

# Global properties of atomic nuclei

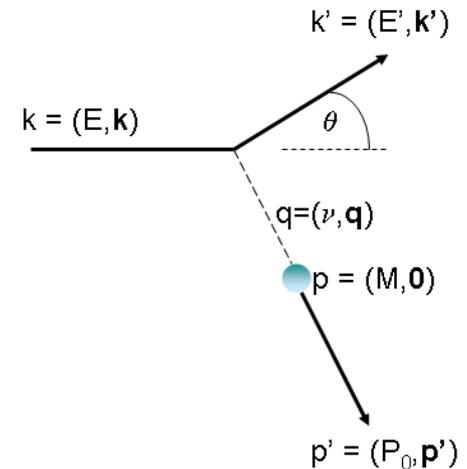
# How to probe nuclear size?

## Electron Scattering from nuclei

For low energies and under conditions where the electron does not penetrate the nucleus, the electron scattering can be described by the Rutherford formula. The *Rutherford formula* is an analytic expression for the differential scattering cross section, and for a projectile charge of  $e$ , it is

$$\left( \frac{d\sigma}{d \cos \theta} \right)_R = \frac{\pi}{2} \left[ \frac{\hbar c Z \alpha}{T_{KE} (1 - \cos \theta)} \right]^2$$

← Kinetic energy of electron



As the energy of the electrons is raised enough to make them an effective nuclear probe, a number of other effects become significant, and the scattering behavior diverges from the Rutherford formula. The probing electrons are relativistic, they produce significant nuclear recoil, and they interact via their magnetic moment as well as by their charge. When the magnetic moment and recoil are taken into account, the expression is called the Mott cross section.

The cross section from elastic electron scattering is:

$$\frac{d\sigma}{d \cos \theta} = \left( \frac{d\sigma}{d \cos \theta} \right)_{\text{point}} |F(q)|^2$$

