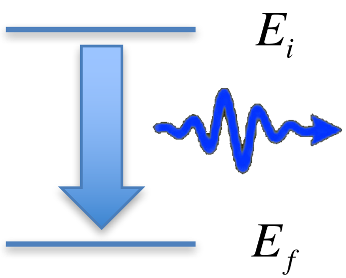
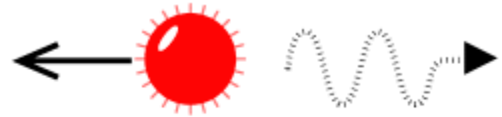
Electromagnetic Decay

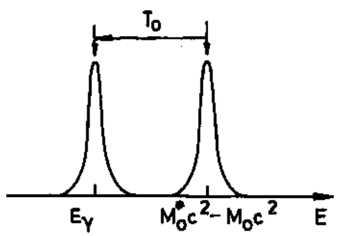
Kinematics of photon emission





$$E_i = E_f + E_\gamma + T_0$$

recoil term



$$T_0 \approx \frac{E_\gamma^2}{2M_0c^2}$$

For A=100 and E_γ=1 MeV, the recoil energy is about 5 eV. But the natural linewidth of the radiation is even smaller.

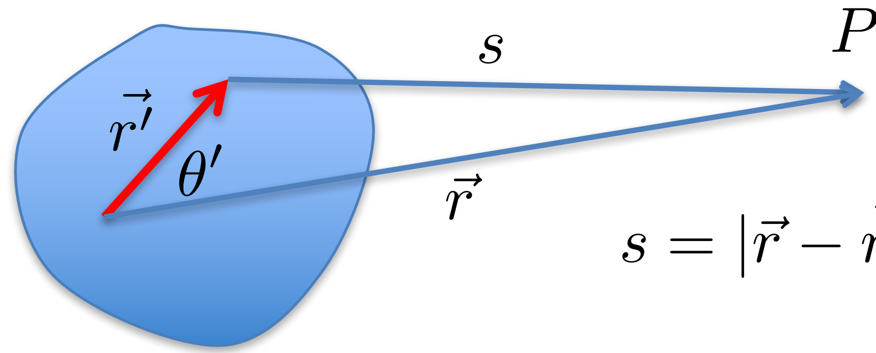
The emission of photons without recoil is possible if one implants the nucleus in a lattice. In such a case, the recoil is taken by the whole lattice and not by a single nucleus. If

$$\hbar\omega_{lattice} >> T_0$$

then, quantum-mechanically, the energy of the emitted gamma radiation takes away the total energy difference (Mössbauer effect – 1958 – or recoilless nuclear resonance fluorescence).

Multipole expansion of electrostatic potential

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{|\vec{r}-\vec{r'}|}\rho(\vec{r'})\, d^3r'$$



$$s = |\vec{r}-\vec{r'}|$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r'})d^3r'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{\vec{p}\cdot\vec{r}}{r^3} + \frac{1}{2}\sum_{i,j} Q_{ij}\frac{x_i x_j}{r^5}\cdots\right)$$

$$\vec{p} = \int \vec{r'}\rho(\vec{r'})d^3r' = \sum_{k=1}^{N} q_k \vec{r'_k}$$ dipole

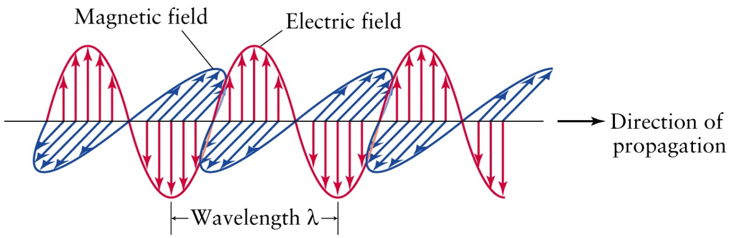
$$Q_{ij} = \int (3x'_i x'_j - r'^2\delta_{ij})\rho(\vec{r'})d^3r'$$ quadrupole

