Systematics of hot giant electric dipole resonance widths

A. Schiller\(^a\) †, M. Thoennessen\(^{a,b}\), K.M. McAlpine\(^{a,b}\) ‡

\(^a\)National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA

\(^b\)Department of Physics & Astronomy, Michigan State University, East Lansing, MI 48824, USA

Giant Electric Dipole Resonance (GDR) parameters for \(\gamma\) decay to excited states with finite spin and temperature have been compiled by two of the authors (nucl-ex/0605004). Over 100 original works have been reviewed and from some 70 of them, more than 300 sets of hot GDR parameters for different isotopes, excitation energies, and spin regions have been extracted. All parameter sets have been brought onto a common footing by calculating the equivalent Lorentzian parameters. Together with a complementary compilation by Samuel S. Dietrich and Barry L. Berman [At. Data Nucl. Data Tables 38, 199-338, (1988)] on ground-state photo-neutron and photo-absorption cross sections and their Lorentzian parameters, it is now possible by means of a comparison of the two data sets to shed light on the evolution of GDR parameters with temperature and spin.

1. INTRODUCTION

Over the last decades, numerous data on hot Giant Electric Dipole Resonance (GDR) parameters have been accumulated. Since the field seems to have matured [1], all present data at this point have been gathered in one comprehensive compilation in a uniform format [2]. By means of a comparison with an earlier compilation [3] on ground-state photo-absorption cross section, it is possible to examine the evolution of GDR parameters with temperature and spin. This proceedings article is organized in the following way: first, we give some theoretical motivation why GDR parameters might depend on temperature. Then, in Section 3 we give an overview of typical experimental techniques used for measuring hot GDR parameters; in Section 4 we describe different ways in which hot GDR parameters are extracted from such experiments through statistical-model calculations. In Section 5 we explain how different sets of GDR parameterizations from the original articles are brought onto a common footing for the compilation [2]. Finally, we investigate simple scaling laws of GDR widths with temperature and spin.

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†Corresponding author: NSCL/MSU, 1 Cyclotron, East Lansing, MI 48824. Phone: (517) 324-8142, Fax: (517) 353-5967, Email: schiller@nscl.msu.edu
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2. THEORETICAL CONSIDERATIONS

A first good understanding of statistical $\gamma$ emission was gained from the works of Brink [4] and Axel [5] who realized that average electric-dipole ($E1$) transition strengths in different energy regimes can be described in a unified fashion by assuming that the GDR can be built on any excited state, and that the GDR properties do not depend on the temperature of the excited state in question.

This so-called Brink-Axel hypothesis has been refined in the past to allow for temperature- and spin-dependent widths. A model to justify such a modification takes into account shape fluctuations of the nucleus. Since the ground-state GDR splits into two components for a nucleus with static deformation, and the splitting depends on the degree of deformation [6], it is reasonable to assume that at finite temperatures, when the nucleus can explore a large volume in deformation space, the GDR response will be an average over different deformations and hence, different splittings. The result within this adiabatic damping model will be a more diffuse and certainly wider GDR than the ground-state GDR.

Quantitatively, assuming a Fermi-gas level density, one can write the nuclear entropy in the microcanonical ensemble as $S = 2\sqrt{a(E - V)}$, where $V$ is some potential energy proportional to the square of the deformation $V = k\beta^2$. The entropy is trivially maximized for $\beta = 0$; expanding $S$ for small $V \ll E$ yields $S \approx 2\sqrt{aE} \left(1 - \frac{V}{2E}\right)$. With $T = \sqrt{E/a}$, the probability distribution to find a nucleus with energy $E$ and deformation $\beta$ becomes $P \approx \exp \left(2\sqrt{aE}\right) \exp \left(-k\beta^2/T\right)$ where the second factor represents a Gaussian distribution of deformations around $\beta = 0$ and with a width of $\sqrt{T/2k}$. Assuming that the splitting of the GDR into two components is roughly proportional to the nuclear deformation, the shape-fluctuation model predicts an increase in width of the GDR roughly proportional to $\sqrt{T}$. Moreover, there is also a potential spin dependence of the GDR width which stems from the possibility of spin-induced deformation. Finally, orientation fluctuations of the nucleus and nuclear-structure effects such as pairing can influence the temperature dependence of the GDR in different energy regimes [7]. Several groups have calculated temperature-dependent GDR widths along these lines [8–10] and a simple scaling law has emerged [8].

