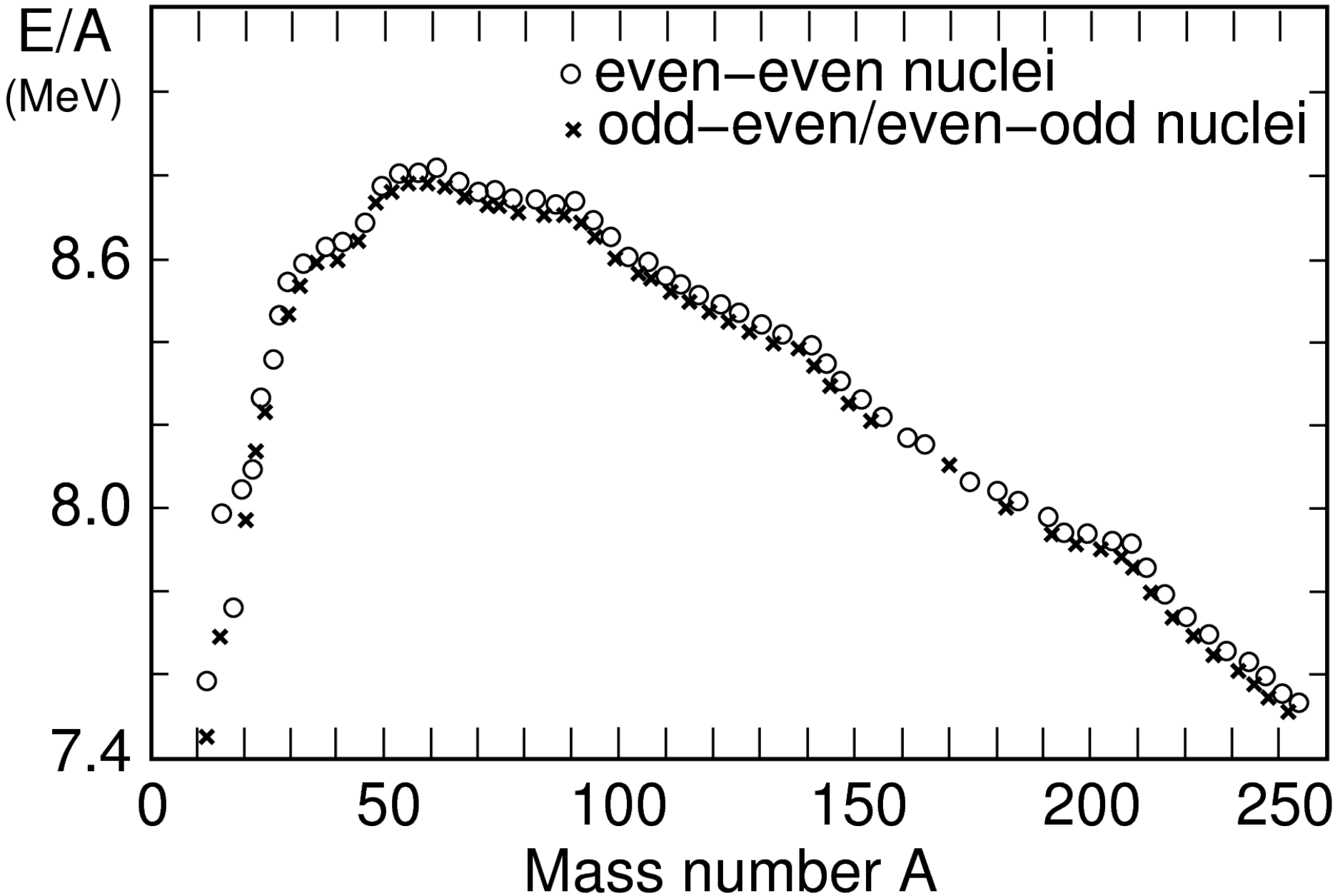
Odd-even effect



Binding (summary)

