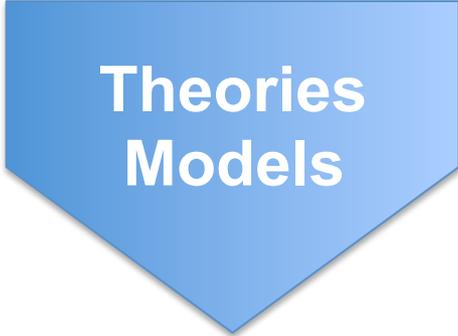


Happy the man who has been able to  
discern the cause of things

Virgil, Georgica

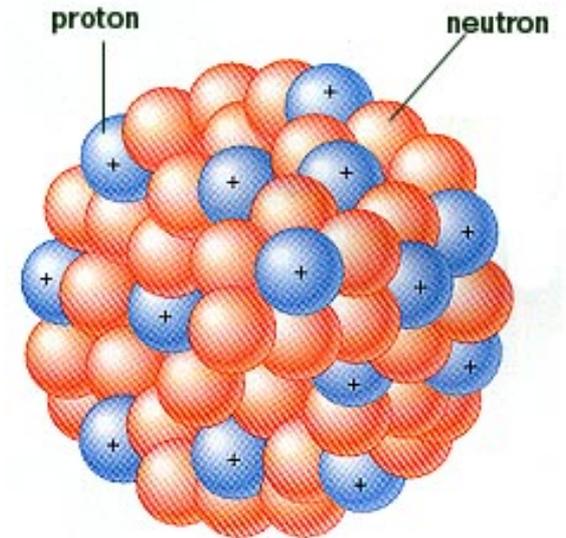


Theories  
Models

- A first rate theory predicts
- A second rate theory forbids
- A third rate theory explains after the facts

Alexander I. Kitaigorodskii

# Modeling the Atomic Nucleus



Theoretical bag of tricks...

# The Nuclear Many-Body Problem

$$\hat{H}\Psi = E\Psi$$

$$\hat{H} = \hat{T} + \hat{V}$$

$\hat{T} = \sum_{i=1}^A \frac{\hat{p}_i^2}{2m_i},$ <p style="text-align: center;"><b>one-body</b></p> <p style="text-align: center;">Kinetic energy</p>	$\hat{V} = \sum_{i<j} \hat{V}_{2b}(i,j) + \sum_{i<j<k} \hat{V}_{3b}(i,j,k)$ <p style="text-align: center;"><b>two-body      three-body</b></p> <p style="text-align: center;">Potential energy</p>
---	--

$$\Psi = \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A; s_1, s_2, \dots, s_A; t_1, t_2, \dots, t_A)$$

**3A nucleon  
coordinates  
in r-space**

**nucleon  
spins:  $\pm 1/2$**

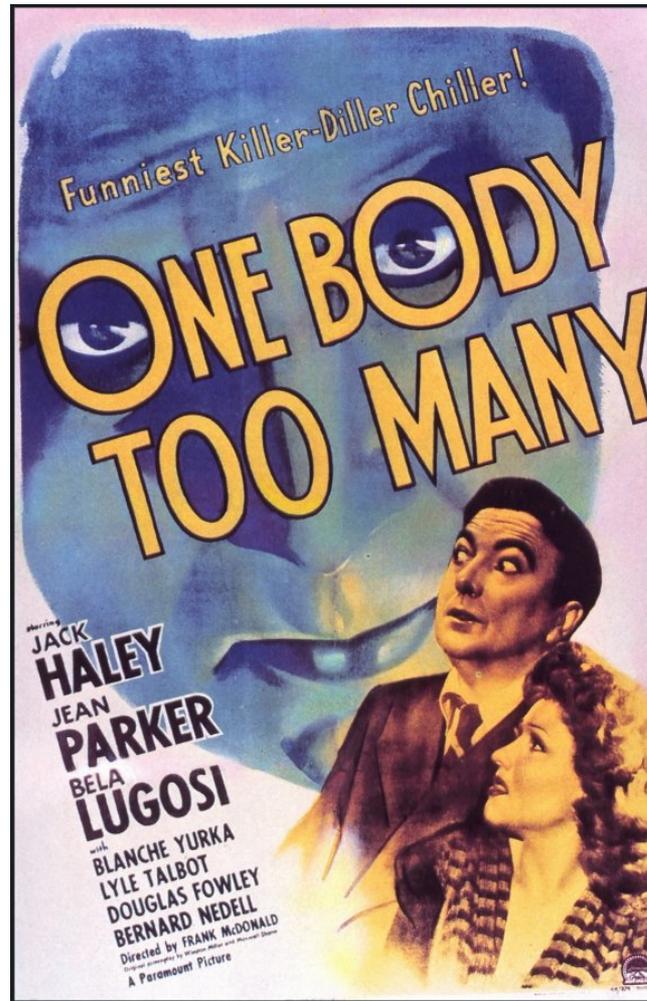
**nucleon  
isospins  
(p or n):  $\pm 1/2$**

