnet, which determines the maximum possible energy of the accelerated ions: \(E/A = K(Q/A)^2\) where \(A\) and \(Q\) are the mass and charge of the ions. The energy per nucleon is proportional to the square of the velocity of the ions: \(E/A \sim v^2\). It then follows that \(K \sim (vA/Q)^2\) or \(\sim (p/Q)^2\) where \(p\) is the momentum of the ions. The ratio \(p/Q\) is also called the rigidity.

These ions have to be bent by the magnetic dipoles of the separator. Setting the centrifugal force equal to the magnetic force \(mv^2/\rho = QvB\), where \(\rho\) is the bending radius and \(B\) the magnetic field, yields \(B\rho = p/Q\), where \(B\rho\) is the bending power of the magnet.

This shows the relationship between the \(K\) value of the cyclotron and the bending power of the fragment separator. The \(B\rho = 6\text{Tm}\) magnets of the A1900 are able to bend particles accelerated from a K1900 cyclotron.

The apparent mismatch (A1900 vs K1200) is due to the fact that the fragments of interest are significantly more neutron-rich and thus more rigid than any primary beam from the K1200 cyclotron. For example, a primary \(^{18}\text{O}\) 120 MeV/nucleon beam has a rigidity of \(B\rho \sim 3.7\text{Tm}\), whereas the produced secondary beam of 100 MeV/nucleon \(^{11}\text{Li}\) has a rigidity of \(\sim 5.5\text{Tm}\).

Figure 5 shows the current layout of the accelerators and fragment separator at the NSCL. It also illustrates the production of a secondary \(^{58}\text{Ni}\) beam from a primary beam of \(^{86}\text{Kr}\) [12].

The ion source is tuned to the charge state yielding the maximum yield (14\(^+\)) of \(^{86}\text{Kr}\) ions. These ions are accelerated in the K500 up to 14 MeV per nucleon. At this energy it is possible to efficiently strip even more electrons in order to increase the charge of the ions before they are injected into the K1200. The higher charge state (34\(^+\)) allows the acceleration of the ions to the final energy
Thus the coupling of the two cyclotrons accelerates the maximum intensity to the maximum energy. Following the K1200 the 86Kr ions strike a thick Be target where the fragmentation occurs. The whole range of produced isotopes is shown in the insert in the lower left corner of Figure 5. The first half of the A1900 is tuned to the optimum rigidity of the fragment of interest. The bending radius is fixed and by tuning the magnetic field it is possible to select isotopes of a given momentum to charge ratio or rigidity. However, due to the broad momentum distribution of the different produced fragments there are still a large number of other isotopes with significantly stronger intensities (center insert) with the same rigidity and further separation is essential. A wedge located at an intermediate image makes use of the fact that isotopes of different Z but with the same rigidity (isotopes with larger (smaller) Z will have larger (smaller) momentum) will have different energy-losses as they pass the wedge. The second half of the A1900 can be tuned to the new (reduced) rigidity of the fragments of interest while eliminating other elements. This results in a 65% pure beam of 78Ni at the end of the separator with the contamination due to neighboring isotopes (right insert).

An excellent tool to calculate the optimum selection of primary beam species and energies, as well as optimum target thickness and wedge for a given isotope is the program LISE++ [13]. It contains the configurations of the fragment production target.
The production of rare isotopes via fast fragmentation has several significant advantages over other methods [4]. The main features are: (i) Economic production of medium-energy beams without re-acceleration; (ii) Increased luminosity from the use of thick secondary targets (by up to a factor of 10,000); (iii) Reduced background from in-flight tracking and identification of individual isotopes in the beam on a particle-by-particle basis; (iv) Efficient particle detection from strong forward focusing; (v) Short beam development times and low losses due to fast (sub-microsecond) and chemistry-independent separation and transport to the experiment.

However, one disadvantage is the poor beam quality and the fact that it is difficult to achieve beams at low energies. Although not critical for studies of the dripline there are many nuclear structure experiments and experiments relevant for nuclear astrophysics which require low energy rare isotope beams of excellent quality.

