Nuclear Arrhenius-Type Plots

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Arrhenius-type plots for a multifragmentation process, defined as the transverse energy dependence of the single-fragment emission probability [\ln(1/p_b) vs 1/\sqrt{E_t}] have been studied by examining the relationship of the parameters \(p_b\) and \(E_t\) to the intermediate-mass fragment multiplicity \(\langle n \rangle\). The linearity of these plots reflects the correlation of the fragment multiplicity with the transverse energy. These plots contain equivalent information as \(N_v\) vs \(\langle n \rangle\) plots and do not provide thermal scaling information about fragment production contained in true Arrhenius plots.  [S0031-9007(97)05259-9]

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About a hundred years ago, the Swedish chemist, Svante Arrhenius, discovered that the rate of chemical reactions increases with temperature [1]. Specifically, the chemical reaction rate constant \(k\) is related to the absolute temperature \(T\) by

\[ k \approx \exp(-E_a/T), \]  

where \(E_a\) is the activation energy of the chemical reaction. The linear relationship between \(\ln(k)\) and \(1/T\) is widely known as Arrhenius plot in chemistry and is associated with thermal equilibrium processes.

Recently, Arrhenius-type plots have been used to study the statistical [2–5] and dynamical [6] properties of fragment emissions in heavy ion reactions. In the intermediate incident energy range, between a few tens of MeV to 100 MeV per nucleon, the production of intermediate mass fragments (IMF) \((3 \leq Z \leq 20)\), also known as multifragmentation, is an important decay mode of highly excited nuclear systems [7]. Calculations suggest that the fragments are produced in the phase coexistence region. Thus, understanding the mechanisms of fragment formation may provide an insight to the liquid gas phase transition in nuclear matter [7].

Several experiments [2–4] gave evidence that in collisions characterized by a given value of the total transverse energy of detected charged particles,

\[ E_t = \sum E_i \sin^2 \theta_i, \]  

the IMF multiplicity distribution may be fitted by a binomial distribution,

\[ P_n^m = \frac{m!}{n!(m-n)!} p_b^n (1-p_b)^{m-n}, \]  

where \(n\) is the IMF multiplicity and the parameter \(m\) is interpreted as the number of times the system tries to emit a fragment. The probability of emitting fragments can be reduced to a single-particle emission probability \(p_b\) for true binomial distributions; the binomial parameters \(m\) and \(p_b\) are related to the mean and variance of the fragment multiplicity distributions according to

\[ \langle n \rangle = mp_b, \]  

\[ \sigma^2_n = \langle n \rangle (1-p_b). \]  

Past investigations found a simple linear relationship between \(\ln(1/p_b)\) and \(1/\sqrt{E_t}\) (nuclear Arrhenius-type plots) for several projectile-target combinations and incident energies [2–4]. By assuming a linear relationship between \(\sqrt{E_t}\) and temperature \(T\) and from the linearity of the observed \(\ln(1/p_b)\) vs \(1/\sqrt{E_t}\) plot, it has been inferred that a thermal scaling of the multifragment processes might be a general property [2–4]. In this picture, the observed “linear” dependence of \(\ln(1/p)\) upon \(1/T\) would be reflecting the dependence of fragment emission probabilities upon a common fragment emission barrier \(B\).

However, unlike chemical reactions, \(p\) and \(T\) were not measured directly in Refs. [2–4]. There, the validity of the Arrhenius-type plots relies on two assumptions: (i) that \(E_t\) is proportional to the excitation energy \(E^*\) and, therefore, should be proportional to \(T^2\), and (ii) that \(p_b\) obtained by fitting the fragment multiplicity distributions is the elementary emission probability \(p\). This Letter will examine the above assumptions and investigate the underlying reasons for the linearity exhibited by the Arrhenius-type plots obtained in many systems.