Eigenstate of angular  
momentum, parity, and  
~isospin

**Bottom line:**

$$2^A \times \frac{A!}{N!Z!} \text{ coupled integro-differential equations in } 3A \text{ dimensions}$$

# The nuclear many-body problem



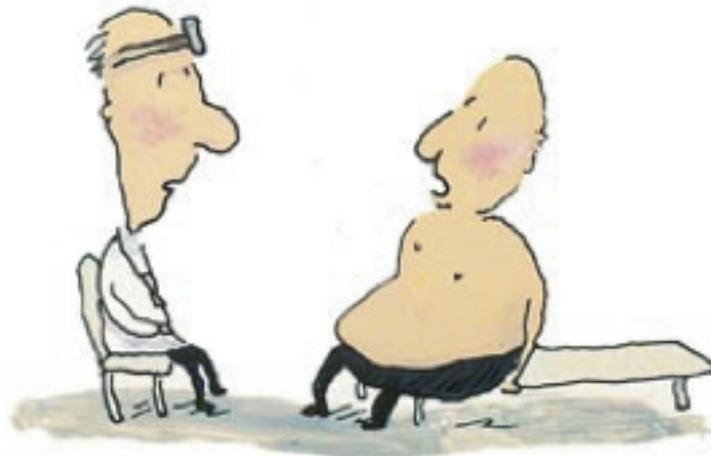
## Weinberg's Laws of Progress in Theoretical Physics

From: "Asymptotic Realms of Physics" (ed. by Guth, Huang, Jaffe, MIT Press, 1983)

