Proton emitters

Unbound states

Discrete (bound) states

daughter

proton

\[ \log_{10} |\phi(r)| \]

quasistationary 1d_{3/2} orbital

\[ Q_p = 1139 \text{ keV} \]

\[ ^{147}\text{Tm} \]

\[ r_B \]

\[ r_2 \]
The landscape of two-proton radioactivity
E. Olsen et al,
PRL 111, 139903 (2013); E: PRL 111, 139903 (2013)
Energy - angle 2D correlation

3-body model prediction

Experiment

45Fe

Y-system
Electromagnetic (gamma) decay
Coupling between nucleons and EM field

- Electromagnetic and weak interactions can be treated as perturbations
- Emission of a $\gamma$-ray is caused by the interaction of the nucleus with an external electromagnetic field
- Besides $\gamma$-decay, electromagnetic perturbation can also induce nuclear decay through *internal conversion* whereby one of the atomic electrons is ejected. This is particularly important for the heavy nuclei.
- The decay can also proceed by *creating an electron-positron pair* (internal pair creation)
- Since the nuclear wave function has a definite angular momentum, the external EM field has to be decomposed in spherical multipoles. The quantization and multipole expansion of EM field is straightforward by *tedious*. 
Electromagnetic Decay
Kinematics of photon emission

\[ E_i = E_f + E_\gamma + T_0 \]
recoil term

For \( A=100 \) and \( E_\gamma=1 \text{ MeV} \), the recoil energy is about 5 eV. But the natural linewidth of the radiation is even smaller.

The emission of photons without recoil is possible if one implants the nucleus in a lattice. In such a case, the recoil is taken by the whole lattice and not by a single nucleus. If

\[ \hbar \omega_{\text{lattice}} \gg T_0 \]

then, quantum-mechanically, the energy of the emitted gamma radiation takes away the total energy difference (Mössbauer effect – 1958 – or recoilless nuclear resonance fluorescence).
Multipole expansion of electrostatic potential

\[
V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{1}{|\vec{r} - \vec{r}'|} \rho(\vec{r}') \, d^3 r'
\]

\[
V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') d^3 r'
\]

\[
V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \cdots \right)
\]

\[
\vec{p} = \int r' \rho(\vec{r}') d^3 r' = \sum_{k=1}^{N} q_k \vec{r}'_k
\]

dipole

\[
Q_{ij} = \int (3x'_ix'_j - r'^2 \delta_{ij}) \rho(\vec{r}') d^3 r'
\]

quadrupole
\[ V = -\frac{1}{c} j_\mu A^\mu \] 

This contains both electric and magnetic interactions

\[ \vec{A}(\vec{r}, t) = \frac{1}{N} \sum_{k, \eta} \left\{ b_{k\eta} \vec{e}_\eta e^{i(\vec{k}\cdot\vec{r} - \omega t)} + b^+_{k\eta} \vec{e}_\eta e^{-i(\vec{k}\cdot\vec{r} + \omega t)} \right\} \]

two polarization states

\[ \vec{A}(\vec{r}, t) = \sum_{\lambda, \mu} \tilde{A}_{\lambda \mu}(\vec{r}, t) \] 

multipole expansion \( \lambda = 1, 2, 3 \ldots \)

But what about \( \lambda = 0 \)?

The typical gamma-rays in nuclear transitions have energies less than 10 MeV, corresponding to wave numbers of the order \( k \sim 1/20 \text{ fm}^{-1} \) or less. The multipole operators give contributions only within the nuclear volume. That is, in most cases \( kr \ll 1 \) and the above series may be approximated by the first term in the expansion alone ("The long-wavelength limit"). We are now ready to calculate the contribution of each multipole order to the transition probability from an initial nuclear state to a final state.

\[ \mathcal{W}(\lambda; J_i \xi_i \rightarrow J_f \xi_f) = \frac{8 \pi (\lambda + 1) k^{2\lambda + 1}}{\lambda [(2 \lambda + 1)!!]^2} \frac{1}{\hbar} B(\lambda; J_i \xi_i \rightarrow J_f \xi_f) \]

transition probability

\[ \mathcal{W}(\lambda; J_i \xi_i \rightarrow J_f \xi_f) = \frac{8 \pi (\lambda + 1) k^{2\lambda + 1}}{\lambda [(2 \lambda + 1)!!]^2} \frac{1}{\hbar} B(\lambda; J_i \xi_i \rightarrow J_f \xi_f) \]

reduced transition probability