Investigations of the low-energy tail of the GDR have also yielded indications for a temperature-dependent GDR width. It was noted by, e.g., Popov [11] in $(n, \gamma\alpha)$ experiments on Sm nuclei that the $\gamma$ strength function tends to approach a finite value for $E_\gamma \rightarrow 0$ for $\gamma$ transitions in the quasicontinuum (below the neutron separation energy). This experimental observation led Kadmenskii, Markushev, and Furman (KMF) to propose a $\gamma$ strength-function model for spherical nuclei with a temperature-dependent width [12] based on the effect of in-medium nucleon-nucleon collisions. The proposed temperature dependence was derived within Migdal’s theory of Fermi liquids and has the form $\Gamma(E_\gamma, T) \propto \left(E_\gamma^2 + 4\pi^2T^2\right)$. The model was later improved by Sirotkin [13] who included the Pauli exclusion principle, and it was extended to deformed nuclei within the framework of the generalized Lorentzian model by Kopecky and Uhl [14] and by inclusion of a coupling term between the $E1$ operator and the quadrupole deformation according to Mughabghab and Dunford [15]. The KMF model (taken at constant temperatures) and
its extensions have been successfully applied to improve $\gamma$ and isomeric production cross sections [16] and they have been used for direct fits of measured low-energy $\gamma$ strength functions [17]. The connection of collisional-damping models with hot GDR parameters is made in [18].

3. EXPERIMENTAL TECHNIQUES

Unlike the measurement of the GDR by ground-state photo-absorption cross-section measurements [3], measurements of hot GDR parameters can be performed in many different ways. One of the simplest ways is by fusion-evaporation reactions where only $\gamma$ rays are detected. Such measurements are the most inclusive reactions, since the high-energy $\gamma$ yield which competes with particle and especially neutron evaporation is representative for a range of different product nuclei, excitation energies, and spins. Moreover, it is not necessarily guaranteed that all detected $\gamma$ rays stem from fusion-evaporation reactions. Other reactions such as inelastic or deep inelastic scattering can compete and yield $\gamma$ rays from target or projectile-like fragments.

To improve the sensitivity of such experiments to the fusion-evaporation reaction channel, typical gates such as, e.g., $\gamma$-multiplicity filters, detection of heavy evaporation residues, and detection of evaporated, light charged particles such as protons or $\alpha$ particles can be performed. The resulting $\gamma$-ray spectra are more exclusive, not only in terms of the product nuclei from which high-energy $\gamma$ rays are emitted, but also in terms of the spin and the excitation-energy range investigated. For example, a gate on different $\gamma$ folds translates rather directly into certain spin regions of the investigated product nucleus. A gate on evaporated light charged particles will not only reduce the average charge and mass of the product nucleus, but it will also reduce its average excitation energy, since the evaporated particles will carry away some part of the initial excitation energy of the compound nucleus; hence applying such a gate will test the GDR at somewhat lower temperatures than the fully inclusive experiment. In the same way, gating on the $\gamma$ sum energy or on specific product nuclei by means of detecting in coincidence discrete, known low-energy $\gamma$ transitions will also influence the average spin and excitation-energy region from which the high-energy $\gamma$ rays are emitted, since one effectively biases the competition between high-energy $\gamma$ decay and neutron evaporation in one or the other direction. Other, more rarely used gating conditions are, e.g., the isomeric $\gamma$ decay by discrete transitions or the $\alpha$ decay of a product nucleus, both of which have similar implications for the average spin and excitation-energy range from which prompt high-energy $\gamma$ rays are observed.

For heavy nuclei, an added difficulty is the possibility of fission of the compound nucleus. Typically, for low spins, the production of an evaporation residue dominates while for high spins fission will become the dominant exit channel. Hence, by gating on evaporation residues or fission fragments, one effectively selects a spin region from which high-energy $\gamma$ emission is observed. In the case of the fission exit channel, one also observes high-energy $\gamma$ emission from the fission fragments themselves, though typically at significantly higher $\gamma$ energies owing to the much lower mass of the fission fragments. Another complication is the fact that high-energy $\gamma$ emission can occur from the compound nucleus (which is desired), or during the saddle-to-scission motion after the nucleus has passed the fission
barrier. Also, for the heaviest nuclei investigated, it is not clear whether a true compound nucleus forms which is confined by some fission barrier or whether one observes direct fission or just the formation of a mononucleus. Where the latter is suspected, the extracted data have not been entered into the compilation [2].