Thus the next generation radioactive beam facility envisioned in the U.S., the Rare Isotope Accelerator (RIA) is based on a fragmentation facility with the capability of target fragmentation. In addition it is planned to produce reaccelerated beams of stopped fast fragmentation products [14]. A layout of the RIA design is shown in Figure 6 [1]. The rare isotopes produced either by target separation at Dubna, GANIL, GSI, MSU, and RIKEN. In addition, it contains many essential tools useful for heavy-ion nuclear physics in general.
fragmentation of stopped projectile fragmentation can either be directly recovered, or used in traps or for laser spectroscopy. Two stages of re-acceleration are envisioned for astrophysics related experiments (<1 MeV/nucleon) and nuclear structure/reactions studies (<15 MeV/nucleon). The exploration of the neutron dripline towards heavier nuclei, however, will be using the fast fragmentation beams directly.

4. Study of Nuclei Beyond the Neutron Dripline

It is commonly believed that the neutron dripline has been reached for all elements up to oxygen. However, this light mass region continues to be of interest because measurements on nuclei beyond the dripline are feasible. In addition, there is always the possibility to observe an unexpected bound isotope.

4.1. NEUTRON CLUSTERS

The most exotic and surprising result recently has been the potential observation of the existence of a four-neutron cluster. A study to measure the break-up of $^{14}$Be found unexpected evidence for a particle stable $^4n$ cluster [15]. If confirmed it certainly would be one of the major discoveries in nuclear physics. The experiment studied the nuclear breakup of $^{14}$Be into $^{10}$Be and four neutrons. During this experiment six events (in coincidence with $^{10}$Be, see Figure 7) were

Figure 7: Evidence for the observation of four-neutron clusters [15].
observed in the neutron detectors that could only be explained by the detection of a correlated cluster of four neutrons. This four neutron cluster would be particle stable and β-decay into $^4$H with a lifetime of $\geq 100$ ns.

4.2. HYDROGEN

The heaviest particle stable hydrogen isotope is tritium $^3$H. A recent study (“Superheavy Hydrogen $^5$H” [16]) of the mass of (unbound) $^5$H represents the heaviest hydrogen isotope observed yet. The width of 1.9 MeV corresponds to an extremely short lifetime of $\sim 3.5 \cdot 10^{-22}$s. $^5$H was observed in the transfer reaction $p(^6$He,$^2$He)$^5$H by detecting the two protons from the decay of $^5$H. In addition tritons from the decay of $^5$H were measured in coincidence in order to eliminate background. Figure 8 shows a schematic of the experimental setup (top) and the derived excitation energy spectrum of $^5$H [16].

An earlier search for $^6$H was unsuccessful [17], however, the possibility to observe the $^7$H with again an even number of neutrons (6) is certainly intriguing. Actually first evidence for a possible resonance in $^7$H has already been reported before the observation of $^5$H [18]. The experiment is very similar to the $^5$H studies; the incident $^6$He beam has to be substituted with a $^8$He beam.

![Experimental setup (top) and reconstructed excitation energy spectrum of $^5$H (left) [16].](image-url)
4.3. HELIUM

Presently $^8\text{He}$ is the heaviest particle stable helium isotope. The masses of unstable $^9\text{He}$ and $^{10}\text{He}$ have also already been measured [19,20]. $^8\text{He}$ is part of the N = 7 isotope chain which attracted special interest early on in the exploration of neutron rich nuclei. The level inversion of the N = 7 nucleus $^{11}\text{Be}$ [21] was first evidence for vanishing of shells in neutron rich nuclei. This $^{11}\text{Be}$ “anomaly” turned out to be a general feature as one approaches the dripline. Figure 9 shows the levels in $^{13}\text{C}$ according to the simple shell model. The 6 protons and the 7 neutrons fill the lowest states with the last single neutron located in the $p_{1/2}$ state. Removing two protons ($^{11}\text{Be}$) changes the level order of the neutrons. The $s_{1/2}$-state falls below the $p_{1/2}$-state. This trend continues in $^{10}\text{Li}$ where the $\nu s_{1/2}$ level couples with the $\pi p_{3/2}$ level to form the 1$^+$ or 2$^+$ ground state. Removing one more proton results again in an $s_{1/2}$ ground state of $^9\text{He}$. Figure 10 shows the energies of the $s_{1/2}$ and $p_{1/2}$ states for the full isotone chain from the bound $^{13}\text{C}$ to the unbound $^{10}\text{Li}$ and $^9\text{He}$. The corresponding wave
functions are indicated in the inserts [22].

The lifetime of the heaviest helium isotope $^{10}$He is almost as short as the lifetime of $^6$H. From the limit of the decay width of $<1.2$ MeV a lifetime limit of $>5.5 \cdot 10^{-22}$s can be derived. It was extracted from the invariant mass spectrum of the detected $^8$He fragment in coincidence with two neutrons, following the proton stripping from $^{11}$Li [20].

