Electromagnetic (gamma) decay
Coupling between nucleons and EM field

- Electromagnetic and weak interactions can be treated as perturbations.
- Emission of a $\gamma$-ray is caused by the interaction of the nucleus with an external electromagnetic field.
- Besides $\gamma$-decay, electromagnetic perturbation can also induce nuclear decay through *internal conversion* whereby one of the atomic electrons is ejected. This is particularly important for the heavy nuclei.
- The decay can also proceed by *creating an electron-positron pair* (internal pair creation).
- Since the nuclear wave function has a definite angular momentum, the external EM field has to be decomposed in spherical multipoles. The quantization and multipole expansion of EM field is straightforward by *tedious*. 

---

![Diagram of $^{60}_{27}$Co and $^{60}_{28}$Ni decay](image)
Electromagnetic Decay
Kinematics of photon emission

\[ E_i = E_f + E_\gamma + T_0 \]

recoil term

For \( A=100 \) and \( E_\gamma=1 \text{ MeV} \), the recoil energy is about 5 eV. But the natural linewidth of the radiation is even smaller.

The emission of photons without recoil is possible if one implants the nucleus in a lattice. In such a case, the recoil is taken by the whole lattice and not by a single nucleus. If

\[ \hbar \omega_{\text{lattice}} \gg T_0 \]

then, quantum-mechanically, the energy of the emitted gamma radiation takes away the total energy difference (Mössbauer effect – 1958 – or recoilless nuclear resonance fluorescence).
Multipole expansion of electrostatic potential

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{1}{|\vec{r} - \vec{r}'|} \rho(\vec{r}') \, d^3 r' \]

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') \, d^3 r' \]

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \ldots \right) \]

\[ \vec{p} = \int \vec{r}' \rho(\vec{r}') \, d^3 r' = \sum_{k=1}^{N} q_k \vec{r}'_k \quad \text{dipole} \]

\[ Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\vec{r}') \, d^3 r' \quad \text{quadrupole} \]
\[ V = -\frac{1}{c} j_\mu A^\mu \]

This contains both electric and magnetic interactions

nuclear current \hspace{1cm} \text{external EM field}

\[ \tilde{A}(\vec{r}, t) = \frac{1}{N} \sum_{k, \eta} \left\{ b_{k\eta} \tilde{e}_\eta e^{i(\vec{k} \cdot \vec{r} - \omega t)} + b_{k\eta}^+ \tilde{e}_\eta e^{-i(\vec{k} \cdot \vec{r} + \omega t)} \right\} \]

two polarization states \hspace{1cm} \text{photon creation operator}

\[ \tilde{A}(\vec{r}, t) = \sum_{\lambda, \mu} \tilde{A}_{\lambda, \mu}(\vec{r}, t) \]

\[ \text{multipole expansion} \hspace{1cm} \lambda = 1, 2, 3, \ldots \]

But what about \( \lambda = 0 \)?

The typical gamma-rays in nuclear transitions have energies less than 10 MeV, corresponding to wave numbers of the order \( k \sim 1/20 \text{ fm}^{-1} \) or less. The multipole operators give contributions only within the nuclear volume. That is, in most cases \( kr \ll 1 \) and the above series may be approximated by the first term in the expansion alone ("The long-wavelength limit"). We are now ready to calculate the contribution of each multipole order to the transition probability from an initial nuclear state to a final state.

\[ \mathcal{W}(\lambda; J_i \xi \rightarrow J_f \xi) = \frac{8\pi(\lambda + 1)k^{2\lambda+1}}{\lambda[(2\lambda + 1)!!]^2} \hbar B(\lambda; J_i \xi \rightarrow J_f \xi) \]

transition probability \hspace{1cm} \text{reduced transition probability}
Electromagnetic Rates

\[ B(\lambda; J_i \xi \rightarrow J_f \xi) = \sum_{\mu M_f} \left| \langle J_f M_f \xi | O_{\lambda \mu} | J_i M_i \xi \rangle \right|^2 = \frac{1}{2J_i + 1} \left| \langle J_f \xi | O_{\lambda \mu} | J_i \xi \rangle \right|^2 \]