In some cases, one employs inelastic or deep inelastic scattering to excite the target or projectile nucleus. By measuring the kinetic energy of at least one of the products, one can reconstruct the reaction kinematics event by event and it is possible to obtain initial excitation-energy indexed coincident high-energy \( \gamma \) spectra with only one beam energy. Otherwise, the excited nucleus can be treated in the same way as a compound nucleus which is formed in a fusion-evaporation reaction.

4. STATISTICAL-MODEL CALCULATIONS AND COMPARISON TO EXPERIMENTAL DATA

High-energy \( \gamma \) spectra are typically analyzed using a statistical-model calculation. In the first step, total fusion cross sections and maximum \( \langle l_0 \rangle \) or average \( \langle I_i \rangle \) angular momenta are determined. Typically, total fusion cross sections can be verified by experiment; maximum angular momenta are calculated by the theory of either Winther [19] or Swiatecki [20]. Average angular momenta can then be determined by \( \langle I_i \rangle = \frac{2}{3} l_0 \). In the next step, the decay of the highly excited compound nucleus is modeled. In many cases, this simply involves a Hauser-Feshbach-type theory [21] into which particle and \( \gamma \) transmission coefficients (sometimes including higher-than-E1 multipolarities) as well as nuclear level densities enter. Typically, particle transmission coefficients are not discussed in great detail in the compiled works. The level-density models are either the P"uhlhofer model [22] (the default in the statistical-model code CASCADE [22]) or the Reisdorf model [23]. The P"uhlhofer model relies on the local Dilg et al. parameterization [24] for excitation energies up to and slightly above the nucleon separation energy, while it interpolates then to a regime where the level-density parameter \( a \) becomes proportional to the nuclear mass number \( A \). The Reisdorf approach builds on the generalized superfluid model by Ignatyuk et al. [25], but it uses a global parameterization for the asymptotic level density parameter \( a \). In one case [26], the level-density model by Fineman et al. [27] is used in the data analysis.

Statistical-model calculations are often adapted to different experimental situations. For light compound nuclei near the \( N = Z \) line, an isospin-dependent formalism is often used. Also, the Wigner energy [28] is sometimes included in the level-density parameterization. For large excitation energies, pre-equilibrium emission due to direct and semi-direct reaction mechanisms are often taken into account. Especially the PEQAG2 code [29] has been developed for this purpose. In the case of fissile compound nuclei, the fission channel and the decay of excited fission fragments are modeled as well. When gating conditions have been applied in the experiment, they are usually reflected in the statistical-model calculation as well, which often implies the need to use a Monte-Carlo simulation tool. In some cases, also asymmetries \( a_2 \) of \( \gamma \) emission are calculated, however, experimental asymmetries have not been entered into the compilation [2] with the exception of their influence on the sign of the quoted deformation.

Typically, calculated high-energy \( \gamma \) spectra are compared to their experimental coun-
terparts, and the $\gamma$ transmission coefficients (parameterized by one- or two-component Lorentzians multiplied by a factor $2\pi E^3_3$) are varied until the best fit is obtained. If absolute values in the high-energy region are compared, the fit is often normalized to the data in an energy region of 3–7 MeV, far below the peak of the GDR. Statistical uncertainties are usually determined in the normal fashion by varying GDR parameters until the quality of the fit deteriorates. Systematic uncertainties are more difficult to estimate. A good way is, e.g., to perform several fits to the experimental data with different level-density parameterizations or differing sets of other input parameters into the statistical-model calculation (say, e.g., those which describe the dynamic of the fission process such as the nuclear viscosity, which governs the timescale of the saddle-to-scission motion). The range of resulting GDR parameters might give a good indication of the size of the systematic error.