4.4. LITHIUM

$^{11}$Li is probably the most famous nucleus which started the current excitement of the field of exotic nuclei. Following the first observation in the sixties [23], the observation of the halo nature of $^{11}$Li established the use of fast fragmentation beams for studies of radioactive isotopes [24]. The halo structure was deduced from the large interaction cross section. These measurements are experimentally the easiest following the initial observation and identification of a new isotope. At high beam energies the survival probability is simply measured by counting the ratio of incoming versus outgoing particles. Knowing the geometric size of the target nuclei it is possible to deduce the size of the beam particles.

Figure 11 shows the increase of the size of $^{11}$Li relative to the lighter Li isotopes (solid line, adapted from Ref. [24]). However, the line does not include $^{10}$Li. As was shown in the previous subsection (Figure 10) $^{10}$Li is unbound. One could thus argue that it has an infinite interaction cross-section because in a hypothetical reaction the number of outgoing $^{10}$Li particle is zero. This is indicated by the dashed line in Figure 11. Maybe it is then not so surprising that $^{11}$Li is so large but that it is bound at all. The reason for the increased binding of $^{11}$Li relative to $^{10}$Li is of course the pairing interacting between the two neutrons.

![Figure 11: RMS-radii of Li isotopes (adapted from [24]). The solid (dashed) line connects the isotopes excluding (including) unbound $^{10}$Li.](image-url)
The most detailed study of the structure of $^{11}\text{Li}$ was a recent angular correlation measurement of the breakup into $^9\text{Li}$ and two neutrons [25]. Reaching the limits of course means that it is not sufficient to identify the supposedly heaviest isotope, but it is necessary to establish the non-existence of even heavier isotopes. The (remote) possibility that $^{13}\text{Li}$ might be bound or at least exhibit a narrow resonance structure similar to $^5\text{H}$ or $^{10}\text{He}$ should be investigated. $^{11}\text{Li}$ could be produced by single proton stripping from $^{14}\text{Be}$ and the resonance energy (if unbound) can then be reconstructed from the breakup into $^{11}\text{Li}$ and two neutrons.

4.5. BERYLLIUM

The difficulty to establish the non-existence of a specific isotope can be illustrated by the example of $^{14}\text{Be}$. In a first measurement $^{13}\text{Be}$ and $^{14}\text{Be}$ had been reported as unbound [26]. However, while $^{13}\text{Be}$ is indeed unbound $^{14}\text{Be}$ was later observed and identified as particle stable [9]. $^{12}\text{Be}$ and $^{14}\text{Be}$ have been shown to be two neutron halo nuclei (see for example [27]). The measurement of the breakup of $^{14}\text{Be}$ into $^{10}\text{Be}$ resulted in the surprising observation of the four-neutron cluster [15].

Again, in order to determine the location of the dripline, it is necessary to go beyond the dripline and identify the first unbound nucleus. $^{12}\text{Be}$ and $^{14}\text{Be}$ are only bound due to the pairing force because $^{11}\text{Be}$ is only bound by ~500 keV and $^{13}\text{Be}$ is unbound. Thus, while it is obvious that $^{15}\text{Be}$ is unbound there is the possibility that $^{16}\text{Be}$ is again bound.

Based on early prediction that $^{16}\text{Be}$ is unbound by ~3 MeV [28] this possibility seems remote. However, the change of the shell structure and the new

![Figure 12: (a) One neutron (squares) and two neutron (circles) separation energies for Be isotopes, (b) Two neutron separation energies for neutron rich light isotopes [29].](image-url)
knowledge about neutron rich nuclei led to improved shell model calculations [30]. Figure 12(a) shows the one neutron (squares) and two neutron (circles) separation energies for beryllium isotopes [28]. While $^{16}$Be will be bound with respect to one neutron emission, it is predicted to be unbound with respect to two neutron emission by only ~600 keV. Figure 12(b) compares the two neutron binding energies of the beryllium isotopes with other light neutron rich nuclei [29,30]. It shows that in addition to $^{16}$Be the calculations also predict $^{19}$B and $^{22}$C to be unbound. However, these nuclei have previously been shown to be bound [9,31].

The search for $^{16}$Be was recently performed at the NSCL at MSU and no events identified as $^{16}$Be were observed establishing that $^{16}$Be is particle unstable and that the neutron dripline in beryllium is located at $^{14}$Be.