$$V = -\frac{1}{c} j_\mu A^\mu$$

nuclear current

external EM field

This contains both electric and magnetic interactions



$$\vec{A}(\vec{r},t) = \frac{1}{N}\sum_{\vec{k},\eta}\left\{ b_{\vec{k}\eta}\vec{\varepsilon}_\eta e^{i(\vec{k}\vec{r}-\omega t)} + b^+_{\vec{k}\eta}\vec{\varepsilon}_\eta e^{-i(\vec{k}\vec{r}+\omega t)} \right\}$$

two polarization states

photon creation operator

$$\vec{A}(\vec{r},t) = \sum_{\lambda\mu} \vec{A}_{\lambda\mu}(\vec{r},t) \longleftarrow$$

multipole expansion λ=1,2,3…

But what about λ=0?

The typical gamma-rays in nuclear transitions have energies less than 10 MeV, corresponding to wave numbers of the order k~1/20 fm^{-1} or less. The multipole operators give contributions only within the nuclear volume. That is, in most cases kr<<1 and the above series may be approximated by the first term in the expansion alone ("The long-wavelength limit"). We are now ready to calculate the contribution of each multipole order to the transition probability from an initial nuclear state to a final state.

$$\mathcal{W}(\lambda; J_i\zeta \rightarrow J_f\xi) = \frac{8\pi(\lambda+1)k^{2\lambda+1}}{\lambda\left[(2\lambda+1)!!\right]^2\hbar} B(\lambda; J_i\zeta \rightarrow J_f\xi)$$

transition probability

reduced transition probability

Electromagnetic Rates

$$B(\lambda; J_i\zeta \rightarrow J_f\xi) = \sum_{\mu M_f} \left| \langle J_f M_f \zeta | O_{\lambda\mu} | J_i M_i \xi \rangle \right|^2 = \frac{1}{2J_i + 1} \left| \langle J_f \zeta \| O_{\lambda\mu} \| J_i \xi \rangle \right|^2$$

$$O_{\lambda\mu}(E\lambda) = \sum_{i=1}^{A} e(i) r_i^{\lambda} Y_{\lambda\mu}(\Omega_i) \qquad \vec{j}_i = \vec{s}_i + \vec{l}_i$$

$$O_{\lambda\mu}(M\lambda) = \sum_{i=1}^{A} \left[g_s(i)\vec{s}_i + g_l(i) \frac{2\vec{l}_i}{\lambda + 1} \right] \vec{\nabla}_i \left[r_i^{\lambda} Y_{\lambda\mu}(\Omega_i) \right]$$

gyromagnetic factors

Selection Rules

$$\frac{\mathcal{W}(\lambda + 1)}{\mathcal{W}(\lambda)} \sim (kR)^2$$

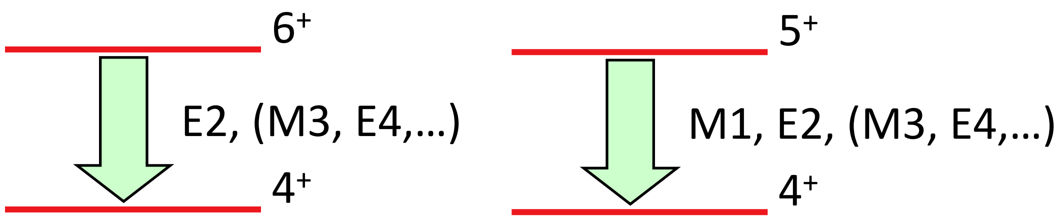
large reduction in probability
with increasing multipolarity order!