- For most nuclei, the binding energy per nucleon is about 8MeV.
- Binding is less for light nuclei (these are mostly surface) but there are peaks for *A* in multiples of 4. (But note that the peak for ^{8}Be is slightly lower than that for ^{4}He.
- The most stable nuclei are in the A~60 mass region
- Light nuclei can gain binding energy per nucleon by fusing; heavy nuclei by fission.
- The decrease in binding energy per nucleon for *A*>60 can be ascribed to the repulsion between the (charged) protons in the nucleus: the Coulomb energy grows in proportion to the number of possible pairs of protons in the nucleus Z(Z-1)/2
- The binding energy for massive nuclei (*A*>60) grows roughly as *A*; if the nuclear force were long range, one would expect a variation in proportion to the number of possible pairs of nucleons, i.e. as A(A-1)/2. The variation as *A* suggests that the force is *saturated*; the effect of the interaction is only felt in a neighborhood of the nucleon.

AMDC
Atomic Mass Data Center

http://amdc.in2p3.fr/web/masseval.html

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The most tightly bound nucleus

Richard Shurtleff and Edward Derringh
Department of Physics, Wentworth Institute of Technology, Boston, Massachusetts 02115

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In many textbooks,[1–3] we are told that ^{56}Fe is the nuclide with the greatest binding energy per nucleon, and therefore is the most stable nucleus, the heaviest that can be formed by fusion in normal stars.

But we calculate the binding energy per nucleon BE/A, for a nucleus of mass number A, by the usual formula,

$$BE/A = (1/A)(Zm_H + Nm_n - M_{\text{atom}})c^2, \qquad (1)$$

where m_H is the hydrogen atomic mass and m_n is the neutron mass, for the nuclides ^{56}Fe and ^{62}Ni (both are stable) using data from Wapstra and Audi.[4] The results are 8.790 MeV/nucleon for ^{56}Fe and 8.795 MeV/nucleon for ^{62}Ni. The difference,

$$(0.005\ \text{MeV/nucleon})(\approx 60\ \text{nucleons}) = 300\ \text{keV}, \qquad (2)$$

is much too large to be accounted for as the binding energy of the two extra electrons in ^{62}Ni over the 26 electrons in ^{56}Fe.

^{56}Fe is readily produced in old stars as the end product of the silicon-burning series of reactions.[5] How, then, do we explain the relative cosmic deficiency of ^{62}Ni compared with ^{56}Fe? In order to be abundant, it is not enough that ^{62}Ni be the most stable nucleus. To be formed by charged-particle fusion (the energy source in normal stars), a reaction must be available to bridge the gap from ^{56}Fe to ^{62}Ni. To accomplish this with a single fusion requires a nuclide with $Z = 2$, $A = 6$. But no such stable nuclide exists. The other possibility is two sequential fusions with ^{3}H, producing first ^{59}Co then ^{62}Ni. However, the ^{3}H nucleus is unstable and is not expected to be present in old stars synthesizing heavy elements. We are aware that there are element-generating processes other than charged-particle fusion, such as processes involving neutron capture, which could generate nickel. However, these processes apparently do not occur in normal stars, but rather in supernovas and post-supernova phases, which we do not address.

We conclude that ^{56}Fe is the end product of normal stellar fusion not because it is the most tightly bound nucleus, which it is not, but that it is in close, but unbridgeable, proximity to ^{62}Ni, which is the most tightly bound nucleus.

[1] Arthur Beiser, *Concepts of Modern Physics* (McGraw-Hill, New York, 1987), 4th ed., p. 421.
[2] Frank Shu, *The Physical Universe* (University Science Books, Mill Valley, CA, 1982), 1st ed., pp. 116–117.
[3] Donald D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (McGraw-Hill, New York, 1968), p. 518.
[4] A. H. Wapstra and G. Audi, Nucl. Phys. A **432**, 1 (1985).
[5] William K. Rose, *Astrophysics* (Holt, Rinehart and Winston, New York, 1973), p. 186.