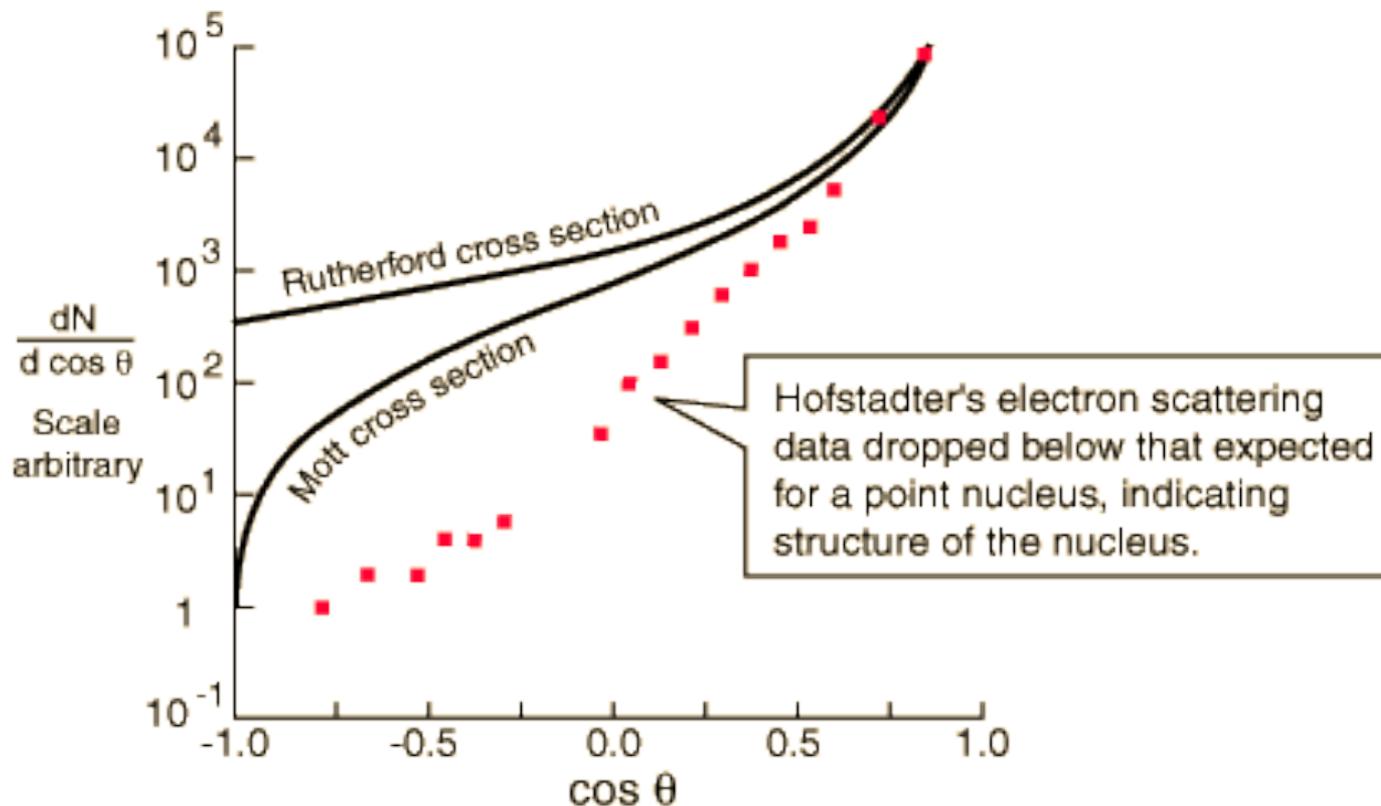
**Mott cross section** **form factor**

**Form factor**

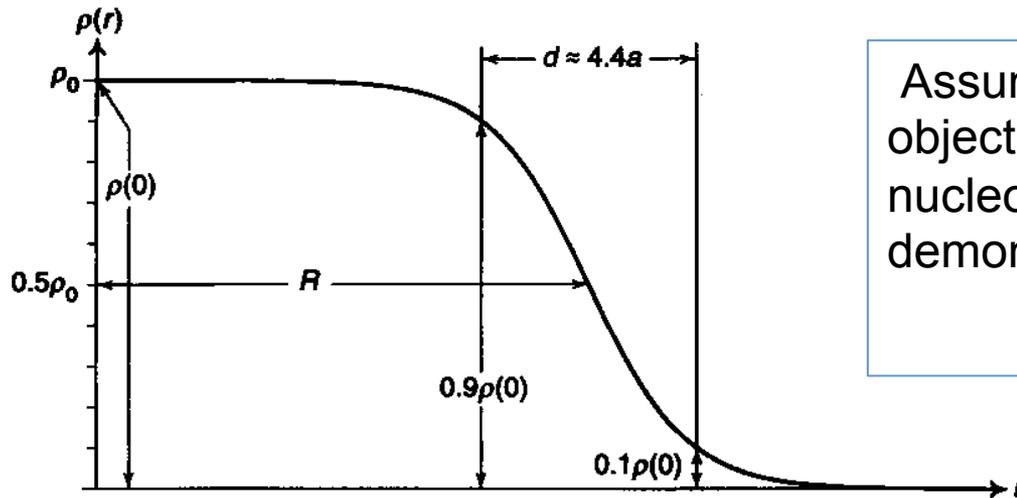
$$F(\vec{q}) = \int d^3r \rho_{\text{ch}}(\vec{r}) e^{i\vec{q}\vec{r}}$$

**$q$**  – three momentum transfer of electron

A major period of investigation of nuclear size and structure occurred in the 1950's with the work of Robert Hofstadter and others who compared their high energy electron scattering results with the Mott cross section. The illustration below from Hofstadter's work shows the divergence from the Mott cross section which indicates that the electrons are penetrating the nucleus - departure from point-particle scattering is evidence of the structure of the nucleus.