$$|J_f - J_i| \leq \lambda \leq J_f + J_i$$

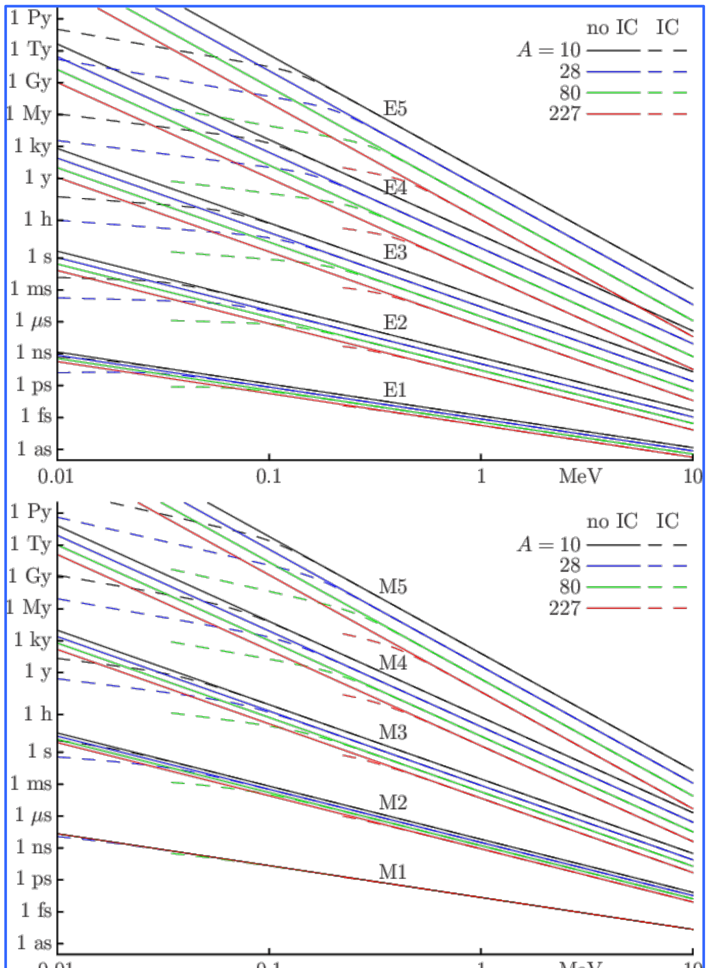
$$P O_{\lambda\mu}(E\lambda) P^{-1} = (-1)^{\lambda} O_{\lambda\mu}(E\lambda), \qquad P O_{\lambda\mu}(M\lambda) P^{-1} = (-1)^{\lambda+1} O_{\lambda\mu}(M\lambda)$$

$$\pi_i \pi_f = (-1)^{\lambda} \qquad \pi_i \pi_f = (-1)^{\lambda+1}$$

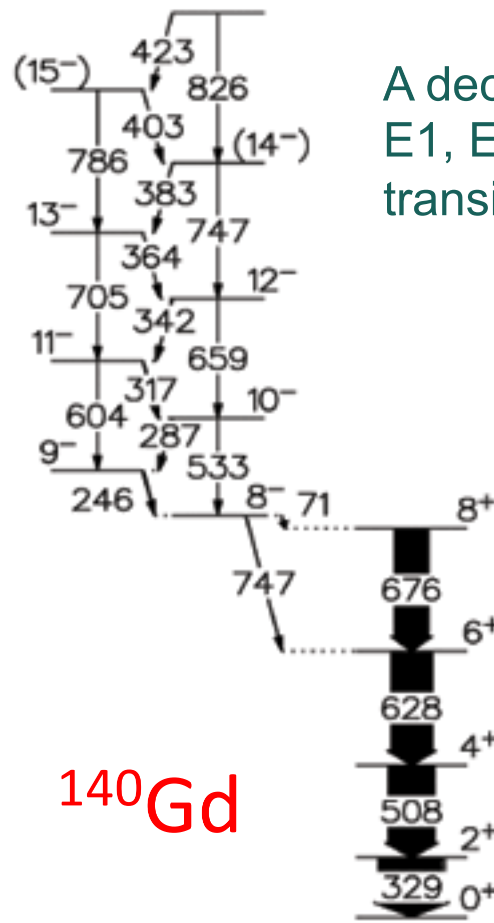
Magnetic transitions are weaker than electric transitions of the same multipolarity



mixed M1+E2 transition



A decay scheme: E1, E2 and M1 transitions



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