**First Law:** "The conservation of Information" (*You will get nowhere by churning equations*)

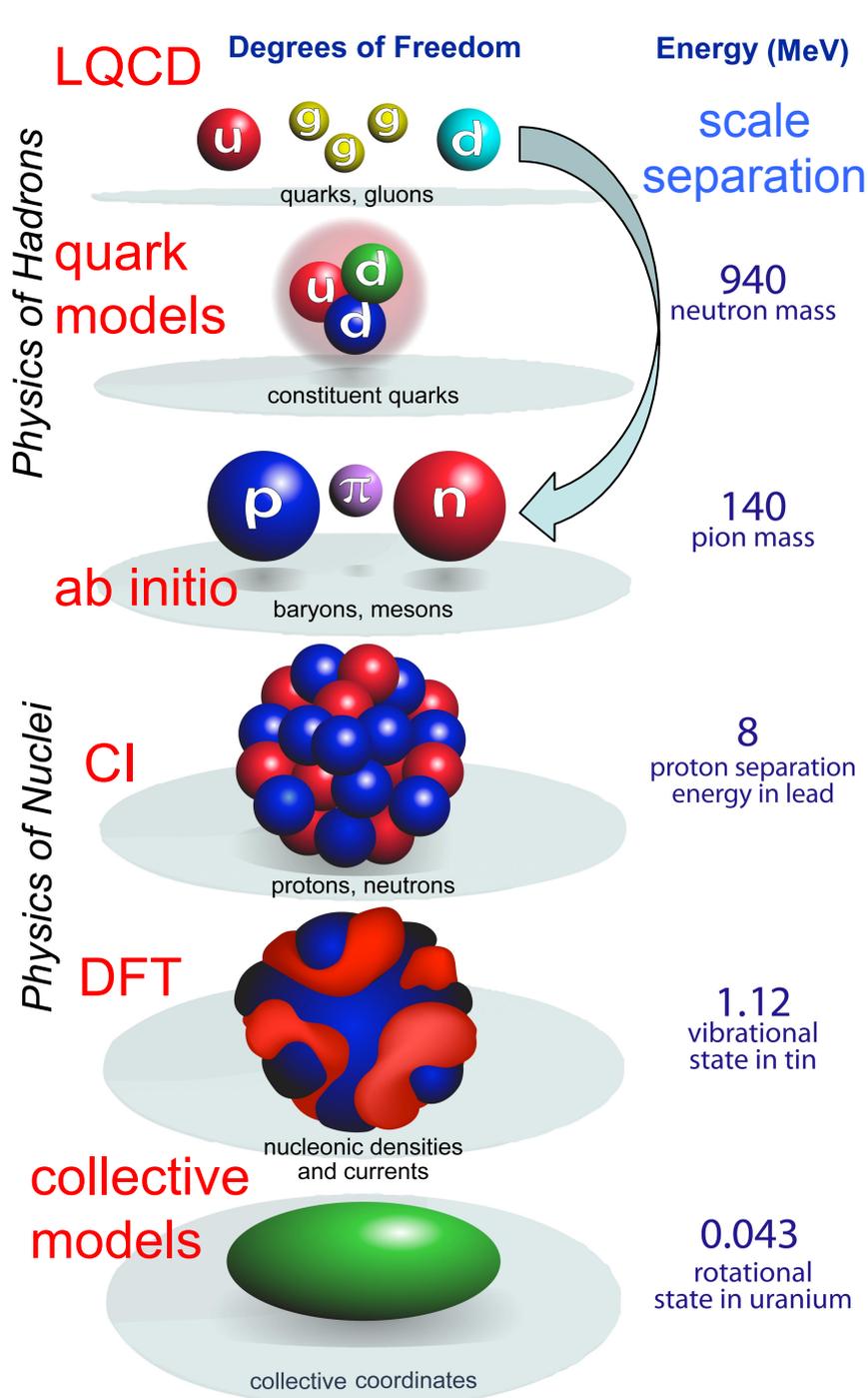
**Second Law:** "Do not trust arguments based on the lowest order of perturbation theory"

**Third Law:** "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"



**Patient:** Doctor, doctor, it hurts when I do this!

**Doctor:** Then don't do that.



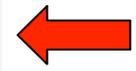
# How are nuclei made?