# Sizes



Assuming the nucleus is a spherical object with a sharp surface and constant nucleonic density  $\rho_0 = 0.16$  nucleons/ $\text{fm}^3$ , demonstrate the relation:

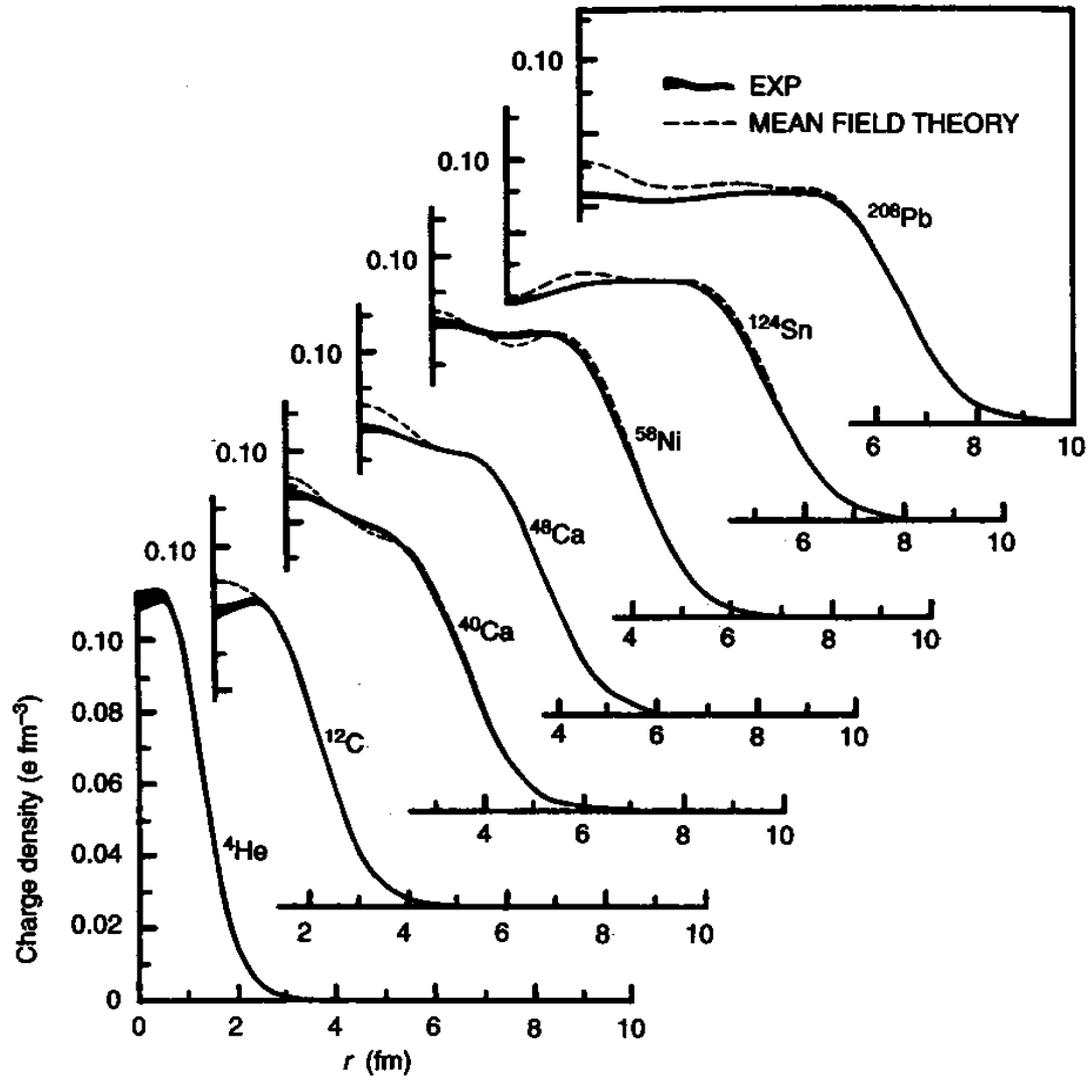
$$R \approx 1.2A^{1/3} \text{ fm}$$

$$\rho(0) = 0.16 \text{ nucleons}/\text{fm}^3$$

$$\rho(r) = \rho_0 \left[ 1 + \exp\left(\frac{r-R}{a}\right) \right]^{-1}$$

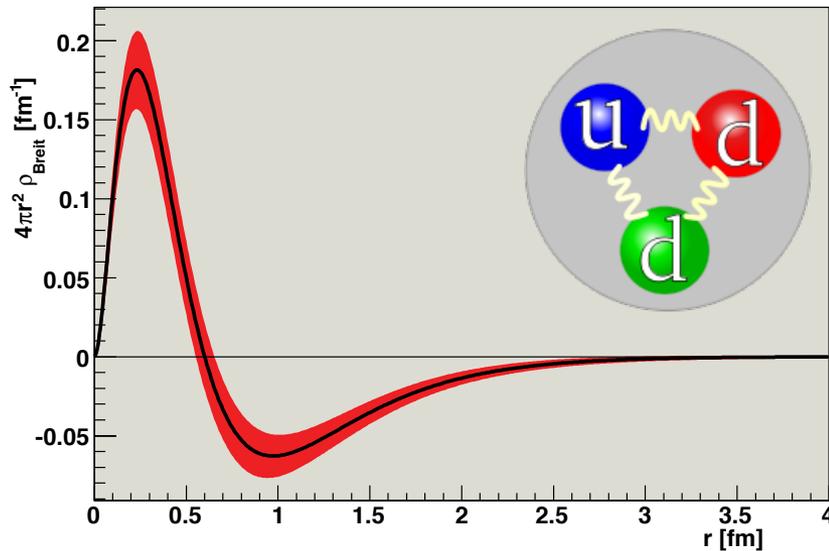
$$R \approx 1.2A^{1/3} \text{ fm}, \quad a \approx 0.6 \text{ fm}$$

# Calculated and measured densities

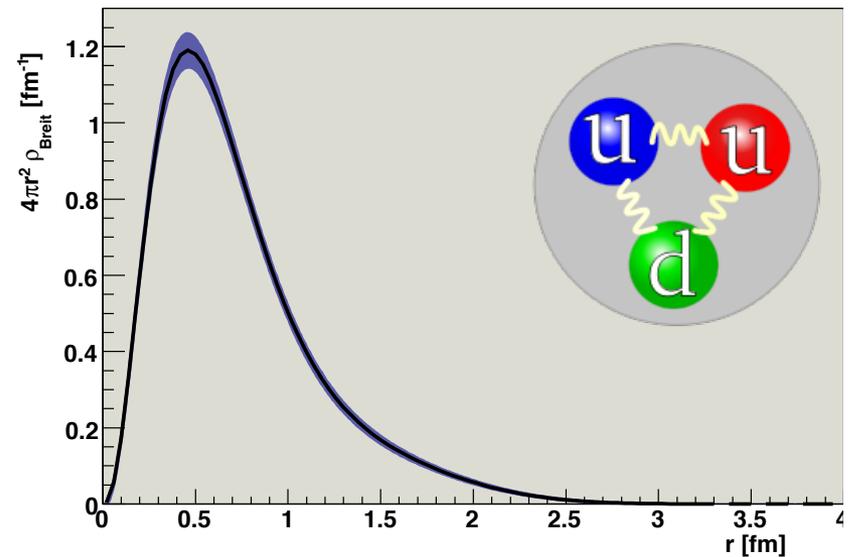


# Protons and neutrons aren't point particles

charge distribution in the neutron



charge distribution in the proton



mass →	2.4 MeV	1.27 GeV	171.2 GeV
charge →	2/3	2/3	2/3
spin →	1/2	1/2	1/2
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top
Quarks	4.8 MeV	104 MeV	4.2 GeV
	-1/3	-1/3	-1/3
	1/2	1/2	1/2
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom

$$\langle R_{\text{ch}}^2 \rangle = \langle R_{\text{pp}}^2 \rangle + \langle r_{\text{p}}^2 \rangle + \frac{N}{Z} \langle r_{\text{n}}^2 \rangle + \frac{3\hbar^2}{4m_{\text{p}}^2 c^2}$$

