GAS LAWS AND KINETIC THEORY

IDEAL GAS LAW

- The pressure of the atmosphere at sea level is $1.01 \times 10^5 N/m^2$.
- Boyle's Law: For a gas at a constant temperature the product of pressure and volume is constant: $PV = constant$, or $P_1V_1 = P_2V_2$.
- Law of Charles and Gay-Lussac: For a gas at a constant pressure the volume is proportional to the temperature: $V/T = constant$, or $V_1/V_2 = T_1/T_2$.
  Remember: The temperature MUST be in Kelvin!!
- Ideal gas law: $PV = nRT$, where $R = 8.314 J/(mol \cdot K)$ is the universal gas constant and n corresponds to the quantity of gas in *moles*.
- The number of *moles* is defined as the ratio of the total mass M of the gas and the *Molecular Mass* (in g/mol) of the gas: $n = M/MolecularMass$. The *Molecular mass* of a gas is different for each gas. For example: Molecular oxygen $(O_2) = 32\ g/mol$, Helium $(He) = 4\ g/mol$ etc.
- 1 *mole* of any gas contains the same number of particles.
  This number is $N_A = 6.02 \times 10^{23} Molecules/mol$ and is called Avogadro's number.

KINETIC THEORY OF GASES

- The temperature of a gas can be related to the internal motion of the molecules.
- The ideal gas approximation yields: $PV = 2/3N\overline{KE}$, where $\overline{KE}$ is the mean kinetic energy of an individual molecule and N is the total number of molecules in the gas.
- The internal energy U of a gas is defined as $U = N\overline{KE}$ and thus $PV = 2/3U$.
- The relation between the mean kinetic energy and the temperature is given by $\overline{KE} = 3/2kT$, where $k = R/N_A$ is the Boltzmann constant: $k = 1.3807 \times 10^{-23} J/K$.
- The *root mean square* velocity is the square root of the mean velocity squared: $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{3P/\rho}$, where ρ is the density of the gas.
- v_{rms} can also be expressed as $v_{rms} = \sqrt{3kT/m}$, where m is the mass of one individual molecule.
- The mass of one molecule can be calculated from Avogadro's number and the Molecular mass of the gas: $m = MolecularMass/N_A$.
- Other helpful relations: The total mass M of a gas is the product of the Molecular Mass and the number of moles n: $M = MolecularMass \times n$
  The total number of gas molecules in a gas N is the product of the number of moles n and Avogadro's number N_A: $N = nN_A$.
- Barometric formula: The pressure at height h above height zero is given by $P = P_0e^{-mgh/kT}$.

THERMODYNAMICS

THERMODYNAMIC SYSTEMS AND ENERGY CONSERVATION

- A thermodynamic system is any collection of objects considered together. Everything else is considered the environment.
- 0^{th} Law: (Thermal Equilibrium) If two objects are in thermal equilibrium with a third object they are also in thermal equilibrium with each other.
- The internal energy is defined as the total energy within the system. If no phase changes are involved the internal energy is proportional to the temperature of the system.
- 1^{st} Law: (Energy Conservation) The change of internal energy ΔU is equal to the heat Q added to the system minus the work W done by the system: $\Delta U = Q - W$.
- Adiabatic: In an adiabatic process no heat is exchanged with the environment (fast process): $Q = 0 \rightarrow \Delta U = -W$.
- Isothermal: In an isothermal process the temperature (internal energy) does not change: $\Delta U = 0 \rightarrow Q = W$.
- Isobaric: In an isobaric process the pressure does not change. The work done by the system is then equal to the product of the pressure times the change of the volume: $W = P\Delta V$.
- In general, the work is equal to the area under the path in the P-V diagram. If a system changes from P_1 and V_1 straight to P_2 and V_2 the work done is $W = 1/2(P_2 + P_1)(V_2 - V_1)$.
- Isochoric: In an isochoric process the volume of a system does not change. If the volume is constant no work is done: $W = 0 \rightarrow \Delta U = Q$.

CARNOT ENGINE

- In a reversible process there is no wasted energy and the system is very nearly in equilibrium at all times.
- A carnot cycle is an idealized reversible process of an ideal gas between a cold and hot reservoir and consists of four reversible processes (two adiabatic and two isothermal). Heat Q_H is taken out of a hot reservoir and some of this energy is converted to work and the rest is added as heat Q_C to the cold reservoir.
- The thermal efficiency of a carnot cycle (Heat engine) is given by the ratio of the resulting work by the heat input: $\textit{Thermal efficiency} = W/Q_H$. The work is equal to the heat flow from the hot to the cold reservoir: $W = Q_H - Q_C$.
- The ratio Q_C/Q_H is proportional to the ratio of the temperatures T_C/T_H (in Kelvin!!) and thus: $\textit{Thermal efficiency} = 1 - T_C/T_H$.
- The coefficient of performance ($c.p.$) of a refrigerator is given by: $c.p. = Q_C/W = T_C/(T_H - T_C)$. The coefficient of performance ($c.p.$) of a heat pump is given by: $c.p. = Q_H/W = T_H/(T_H - T_C)$.

ENTROPY

- 2^{nd} Law: Heat cannot, by itself, pass from a colder to a warmer body.
- The entropy ΔS is defined as the ratio of absorbed Heat Q divided by the Temperature T in a reversible process: $\Delta S = Q/T$.