Origin of elements, isotopes

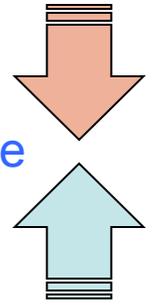
Hot and dense quark-gluon matter

Hadron structure

Resolution



Hadron-Nuclear interface



Effective Field Theory



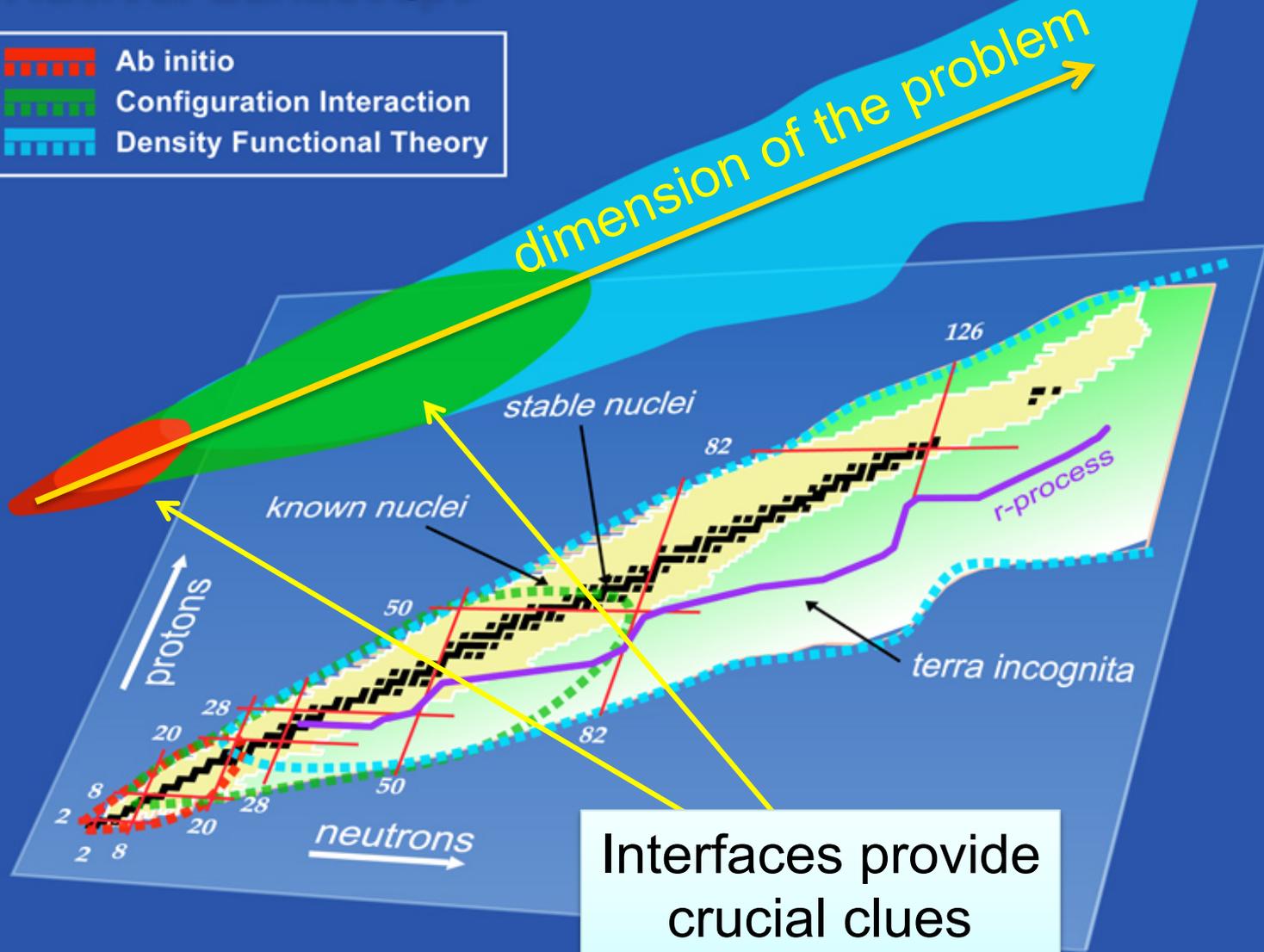
Nuclear structure  
Nuclear reactions  
New standard model

Applications of nuclear science

To explain, predict, use...

# Nuclear Landscape

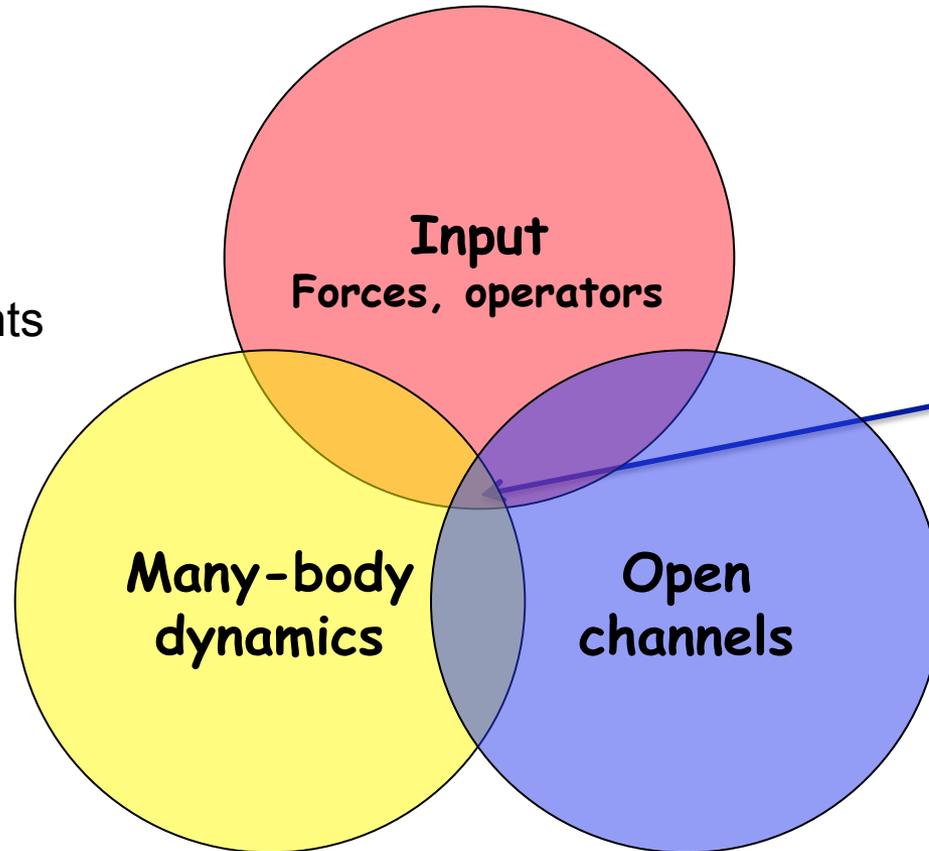
- Ab initio
- Configuration Interaction
- Density Functional Theory



