MAGNETIC FIELD DUE TO A CURRENT

- The magnetic field $d\vec{B}$ at a distance $r$ from a wire due to a current $i$ inside a piece $d\vec{s}$ of the wire is given by Bio-Savart’s law: $d\vec{B} = \mu_0 i d\vec{s} \times \vec{r} / (4\pi r^3)$
  The constant: $\mu_0 = 4\pi \times 10^{-7} \text{Tm/A}$ is called the permeability of free space.

- Magnetic field of a long straight wire at a distance $r$ from the wire: $B = \mu_0 i / (2\pi r)$

- Magnetic field inside of a long straight wire with radius $R$ at a distance $r$ from the center: $B = \mu_0 i r / (2\pi R^2)$

- Magnetic field at a center of a circular arc: $B = \mu_0 i \Phi / (4\pi R)$, where $\Phi$ is measured in radians.
  Thus the magnetic field at the center of a current loop is: $B = \mu_0 i / (2R)$

- Magnetic field of a coil along the symmetry axis $z$ through the center: $B = \mu_0 \mu / (2\pi z^3)$, where $\mu = NiA$ is the magnetic dipole moment of the coil and $N$ is the number of turns in the coil.

- Right Hand Rule: The direction of the field curves around with the fingers of the right hand when the thumb points in the direction of the current.

FORCE BETWEEN TWO WIRES

- The force on a wire which carries a current $i_1$ due to the magnetic field of another (parallel) wire with current $i_2$ ($B = \mu_0 i_2 / (2\pi d)$) is given by: $F = \mu_0 L i_1 i_2 / (2\pi d)$.
  $d$ is the distance between the two wires and $L$ is the length of the wires.

- If the current in both wires are in the same direction, the force is attractive, and if the currents are in opposite directions the force is repulsive.

MAGNETIC FIELD OF A SOLENOID AND TOROID

- A solenoid is a long straight coil of tightly wound wire. The magnetic field inside the solenoid is directed along the center of the coil: $B = \mu_0 i n$, where $n$ is the number of turns per unit length.
  Thus, if $N$ is the total number of turns of the solenoid of length $L$, then $n = N/L$.

- A toroid is a solenoid bent into the shape of a doughnut. The magnetic field at the center of the toroid is: $B = \mu_0 i N / (2\pi r)$, where $N$ is the total number of turns and $r$ is the radius of the toroid.

AMPERE’S LAW

- For certain situations which involve symmetries it is easier to use Ampere’s law rather than Biot-Savart’s law. The integral of the scalar product of the magnetic field $\vec{B}$ and pathsegment $d\vec{s}$ over a closed imaginary loop is proportional to the enclosed current: $\oint \vec{B} d\vec{s} = \mu_0 i_{enc}$. 
INDUCTION

- A current $I$ generates a magnetic field $B$. Can a magnetic field also generate a current? Yes and No. A constant (in time) magnetic field does not generate a current, but changes in the field do.

- Faraday’s observations: An emf can be generated in a loop of wire by:
  (i) holding it close to a coil (solenoid) and changing the current in the coil,
  (ii) keeping the current in the coil steady, but moving the coil relative to the loop.
  (iii) moving a permanent magnet in or out of the loop.
  (iv) rotating the loop in a steady magnetic field.
  (v) changing the shape of the loop in the field.

- The Magnetic flux is defined as $\Phi_B = \int \vec{B} \cdot d\vec{A}$. If $B$ is constant over the area, then the flux is given by $\Phi_B = BA \cos \theta$. $\theta$ is the angle between $\vec{B}$ and $\vec{A}$, the “normal” to the surface.

- A change in the magnetic flux produces a potential difference (and via Ohm’s law a current) in a coil: $\mathcal{E} = -Nd\Phi_B/dt$. $\Phi$ is the flux through the coil and $N$ is the number of turns of the coil.

- Thus, if $B$ is constant within the coil: $\mathcal{E} = -Nd(BA \cos \theta)/dt$. In most cases, only one variable depends on the time:
  (1) $A, \theta$ constant: $\mathcal{E} = -NA \cos \theta dB/dt$
  (2) $B, \theta$ constant: $\mathcal{E} = -NB \cos \theta dA/dt$
  (3) $A, B$ constant: $\mathcal{E} = -NAB d(\cos \theta)/dt$

- Why the negative sign? **Lenz’s law:** An induced emf gives rise to a current whose magnetic field opposes the change in flux that produced it.

- The magnitude of the emf of a moving conductor in a perpendicular magnetic field is given by: $\mathcal{E} = BV_{||}$, where $L$ is the length of the conductor and $v$ is the velocity (perpendicular to the field) of the conductor ($v = dx/dt$).

- Faraday’s law of induction can also be expressed in terms of an electric field: $\oint \mathcal{E} d\vec{s} = -d\Phi_B/dt$.

INDUCTANCE AND SELFINDUCTION

- The inductance is defined as $L = N\Phi/i$, Units: 1 henry = 1 H = 1 Vs/A.

- The inductance per unit length of a solenoid is $L/l = \mu_0 n^2 A$.

- A changing current in a coil generates a self-induced emf in the coil: $\mathcal{E}_L = -LdI/dt$.

- An RL-circuit consists of an inductor and a resistor in series: $LdI/dt + Ri = \mathcal{E}$. The current rises according to $i = \mathcal{E}/R(1 - e^{-t/\tau_L})$ where $\tau_L = L/R$ is the inductive time constant. After a connection is broken the current decreases as $i = i_0 e^{-t/\tau_L}$.

- The energy stored in an inductor is given by $U_B = Li^2/2$. The energy density of a coil is given by $u_B = B^2/(2\mu_0)$.

- The mutual inductance between two coils is defined as $M$ and $E_2 = -Mdi_1/dt$ and $E_1 = -Mdi_2/dt$. 