The assumption that temperature is proportional to the square root of the excitation energy, \(T \propto \sqrt{E^*}\), is valid for compound nuclei formed at low-to-moderate temperatures. Some relationship between the transverse energy and excitation energy, and therefore temperature, may also be obtained for compound nuclei, provided adjustment is made for the Coulomb barrier and for neutron emission [8]. For the intermediate-energy heavy ion reactions such as \(^{36}\text{Ar} + ^{197}\text{Au}\) at \(E/A = 35\) to 110 MeV, where linearity of the Arrhenius-type plots has been observed, however, the final states contain fast particles emitted from the overlap region of projectile and target...
as well as delayed emission from projectilelike and targetlike residues. In particular, particle production from the overlap participant region dominates in central collisions [9–12], and the transverse energy from this region is strongly affected by the collective motion [13]. There has never been any unambiguous experimental evidence supporting that \( \sqrt{E_t} \) is proportional to the temperature and, in fact, the evidence is to the contrary. Theoretical models such as the statistical multifragmentation model suggest that \( T \) need not be proportional to \( \sqrt{E_t} \) [5]. The temperature measurements using both the excited states populations and isotope yield ratios show that the temperature dependence on charged particle multiplicities for Au + Au [14–16] and Ar + Au reactions at 35A MeV is very weak [17]. Since \( E_t \) is directly proportional to the charged particle multiplicities [18], these temperature measurements thus imply that the assumption, \( T \propto \sqrt{E_t} \), is not valid. Based on this argument alone, the \( \ln(1/p_b) \) vs \( 1/\sqrt{E_t} \) plot is not the Arrhenius plot analogous to that observed in chemical reactions. The breakdown of the \( T \propto \sqrt{E_t} \) assumption suggests that the previous interpretation of thermal scaling on emission probabilities, charge distributions, and azimuthal correlations [2–4,19–22] should be reexamined.

Next, we will examine the assumptions used to extract the fragment emission probability. There is no a priori reason for the emitted fragments to prefer binomial statistics [2–4] or Poissonian statistics [23]. In Poissonian statistics, the probability of emitting \( n \) fragments is
\[
P_p(n) = \frac{\lambda^n}{n!} e^{-\lambda},
\]
where \( \lambda = \langle n \rangle \) is the mean. The major difference between the binomial and Poisson distributions is the ratio of the variance to the mean, \( \sigma^2/\langle n \rangle \), where \( \sigma^2/\langle n \rangle = 1 \) for Poisson distribution and \( <1 \) for binomial distribution. It has been demonstrated that constraints from conservation laws reduce the width of the Poisson distributions to much less than one [24]. For example, if charge conservation constraint is applied to a Poissonian distribution, Eq. (7) is modified to (see Appendix)
\[
P_p(n, \alpha) \propto \frac{\lambda^n}{n!} e^{-\lambda} e^{-\alpha(n-\lambda)^2},
\]
where \( \alpha \) is the charge constraint factor.

For small \( \alpha \), the mean fragment multiplicity for Eqs. (7) and (8) are nearly the same; \( \langle n \rangle = \lambda \). Figure 1 shows three modified Poisson distributions of Eq. (8) (solid and open points) for \( \lambda = 3,6, \) and \( 10 \), and \( \alpha = 0.1 \). To illustrate that distributions such as Eq. (8) whose values of \( \sigma^2/\langle n \rangle \) are less than one can be described by binomial distributions, we used Eqs. (4) and (5) to determine the binomial parameters \( m \) and \( p_b \) whose values are listed in Fig. 1. The solid and dashed lines are binomial distributions of Eq. (3). The agreement between the two distributions is very good. However, in this context, \( m \) and \( p_b \) are mainly fit parameters used to describe the modified Poisson distributions of Eq. (8), and \( p_b \) is not an elementary emission probability. If the small values of \( \sigma^2/\langle n \rangle \) reflect the constraints of conservation laws of Eq. (8) observed in Ref. [24], the reducibility of fragment emission to binomial distributions shown by Refs. [2–4] does not imply any fundamental significance for the parameters \( m \) and \( p_b \) thereby extracted. If \( p_b \) is not an elementary emission probability, then the second assumption used in constructing the nuclear Arrhenius plots also breaks down.

Even though the \( \ln(1/p_b) \) vs \( 1/\sqrt{E_t} \) plots constructed in heavy ion reactions are not true Arrhenius plots, analyses of many systems [2–4,25] suggest that the approximate linearity of such plots may be universal. To explain this appealing systematics, we examine the correlations between the observable \( E_t \), parameter \( p_b \), and the fragment multiplicity \( n \). Since \( E_t \) is obtained from the measured energies of both the light particles and fragments [see Eq. (2)], the energy and multiplicities are related by
\[
E_t = (N_C - \langle n \rangle)E_t^{LP} + \langle n \rangle E_t^{IMF},
\]
where \( N_C \) is the total charged multiplicities, \( E_t^{LP} \) and \( E_t^{IMF} \) are the average transverse energy of a light particle and an IMF, respectively. The second term on the right-hand side of Eq. (9) is an autocorrelation between \( E_t \) and \( \langle n \rangle \). At \( \langle n \rangle = 4 \), IMF’s contribute about 20% of the calculated \( E_t \) in Ref. [4]. The contributions from the IMF autocorrelation may vary with reactions but, in general, they do not exceed 50% of \( E_t \) [26,27]. In addition to the trivial autocorrelation [6,25,27], the dependence of \( E_t \) on \( \langle n \rangle \) comes from the dependence of \( N_C \) on \( \langle n \rangle \) which can be approximated by [9–12]
\[
N_C = a' + b'\langle n \rangle,
\]
where \( a' \) corresponds to the typical number of light charged particles emitted before any IMF is emitted and
The number of light charged particles emitted for each IMF emitted. The nearly linear dependence of \( N_C \) as a function of \( \langle n \rangle \) can be understood as a result of the participant mass changing with impact parameter. There is some nonlinear dependence of \( \langle n \rangle \) at very high \( N_C \), where \( \langle n \rangle \) saturates for central collision, and at very low \( N_C \), where fluctuations in a small value of \( n \) prevent a sharp cutoff in \( N_C \). Except for very large and small values of \( N_C \), Eq. (9) can then be rewritten into