$$\sqrt{\langle r_{\text{p}}^2 \rangle} = 0.8775(51) \text{ fm}$$

$$\langle r_{\text{n}}^2 \rangle = -0.1149(27) \text{ fm}^2$$

relativistic Darwin-Foldy correction

# Proton size puzzle

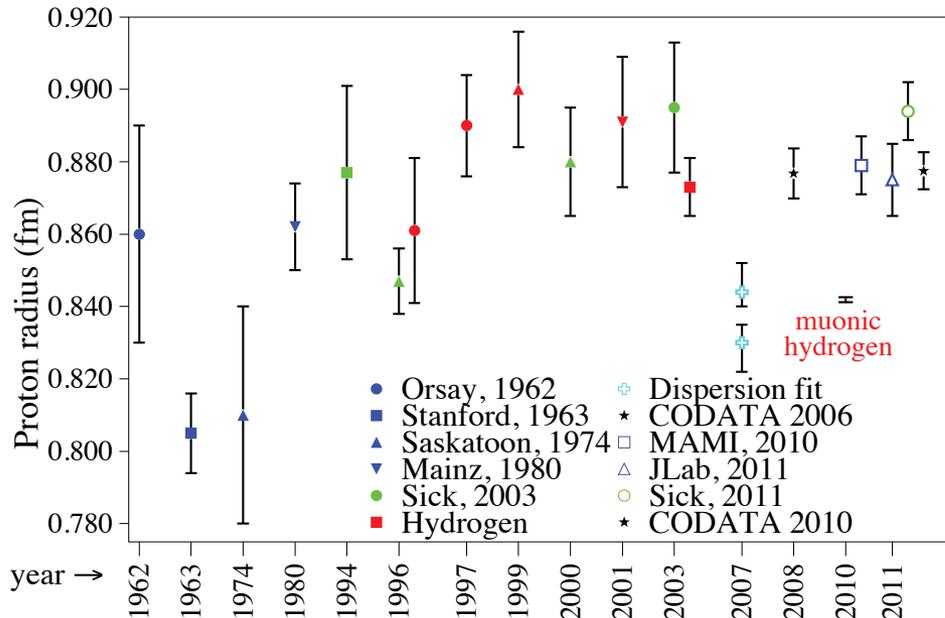
<http://www.newscientist.com/article/dn19141-incredible-shrinking-proton-raises-eyebrows.html>

<http://www.psi.ch/media/proton-size-puzzle-reinforced>

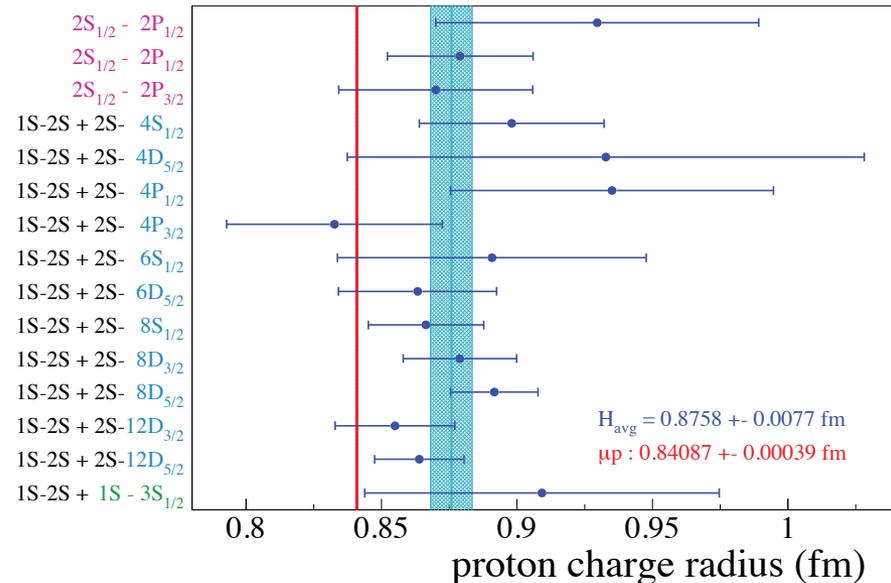
Muon has a mass of 105.7 MeV, which is about 200 times that of the electron