Nuclear liquid drop

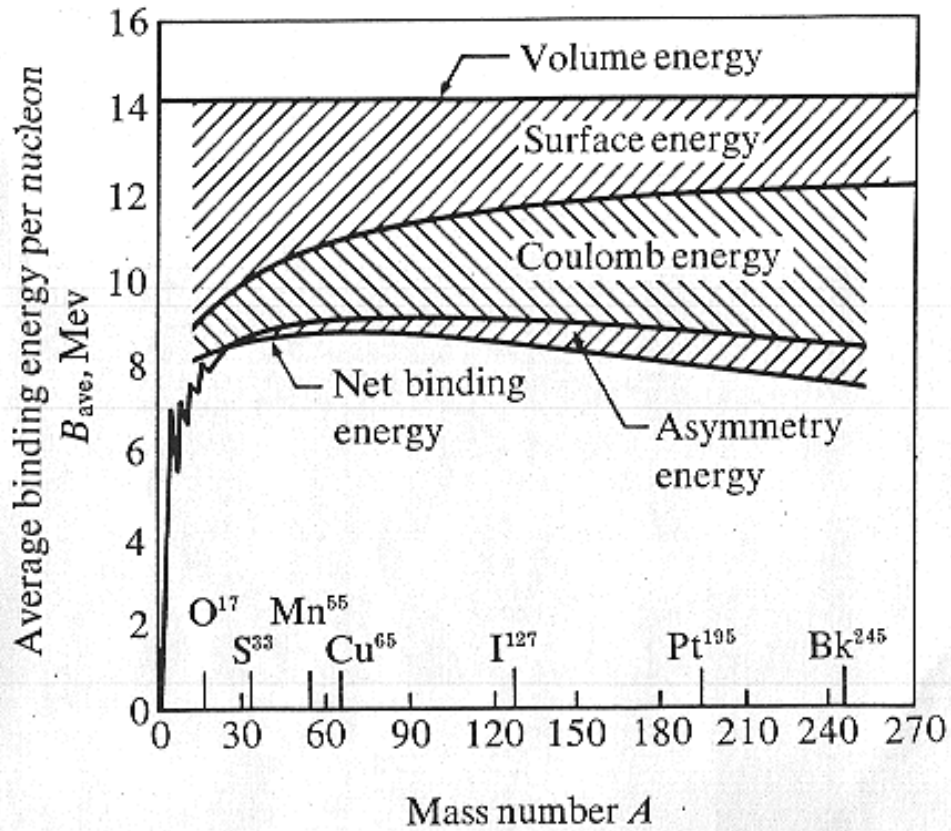
The semi-empirical mass formula, based **on the liquid drop model**, considers five contributions to the binding energy (Bethe-Weizacker 1935/36)

$$B = a_{vol}A - a_{surf}A^{2/3} - a_{sym}\frac{(N-Z)^2}{A} - a_C\frac{Z^2}{A^{1/3}} - \delta(A)$$