\[
E_t = a + b \langle n \rangle = b(a/b + \langle n \rangle),
\]

where \( a = a' E_t^{1P} \) is the threshold transverse energy associated with light particles emitted before any IMF and \( b = (b' - 1) E_t^{1P} + E_t^{1MF} \).

The binomial fit parameter \( m \) is nearly constant as a function of the transverse energy \( E_t \) [2–4]. Thus the plots of \( \ln(1/p_b) \) vs \( 1/\sqrt{E_t} \) can now be reduced to \( \ln(1/\langle n \rangle) \) vs \( 1/\sqrt{a/b + \langle n \rangle} \) according to Eqs. (4) and (11). Figure 2 shows the dependence of \( 1/\langle n \rangle \) and \( 1/\sqrt{a/b + \langle n \rangle} \) for \( \langle n \rangle \) ranging from 0.25 to 5.0, typical values observed in multifragmentation of heavy ion reactions [2–4,9–12]. For \( a/b = 0 \) the curve is concave, and for \( a/b = 1 \) the curve becomes slightly convex. In the middle region, where \( a/b = 0.5 \), the curve is nearly linear. Thus the linearity of the Arrhenius-type plots merely reflects the correlation of the fragment multiplicity with \( E_t \), when the value of \( a/b \) is about 0.5 which naturally arises from the intrinsic linear dependence of \( \langle n \rangle \) on \( E_t \).

To illustrate the self-correlation effect in nuclear Arrhenius-type plots, the published data for the Ar + Au collisions at \( E/A = 110 \text{ MeV} \) [2,4] are plotted as filled points in the left panel of Fig. 3. The solid line is the self-correlation of \( 1/\langle n \rangle \) and \( 1/\sqrt{a/b + \langle n \rangle} \), scaled according to Eqs. (4) and (11) using the experimentally determined values of \( m = 12 \) [2,4], \( a/b = 0.5 \), \( b = 213 \text{ MeV} \) [2,4,29]. The good agreement between the data and the self-correlation confirms that the linearity observed in the Arrhenius-type plot comes mainly from the linear dependence of \( E_t \) (or \( N_C \)) on \( \langle n \rangle \) with a nonzero offset in Eq. (11) and from the limited range of \( \langle n \rangle \).

The nonzero value of \( a \) arises from Eq. (10) because \( a' \) is not zero. One would expect that a relation between \( 1/\langle n \rangle \) and \( 1/\sqrt{N_C} \) similar to those shown in Fig. 2 should be observed. The right panel of Fig. 3 shows the plot of \( 1/\langle n \rangle \) vs \( 1/\sqrt{N_C} \) for \( \text{Ar} + \text{Au} \) reaction at \( E/A = 110 \text{ MeV} \) [2,4,28]. The linearity of the plot is comparable to most Arrhenius-type plots [2–4,25]. Since \( \langle n \rangle \) and \( N_C \) are much less affected by the energy resolution or thickness of the detectors [27], they are better observables than \( p_b \) and \( E_t \) used in the Arrhenius-type plots. Figure 3 suggests that the Arrhenius-type plots contain essentially the same information as the much simpler plots of the IMF multiplicity, \( \langle n \rangle \), as a function of charged particle multiplicity \( (N_C) \) published in many studies [9–12].

In summary, without invoking the interpretation of fragment emission probability in binomial distributions or the temperature dependence of \( E_t \), the linearity of the Arrhenius-type plots can be reproduced from the linear dependence of \( E_t \) (or \( N_C \)) on fragment multiplicity \( \langle n \rangle \). \( \ln(1/p_b) \) vs \( 1/\sqrt{E_t} \) are not true Arrhenius plots. They carry the same information as \( N_C \) vs \( \langle n \rangle \) plots.