number of nuclei < number of processors!

# Theory of nuclei is demanding

- rooted in QCD
- insights from EFT
- many-body interactions
- in-medium renormalization
- microscopic functionals
- low-energy coupling constants optimized to data
- crucial insights from exotic nuclei



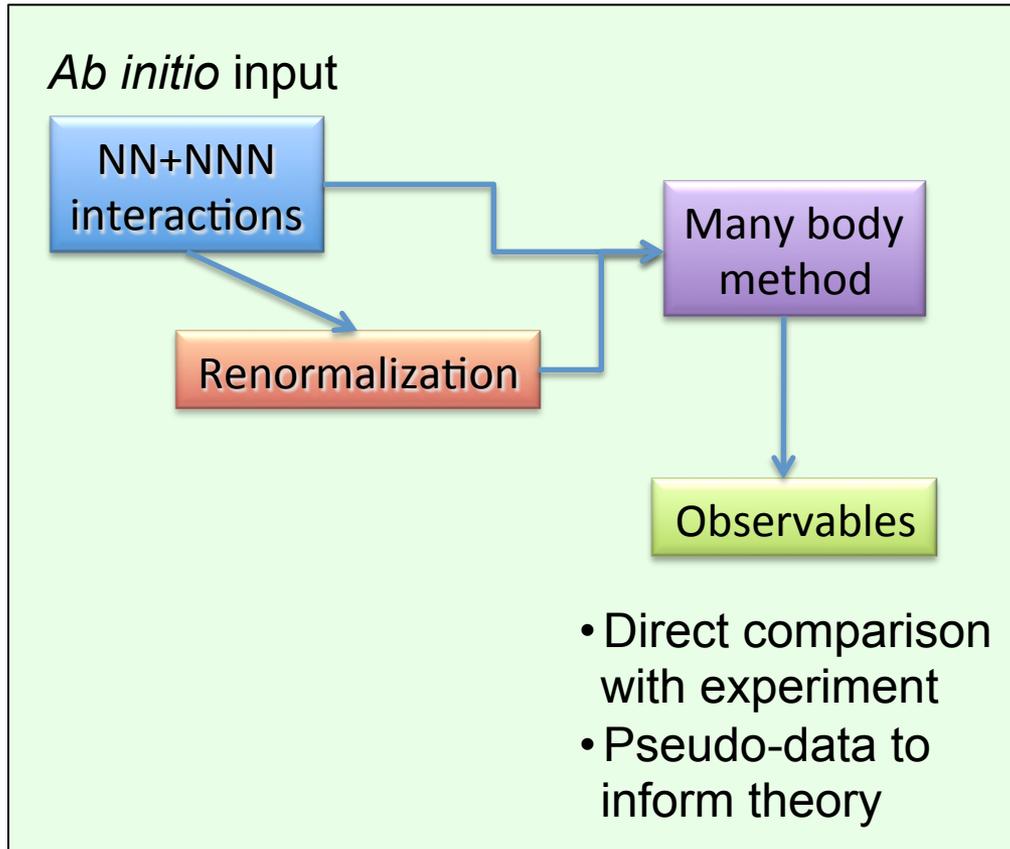
$^{11}\text{Li}$   
 $^{100}\text{Sn}$   
 $^{240}\text{Pu}$   
 $^{298}\text{U}$