15.68 18.56 28.1 0.717

pairing term

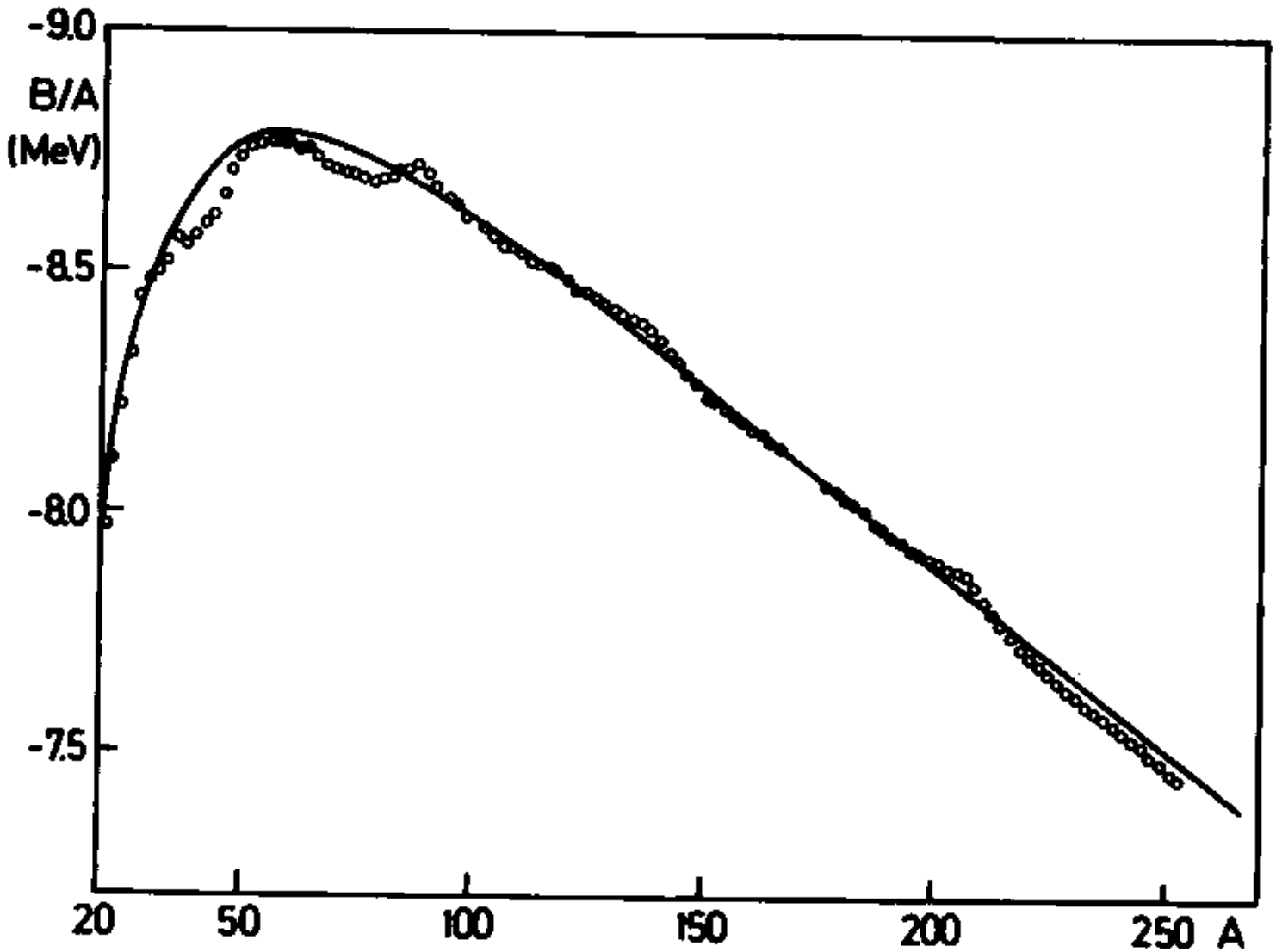
$$\delta(A) = \begin{cases} -34\,A^{-3/4} & \text{for even - even} \\ 0 & \text{for even - odd} \\ 34\,A^{-3/4} & \text{for odd - odd} \end{cases}$$