Appendix.—When many independent processes contribute to the multiplicity of certain particles, each with a small probability, then the multiplicity is expected to obey a Poisson distribution. Within the grand canonical ensemble, a Poisson distribution also follows. In general, the Poisson distribution is associated with a large overall system compared to the subsystem studied. However, in heavy ion reactions, the systems studied are always finite, constrained by conservation laws such as the overall energy, charge, and mass conservation within the participant region.

![FIG. 2. Dependence of 1/n on 1/\sqrt{a/b + \langle n \rangle} for a/b = 0, 0.5, and 1.0.](image)

![FIG. 3. Left panel: Arrhenius-type plot for the Ar+Au reaction at E/A = 110 MeV [2,4]. The solid line is the self-correlation of 1/\langle n \rangle as a function of 1/\sqrt{0.5 + \langle n \rangle} scaled according to Eqs. (4) and (11) with the experimental values of m = 12, b = 213 MeV [2,4,29]. Right panel: dependence of 1/\langle n \rangle on 1/\sqrt{N_C}.](image)
To investigate the minimal effect of the above-mentioned constraints on the multiplicity distribution of IMF, we first consider a situation where probabilities of emitting various individual particles are independent, with multiplicities being governed by a Poisson distribution in the absence of any constraint. When multiplicities of individual IMFs are governed by a Poisson distribution, then the overall multiplicity of IMFs is also governed by a Poisson distribution [30]. For simplicity, we next impose only a single constraint, that of the charge conservation, and, possibly, to some extent with the amount of IMF. The constraint modifies the Poisson distribution to

\[ P_m(n) \propto \sum_{\{n_{\tau}: \tau \in \text{IMF}\}} \delta_{n} \sum_{\mu} n_{\mu} \prod_{\tau} [P^\tau_{\mu}(n_{\tau})] \times \sum_{Z_{\text{oth}}} P_{\text{oth}}(Z_{\text{oth}}) \delta_{Z_{\text{oth}}} \sum_{\nu} n_{\nu} \zeta_{\nu}, \]  

(12)

where \( P^\tau_{\mu} \) is a Poisson distribution for fragment \( \tau \), \( P_{\text{oth}} \) is the charge distribution of all particles other than IMF, and \( Z \) is the charge of the emitting source, \( Z = \langle Z_{\text{oth}} \rangle + \sum_{\nu} (n_{\nu}) \zeta_{\nu} \). If the system emits much more particles other than IMF, from the central-limit theorem, \( P_{\text{oth}} \) is expected to be close to a Gaussian function,

\[ P_{\text{oth}}(Z_{\text{oth}}) \propto \exp \left( -\frac{(Z_{\text{oth}} - \langle Z_{\text{oth}} \rangle)^2}{2\sigma^2(Z_{\text{oth}})} \right). \]

(13)

The dispersion in Eq. (13) should be associated primarily with light (\( Z = 1 \) and \( Z = 2 \)) particles in the participant region, and, possibly, to some extent with the amount of charge that the spectator matter carries off. From the central-limit theorem, given Poisson distributions, the dispersion is then \( \sigma^2(Z_{\text{oth}}) \geq (n_{Z=1}) + 4(n_{Z=2}) \). Substituting Eq. (13) into Eq. (12) we get

\[ P_m(n) \propto \sum_{\{n_{\tau}: \tau \in \text{IMF}\}} \delta_{n} \sum_{\mu} n_{\mu} \prod_{\tau} [P^\tau_{\mu}(n_{\tau})] \times \exp \left( -\frac{\sum_{\nu} (n_{\nu} - \langle n_{\nu} \rangle \zeta_{\nu})^2}{2\sigma^2(Z_{\text{oth}})} \right). \]

(14)

Finally, to assess the effect of the constraining factor in Eq. (14) we approximate the charge in the exponential by its average value, \( \zeta_{\nu} = \langle \zeta \rangle = \sum_{\mu} \langle n_{\mu} \zeta_{\mu} \rangle / \langle n \rangle \), and obtain

\[ P_m(n) \propto P_p(n) \exp\left[ -\alpha (n - \langle n \rangle)^2 \right], \]

(15)

where \( \alpha = \langle \zeta \rangle^2 / 2\sigma^2(Z_{\text{oth}}) \leq \langle \zeta \rangle^2 / 2(n_{Z=1}) + 4(n_{Z=2}) \). Charged particle multiplicities measured in Ref. [29] suggest \( \alpha \leq 0.2 \).

The main effect of the constraint in Eq. (15) is to narrow the multiplicity distribution, compared to the Poisson distribution. Depending on correlations between charge, mass, and energy within the rest of the system, the other constraints may affect the multiplicity distributions further [5].

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