\[ O_{\lambda \mu}(E\lambda) = \sum_{i=1}^{A} e(i) r_i^\lambda Y_{\lambda \mu}(\Omega_i) \]

\[ O_{\lambda \mu}(M\lambda) = \sum_{i=1}^{A} \left[ g_s(i) \bar{s}_i + g_l(i) \frac{2\bar{l}_i}{\lambda + 1} \right] \bar{\nabla}_i [ r_i^\lambda Y_{\lambda \mu}(\Omega_i) ] \]

gyromagnetic factors

Selection Rules

\[ \frac{W(\lambda + 1)}{W(\lambda)} \sim (kR)^2 \]

large reduction in probability with increasing multipolarity order!

\[ |J_f - J_i| \leq \lambda \leq J_f + J_i \]

\[ PO_{\lambda \mu}(E\lambda)P^{-1} = (-1)^\lambda O_{\lambda \mu}(E\lambda), \quad PO_{\lambda \mu}(M\lambda)P^{-1} = (-1)^{\lambda+1} O_{\lambda \mu}(M\lambda) \]

\[ \pi_i \pi_f = (-1)^\lambda \]

\[ \pi_i \pi_f = (-1)^{\lambda+1} \]
Magnetic transitions are weaker than electric transitions of the same multipolarity.

\[ 6^+ \rightarrow E2, (M3, E4, \ldots) \]

\[ 5^+ \rightarrow M1, E2, (M3, E4, \ldots) \]

Mixed M1+E2 transition.

A decay scheme: E1, E2 and M1 transitions.

\[ ^{140}\text{Gd} \]
What is the radiative width corresponding to:

a) 1 fs
b) 1 ps
c) 1 ns
d) 1 µs

gamma decays?
Internal conversion

An atomic electron is ejected instead of gamma-ray (atomic Auger effect). Atomic electrons, especially those in the inner orbits, such as K- and L-orbits, spend a large fraction of their time in the vicinity of the nucleus, the source of the EM field.

\[ T_{e^-} = (E_i - E_f) - B_n \]

Electron kinetic energy Electron binding energy

Virtual photon exchange. This process can occur between I=0 states!

Electron spectrum: internal conversion + beta-decay background

Important for heavy nuclei (large Z): it goes as \( Z^3 \) !!!
Internal pair production

An electron-positron pair is emitted in the place of gamma-ray. This can happen if the energy of the decay

\[ E_\gamma > 2m_e c^2 = 1.02 \text{MeV} \]

Usually, this process is several orders of magnitude retarded compared to allowed gamma-ray decays. The pair production rate is largest where the internal conversion rate is smallest. That is in the region of low atomic number and high transition energies.
2-gamma decay

\[ \gamma \gamma = 2E1+2M1 \]

M1 and E1 transitions are similar in strength!

Best candidates: \(^{16}\text{O}, \, ^{40}\text{Ca}, \, ^{90}\text{Zr}\)

Provides important structural information about nuclear electric polarizability and diamagnetic susceptibility.

Another interesting candidate is \(^{137}\text{Ba}\). Why?
\( \gamma \)-ray laser?

Hafnium-178m has a long half-life of 31 years and a high excitation energy of 2.4 MeV. As a result, 1 kilogram of pure \(^{178m}\text{Hf}\) contains approximately \(10^{12}\) J of energy. Some estimates suggest that, with accelerated decay, 1 gram of 100-percent isomeric \(^{178m}\text{Hf}\) could release more energy than the detonation of 200 kilograms of TNT.

2008 LLNL report: “Our conclusion is that the utilization of nuclear isomers for energy storage is impractical from the points of view of nuclear structure, nuclear reactions, and of prospects for controlled energy release. We note that the cost of producing the nuclear isomer is likely to be extraordinarily high, and that the technologies that would be required to perform the task are beyond anything done before and are difficult to cost at this time.”

https://str.llnl.gov/str/JulAug05/Becker.html
http://defensetech.org/2006/06/13/superbomb-or-crapshoot/
http://www.theguardian.com/science/2008/aug/14/particlephysics.research