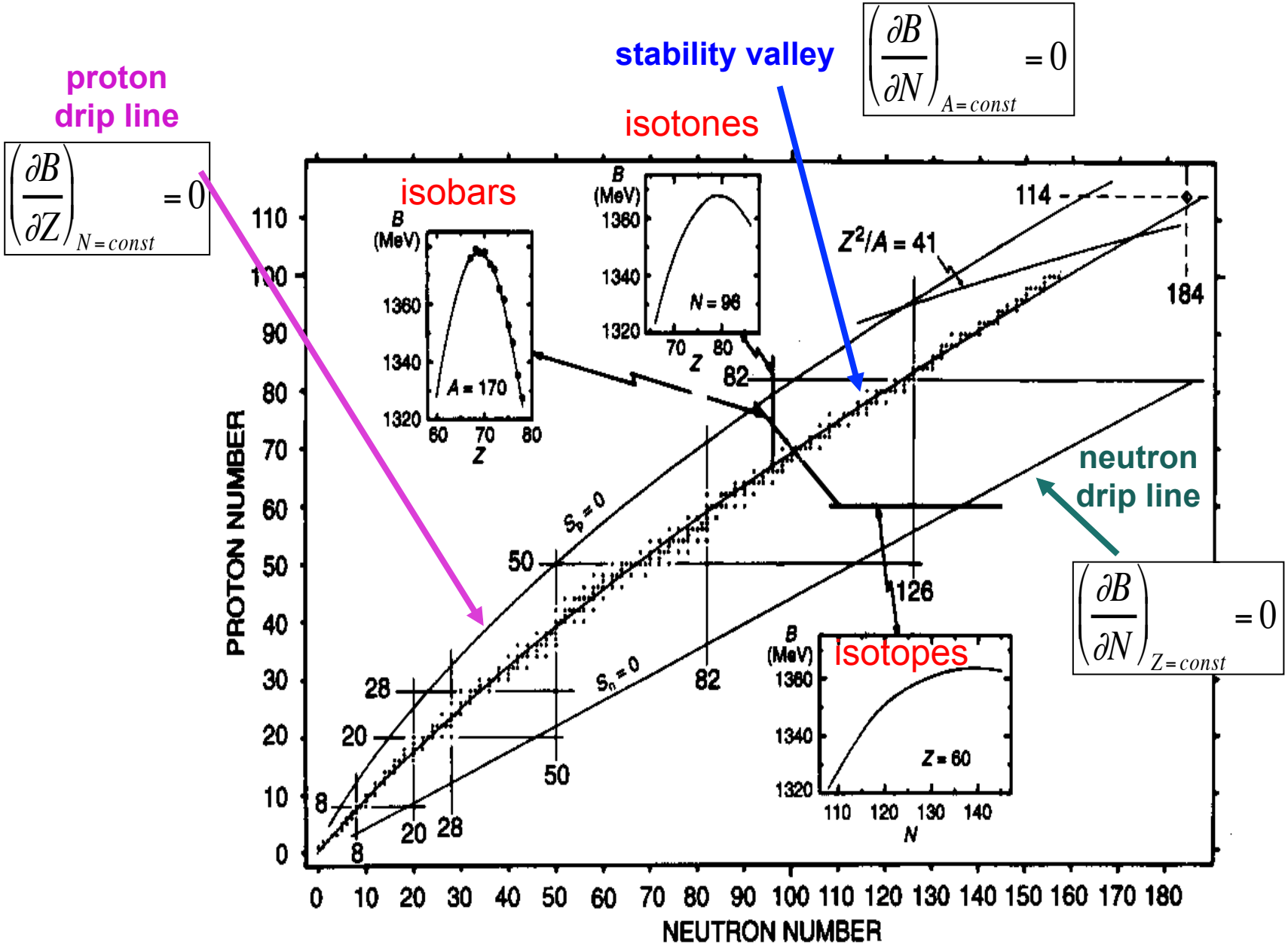


Leptodermous expansion

B. Grammaticos, Ann. Phys. (NY) 139, 1 (1982)

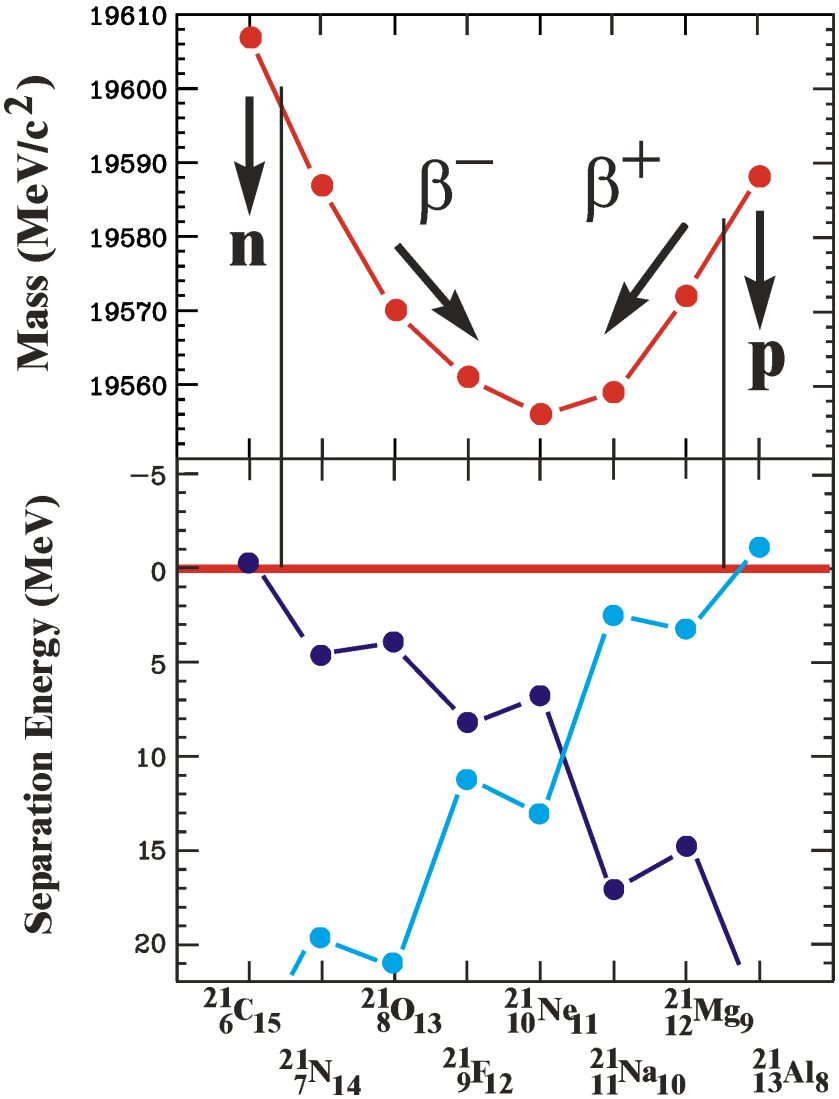
The semi-empirical mass formula, based **on the liquid drop model**, compared to the data




proton drip line
$\left(\frac{\partial B}{\partial Z}\right)_{N=const} = 0$
stability valley
$\left(\frac{\partial B}{\partial N}\right)_{A=const} = 0$
isotones
isobars
neutron drip line
$\left(\frac{\partial B}{\partial N}\right)_{Z=const} = 0$
isotopes
PROTON NUMBER
NEUTRON NUMBER
$Z^2/A = 41$
$S_p = 0$
$S_n = 0$
A = 170
N = 96
Z = 60
B (MeV)
Z
N
8
20
28
50
82
126
114
184

Separation energies

A = 21 isobaric chain



one-nucleon separation energies

$$S_{1n} = B(N,Z) - B(N-1,Z)$$

$$S_{1p} = B(N,Z) - B(N,Z-1)$$

two-nucleon separation energies

$$S_{2n} = B(N,Z) - B(N-2,Z)$$

$$S_{2p} = B(N,Z) - B(N,Z-2)$$

Using http://www.nndc.bnl.gov/chart/ find one- and two-nucleon separation energies of ^{8}He, ^{11}Li, ^{45}Fe, ^{141}Ho, and ^{208}Pb. Discuss the result.