- many-body techniques
  - direct *ab initio* schemes
  - symmetry breaking and restoration
- high-performance computing
- interdisciplinary connections

- nuclear structure impacted by couplings to reaction and decay channels
- clustering, alpha decay, and fission still remain major challenges for theory
- unified picture of structure and reactions

# *Ab initio* theory for light nuclei and nuclear matter

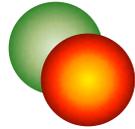
***Ab initio*: QMC, NCSM, CCM,...**  
(nuclei, neutron droplets, nuclear matter)



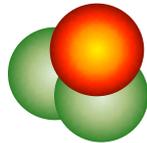
## Input:

- Excellent forces based on the phase shift analysis and few-body data
- EFT based nonlocal chiral NN and NNN potentials
- SRG-softened potentials based on bare NN+NNN interactions

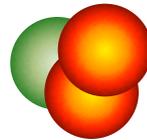
# Few-nucleon systems



$A=2$ : many years ago...



${}^3\text{H}$ : 1984 (1% accuracy)



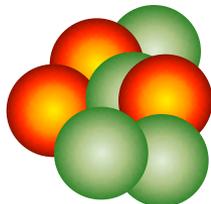
${}^3\text{He}$ : 1987



${}^4\text{He}$ : 1987



${}^5\text{He}$ : 1994 (n- $\alpha$  resonance)



$A=6,7,..12$ : 1995-2011

# Green's Function Monte Carlo (imaginary-time method)

$$|\psi_0\rangle = \lim_{\tau \rightarrow \infty} e^{-(\hat{H} - E_0)\tau} |\psi_V\rangle$$

trial wave function

$$|\psi(\tau)\rangle = e^{-(\hat{H} - E_0)\tau} |\psi_V\rangle$$

$$|\psi(0)\rangle = |\psi_V\rangle, \quad |\psi(\infty)\rangle = |\psi_0\rangle$$

$$\tau = n\Delta\tau \quad \Rightarrow \quad |\psi(\tau)\rangle = \left[ e^{-(\hat{H} - E_0)\Delta\tau} \right]^n |\psi_V\rangle$$

- Quantum Monte Carlo (GFMC) <sup>12</sup>C
- No-Core Shell Model <sup>14</sup>F, <sup>14</sup>C
- Faddeev-Yakubovsky
- Lattice EFT <sup>12</sup>C (Hoyle)
- Coupled-Cluster Techniques <sup>17</sup>F, <sup>56</sup>Ni
- Fermionic Molecular Dynamics
- ...