Other sources of systematic errors concern the experimental conditions. Some of the most important problems there involve inefficient neutron-$\gamma$ discrimination (often done by time-of-flight techniques, or by simply considering $\gamma$ rays only at backward angles), contamination of high-energy $\gamma$ spectra by cosmic rays (which can be greatly reduced by coincidence measurements), target impurities, pile-up, and add-back issues. Add back is a technique often used for an array of small detectors where for high-energy $\gamma$ rays one observes a significant amount of (i) Compton scattering from one detector into a neighboring one, and (ii) pair production with subsequent annihilation $\gamma$ rays being detected in neighboring detectors. The add-back technique remedies this situation by adding the deposited energies in neighboring detectors and (rightly) consider such events as stemming from one single $\gamma$ ray. Pile up is a problem especially for large detectors, where two or more coincident $\gamma$ rays hit the same detector and their energies add up and are falsely registered as one high-energy $\gamma$ ray. If one applies the add-back technique, however, pile up can also occur when two coincident $\gamma$ rays hit two neighboring detectors. Obviously, for any given detector array, there is an optimal balance between the benefit of applying the add-back technique and the possible distortions of the high-energy $\gamma$ spectrum due to pile up. When this balance is not met, the extracted data have not been entered into the compilation [2].

Some other physical background involves nuclear bremsstrahlung which is emitted in the first moments of the fusion process where individual nucleons of the projectile are greatly de-accelerated in the proximity of target nucleons and emit high-energy $\gamma$ rays. These $\gamma$ rays can either be modeled (and hence subtracted from the high-energy $\gamma$ spectrum which is to be fitted by the statistical-model calculation), or they are simply considered as a source of systematic error. Another possible source of high-energy $\gamma$ rays is the pre-equilibrium $\gamma$ emission during the formation of the compound nucleus which has been investigated by measuring high-energy $\gamma$ spectra for (isospin) symmetric and asymmetric reactions. Since in the extreme, such reactions essentially probe the di-nuclear system and not a compound nucleus, the resulting data have not been entered into the compilation [2]. In general, the compilation [2] focuses more on low-energy data to avoid complications due to non-compound sources of high-energy $\gamma$ rays, hence, data concerning the saturation or increase of the GDR width at very high excitation energies have typically been omitted.
5. DATA TREATMENT

In the data treatment for the compilation [2], the first step was to determine the target isotope from the context (for those few articles where it is not stated explicitly). Ranges of laboratory energy $E_{\text{lab}}$, initial excitation energy $E_{\text{ex}}$, and initial spin $I_i$ were replaced by their central values. Average initial spins $\langle I_i \rangle$ were determined from maximum spins $l_0$ by means of $\langle I_i \rangle = \frac{3}{2}l_0$ for the case of fusion-evaporation reactions but irrespective of gating conditions. Ranges in final spin $I_f$ after GDR $\gamma$ emission were replaced by central values as well; the widths of the $I_f$ ranges were converted into FWHM values which were preferred in this case over regular uncertainties due to the often non-Gaussian distribution of final spins.

Hot GDR parameterizations can take many different forms. The most common is probably the parameterization in terms of a centroid $E$, width $\Gamma$, and maximum $\sigma$ of an equivalent Lorentzian photon-absorption cross section. The maximum $\sigma$ is often formulated as a fraction $S$ of the Thomas-Reiche-Kuhn (TRK) sum rule which describes the integral $\frac{2}{\pi}\sigma\Gamma = 60\frac{N}{A}$ MeV mb of the Lorentzian in terms of neutron $N$, proton $Z$, and mass $A$ numbers [3]. For two-component Lorentzian parameterizations, often the total $S_1 + S_2$, and either the ratio $S_2/S_1$ or the relative fraction $F_2 = S_2/(S_1 + S_2)$ are given. In either case, the given parameters were converted into $S_1$ and $S_2$ values. Sometimes, the total is assumed to fulfill the TRK while only the ratio or the relative fraction $F_2$ is determined by the fit. In these cases, the calculated errors are correlated and are marked by an asterisk in the compilation [2]. In other cases, no information is given on the total. In those cases, the fraction $S_2$ is given in terms of the fraction $S_1$ in the compilation [2].

In case of GDR widths, some authors reduce their number of fit parameters by introducing a phenomenological relation between the widths and the centroids of a two-component Lorentzian according to $\Gamma = cE^2$, where $c$ becomes the fit parameter. In such cases, the individual widths $\Gamma$ including their errors were calculated. However, the errors are again correlated and marked by an asterisk in the compilation [2]. In the case two GDR centroids, some authors give the average $E_{\text{ave}} = \frac{S_1E_1 + S_2E_2}{S_1 + S_2}$ and the ratio $E_2/E_1$. Sometimes this ratio is replaced by an average deformation $\beta$ which can be related to $E_2/E_1$ by either $\beta = \frac{2}{3}\sqrt{\frac{4\pi}{5}} \ln \frac{E_2}{E_1}$ or $\beta = \sqrt{\frac{4\pi}{5}} \frac{E_2/E_1 - 1}{E_2/E_1 + 0.8065}$, where $E_\perp$ and $E_\parallel$ denote the centroids of the GDR components due to oscillations perpendicular and parallel to the symmetry axis. In the case where the original article does not mention which of the two formulas applies, the two formulas result in values for $E_1$ and $E_2$ within 0.2 MeV of each other, a difference far less than the quoted statistical error.