4.6. BORON TO FLORINE

The main interest in the mass region between boron and fluorine towards the neutron dripline is due to the changes of shell structure. While the N = 8 is observed to disappear for beryllium [32,33], evidence for the disappearance of the N = 20 shell is the non-existence of $^{26}$O and $^{28}$O [34,35]. Recently a survey of neutron separation energies clearly showed the emergence of a new shell for oxygen isotopes at N = 16 [36]. This opened up the possibility that the heaviest boron isotope might be $^{21}$B instead of $^{19}$B. A recent experiment, however, could not find any evidence for the existence of $^{21}$B establishing $^{19}$B as the heaviest stable boron isotope [37].

The next three elements beyond boron are carbon, nitrogen and oxygen. The heaviest isotopes for these three elements all have 16 neutrons ($^{22}$C, $^{23}$N, and $^{24}$O) which is additional support for the existence of a shell closure at N = 16. The N = 16 (sub) shell closure has recently been explained by detailed shell model calculations [38]. It leads to the superficially surprising effect that the addition of one proton (from oxygen to fluorine) leads to the binding of at least six additional neutrons. While $^{24}$O (N = 16) is the last bound oxygen isotope $^{31}$F (N = 22) has been observed to be stable [39].

The effect can be qualitatively demonstrated without the shell model calculations as shown in Figure 13. Shell closures can easily be identified by a dramatic drop of the binding energy for nuclei with two neutrons beyond a shell closure, followed by a more flat distribution with increasing numbers of neutrons. This is obvious because it is difficult to separate two neutrons from a closed shell or with only one neutron in the next shell. However, once two or more neutrons are present in the next shell it is easier to remove two neutrons. This discontinuity in the distribution is indicated for the N = 8 shell at N = 10 in Figure 13. The disappearance of this shell is clearly visible.
Similarly the transition from the N = 20 (at N = 22) to N =16 (at N = 18) occurs from Mg to F. The two neutron binding energies for $^{29}F$ and $^{31}F$ are not known, however, the fact that these isotopes are bound indicates that they have to be larger than zero. In contrast the two neutron separation energies for $^{26}O$ and $^{28}O$ have to be negative as indicated because these isotopes are unbound. The drop of the two neutron binding energy due to the N = 16 shell falls just above zero for fluorine isotopes while for oxygen isotopes it drops just below zero. The subsequent almost constant separation energy leaves the fluorine isotopes bound while the oxygen isotopes are unbound leading to the additional binding of at least six neutrons for fluorine. It could even be that it could bind an additional eight neutrons, because it is not yet known if $^{33}F$ is bound or unbound. Thus the limit of current knowledge of the dripline ends at oxygen.

5. Summary and Conclusion

The exploration of the limit of stability along the neutron dripline continues to be pushed towards heavier and heavier elements. The very recent experimental activities to revisit the dripline for elements lighter than oxygen did not find any
new isotopes. Thus the dripline up to oxygen has been confirmed. However, it is still not clear if $^{31}$F is indeed the heaviest stable fluorine isotope. Most recent searches in this mass region have observed $^{34}$Ne and $^{37}$Na for the first time [40,41]. Although these are the heaviest observed isotopes of these elements it is not clear if they are at the dripline. Even heavier isotopes could still be bound. The fully upgraded coupled cyclotron facility at the NSCL should be able to explore the dripline up to sulfur and if RIA becomes a reality the dripline could potentially be determined up to zinc (Z = 30) and maybe even again around zirconium (Z = 40) as shown in Figure 14 [42].

The quest to observe isotopes that have never been observed before is certainly a strong driving force behind the search for the dripline. It is followed by the need to understand the structure of these newly observed isotopes. Will these nuclei reveal new insights in the nuclear forces? Can they be explained by simple extrapolations of the present knowledge or will they force modifications of the current theories? Finally, what are the implications of the results of these measurements on other fields, most notably nuclear astrophysics, where the

Figure 14: Predicted yields of isotopes from the proposed Rare Isotope Accelerator RIA [42].
EXPLORATIONS ALONG THE DRIPLINES

calculations of the r-process depends on nuclear quantities like the mass and lifetimes of very neutron-rich nuclei. These questions demonstrate the necessity for a basic understanding of nuclei which is the main justification for a next generation rare isotope facility like RIA.

Acknowledgments

This work was supported by the U.S. National Science Foundation under grant PHY 01-10253.

References

http://groups.nscl.msu.edu/lise/spectrometer.html
[38] Y. Utsuno *et al.*, Phys. Rev. **C64** (2001) 011301