Bohr radius: 
$$a_0 = \frac{\hbar}{m_e c \alpha}$$

Proton radius determinations over time



Proton charge radii obtained from hydrogen spectroscopy



# Isotope Shift

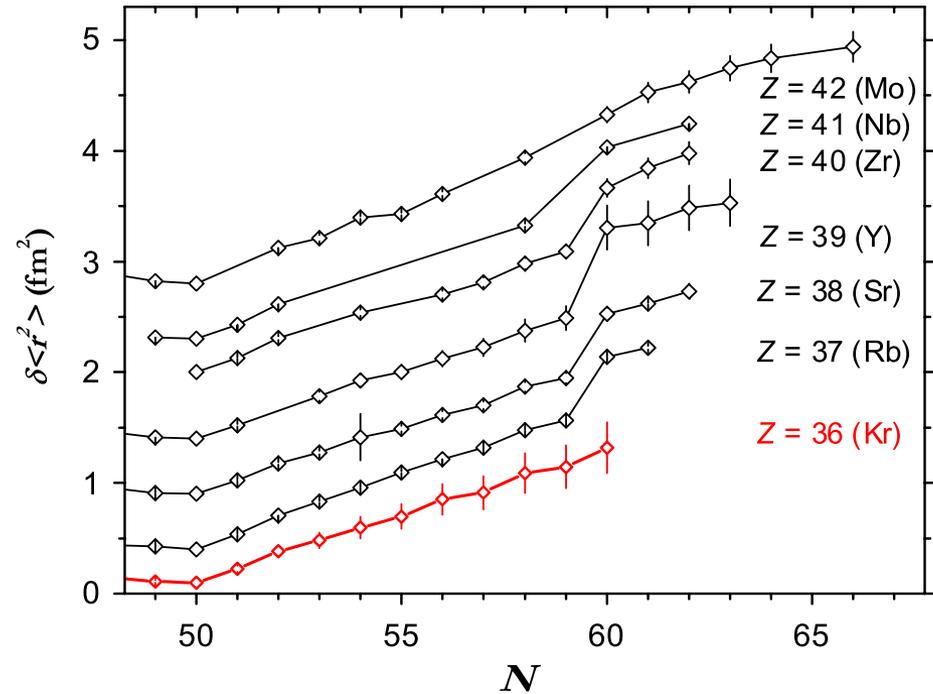
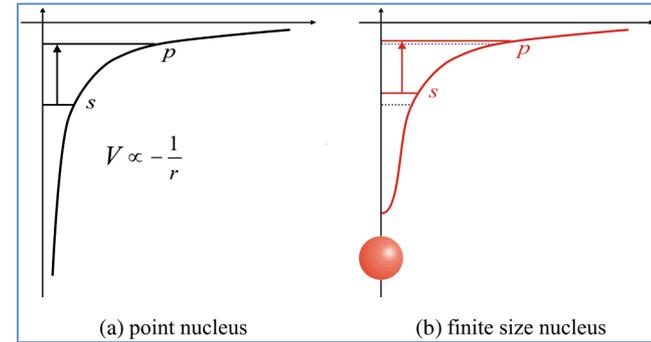
## Laser trapping of exotic atoms. RMP 85, 1383 (2013)

TABLE I. Contributions to the electronic binding energy and their orders of magnitude in atomic units.  $a_0$  is the Bohr radius,  $\alpha \approx 1/137$ . For helium, the atomic number  $Z = 2$ , and the mass ratio  $\mu/M \sim 1 \times 10^{-4}$ .  $g_I$  is the nuclear  $g$  factor.  $\alpha_d$  is the nuclear dipole polarizability.

Contribution	Magnitude
Nonrelativistic energy	$Z^2$
Mass polarization	$Z^2 \mu/M$
Second-order mass polarization	$Z^2 (\mu/M)^2$
Relativistic corrections	$Z^4 \alpha^2$
Relativistic recoil	$Z^4 \alpha^2 \mu/M$
Anomalous magnetic moment	$Z^4 \alpha^3$
Hyperfine structure	$Z^3 g_I \mu_0^2$
Lamb shift	$Z^4 \alpha^3 \ln \alpha + \dots$
Radiative recoil	$Z^4 \alpha^3 (\ln \alpha) \mu/M$
Finite nuclear size	$Z^4 \langle r_c/a_0 \rangle^2$
Nuclear polarization	$Z^3 e^2 \alpha_d / (\alpha a_0^4)$

$$\alpha = \frac{1}{137}$$

$\mu$ =reduced electron mass



Difference in mean-square charge radii for the  $N \sim 60$  region, PRL 105, 032502 (2010)