In the treatment of errors, the first step was to add quadratically statistical and systematic errors (when quoted separately). Where a range of systematic uncertainties is given, the center value of that range has been adopted as a representative systematic uncertainty. When an uncertainty is given in terms of a FWHM, it has been converted by $\sigma = \text{FWHM}/\sqrt{8 \ln 2}$. In general, rigorous error propagation was performed. However, no original work gives the full covariance matrix for the fitted GDR parameters. Therefore, the derived errors are only representative of the true errors under the assumption that the originally fitted parameters are fully uncorrelated. In cases where the number of calculated parameters is larger than the number of originally fitted ones, an asterisk in the compilation [2] denotes the correlations which were introduced to the errors.
cases where GDR parameters have been held fixed during the fit, a little ‘f’ is given in the compilation [2] instead of an error. Some works do not quote errors at all. In such cases no attempt has been made to estimate the errors. In a few cases where $E_1$ and $E_2$ are given without errors, while $E_{\text{ave}}$ is given with error, this error has not been translated into errors of the individual values, and hence $E_1$ and $E_2$ remain without errors.

6. GDR WIDTH SYSTEMATIC

The compilation [2] is then used as a starting point to create systematics of GDR widths. In the first step, FWHMs were calculated in the case of two- (or three-) component fits of the hot GDR. The errors of the FWHMs were determined by randomizing the Lorentzian fits within the quoted error bars of the parameters. In cases where the original article gives a FWHM, our calculated error bars are larger by about a factor of $\sim 2$ than the quoted ones which indicates a certain amount of anticorrelation of errors.

Figure 1. Ratio of calculated and experimental hot GDR width as function of average spin at which GDR decay occurs.

Ground-state widths are taken from the compilation [3]. Also here, in the case of two-
component Lorentzians, the FWHMs were calculated. Since hot GDR parameters are typically measured in fusion-evaporation reactions, i.e. for product nuclei several units shifted to the proton-rich side away from the valley of stability, while ground-state GDR parameters are measured in photo-neutron reactions on stable targets, it is rare to find hot and cold GDR parameters for one isotope. Therefore, in order to compare hot and cold GDR FWHMs for a larger sample, we do not require an isotopic match, but only a match in mass number. Moreover, in cases where a specific mass number does not appear in the ground-state GDR compilation, we have estimated the ground-state FWHM by linear interpolation between the two closest isotopes in mass.

Figure 2. Same as Fig. 1 but as function of temperature of the hot GDR.

In our investigation, we follow the scaling law of [8]. In Fig. 1, we plot the theoretical FWHMs (calculated according to [8] using the ground-state FWHMs from [3]), divided by the experimental FWHMs for the hot GDR from the compilation [2] as function of average spin at which the hot GDR decay occurs. The data scatter up to 40% around unity, but there is no residual trend discernible in the data. The situation is somewhat different when the same ratio is plotted as function of temperature of the hot GDR (see Fig. 2). A slight downward trend of the ratio with temperature might be discussed. If substantiated, this could indicate an overestimation by the Kusnezov scaling law of hot GDR widths at low temperatures. We will increase our sample size by adding data from the compilation [2] to the plot for which either spin or temperature has not been given in the original article. For this reason, we will calculate these values from simple estimates.
using fusion-evaporation reaction models.

7. CONCLUSIONS

We have discussed a new compilation of hot GDR parameters. This compilation is used to test simple scaling laws of hot GDR widths with spin and temperature. It is found that with a limited data set (original articles which quote average spin and temperature for hot GDR decay), the dependence of the GDR width on spin is described well, while its dependence on temperature could possibly be improved. We are about to extend our data set by including data for which the original articles do not quote a spin or a temperature for the hot GDR decay. In those cases we will estimate these values ourselves. This is, however, still a work in progress.

REFERENCES

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