ELECTROSTATICS

ELECTRIC CHARGE

- Electric charge is a property of atomic particles, the electron and the proton, which make up atoms (together with neutrons).
- The standard unit of charge is the Coulomb (C).
- Electric charge and mass of particles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Electric Charge</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>$-e = -1.6 \cdot 10^{-19}$ C</td>
<td>$m_e = 9.11 \cdot 10^{-31}$ kg</td>
</tr>
<tr>
<td>Proton</td>
<td>$+e = 1.6 \cdot 10^{-19}$ C</td>
<td>$m_p = 1.672 \cdot 10^{-27}$ kg</td>
</tr>
<tr>
<td>Neutron</td>
<td>0</td>
<td>$m_n = 1.674 \cdot 10^{-27}$ kg</td>
</tr>
</tbody>
</table>

- Law of charges: *Like charges repel, and unlike charges attract.*
- An electric charge $q$ is a charge which is an integer multiple of the fundamental charge constant $e = 1.6 \cdot 10^{-19}$ C, $q = n e$. Electric charge is quantized.
- The net charge of an object is the difference between the number of protons and electrons in it times the elementary charge constant.
- Law of conservation of net charge: *The net charge of an isolated system remains constant.*
- Electric charge transfer is a transfer of electrons.

| Charging positively: Removal of electrons from an object | Charging negatively: Addition of electrons to an object |

ELECTRIC FORCE

- The mutual electrostatic forces on two point charges are equal and opposite, pointing to (away from) the other particle for unlike (like) charges.
- Coulomb’s Law
  The electrostatic force between two charges $q_1$ and $q_2$ separated by a distance $r$ is:
  $$F = \frac{k q_1 q_2}{r^2}$$
  $$k = 8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2$$
- Charges interact pairwise via Coulomb force. The superposition principle is valid:

  *The net force acting on any charge is the vector sum of the forces due to each of the remaining charges in a given distribution.*
ELECTRIC FIELD

- Test charge = charge which feels the force of other charges, but exerts no force on them. (mathematical construction)

- Electric field, $\vec{E} = \text{force per unit test charge}$: $\vec{E} = \vec{F}/q_0$.
  
  SI-unit of the $\vec{E}$-field: N/C.

- Electric field of a point charge:
  - Force between two point charges: $F = kQq_0/r^2$.
  - $E$-field felt by test charge $q_0$ at $r$ due to the presence of $Q$ is then: $E = F/q_0 = kQ/r^2$.
  - Direction of $\vec{E}$ = direction of $\vec{F}$.
  - Unit positive test charge would be attracted to a negative charge. $\vec{E}$-field points towards a negative point charge and away from a positive point charge.

- Superposition of electric fields: $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$, $\vec{E} = \vec{F}/q_0 \rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

- Rules for electrical field lines:
  - The lines are directed pointing away from the positive and towards the negative charges.
  - At any given point in space, the tangent to the line is the direction of the $\vec{E}$-field at that point.
  - The number of lines drawn to or from a charge is proportional to the magnitude of the charge.

- Consequences of these rules:
  - In the immediate vicinity of a point charge, field lines are radially directed.
  - Field lines do not intersect in a charge-free region.
  - Field lines do not begin or end in a charge-free region.

- Density of field lines (number of field lines per unit area) is proportional to the $\vec{E}$-field; and by convention, the total number of field lines is proportional to the charge $q$.

ELECTRIC DIPOLE

- An arrangement of two equal but opposite charges $q$ separated by a fixed distance $d$ is called a dipole.
  
  In a uniform field $E$, a fixed dipole is subject to a torque: $\tau = qdE\sin\theta$. $\theta$ is the angle between the dipole direction and the field. $p = qd$ is the dipole moment.

- The field of a dipole along the dipole axis at large distances ($z >> d$) is: $E = 2kp/z^3$

- The field of a dipole along an axis perpendicular to the dipole axis at large distances ($x >> d$) is: $E = kp/x^3$

CONTINUOUS CHARGE DISTRIBUTIONS

- The linear charge density is defined as $\lambda = Q/L$
  
  The surface charge density is defined as $\sigma = Q/A$
  
  The volume charge density is defined as $\rho = Q/V$

- The electric field along the $z$-axis of a charged ring is: $E = kqz/(z^2 + R^2)^{3/2}$

- The electric field along the $z$-axis of a charged disk is: $E = \sigma/2\varepsilon_0(1 - z/\sqrt{z^2 + R^2})$