# Nucleon-Nucleon Interaction

NN, NNN, NNNN,..., forces

**GFMC calculations tell us that:**

$$\langle V_\pi \rangle / \langle V \rangle \sim 70 - 80\%$$

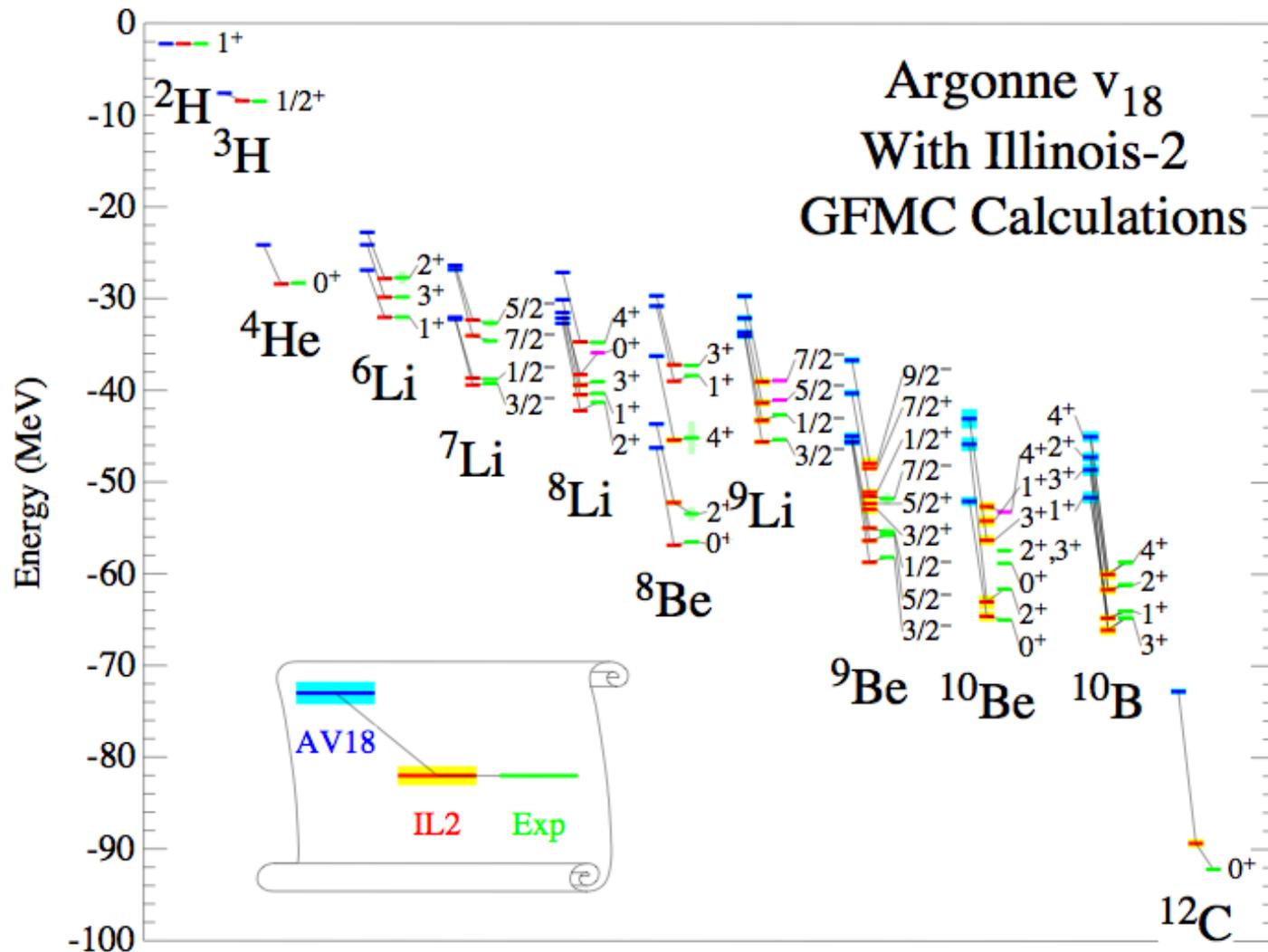
$$\langle V_\pi \rangle \sim -15 \text{ MeV/pair}$$

$$\langle V^R \rangle \sim -5 \text{ MeV/pair} \quad \text{short-range}$$

$$\langle V^3 \rangle \sim -1 \text{ MeV/three} \quad \text{three-body}$$

$$\langle T \rangle \sim 15 \text{ MeV/nucleon}$$

$$\langle V_C \rangle \sim 0.66 \text{ MeV/pair of protons}$$

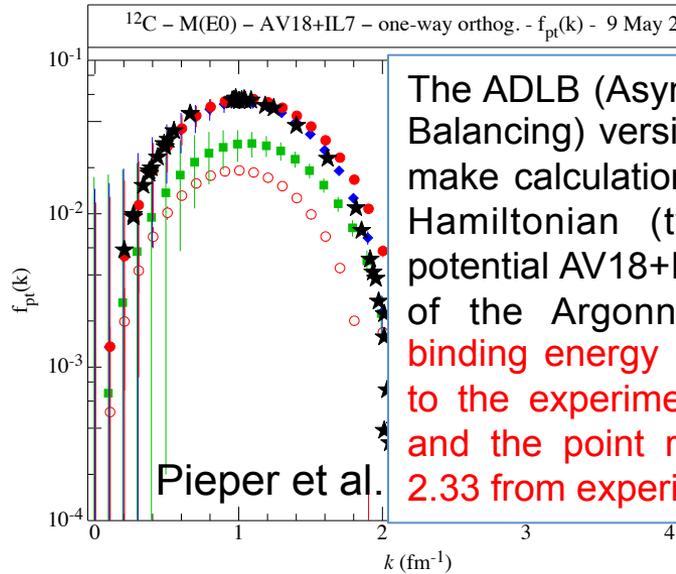
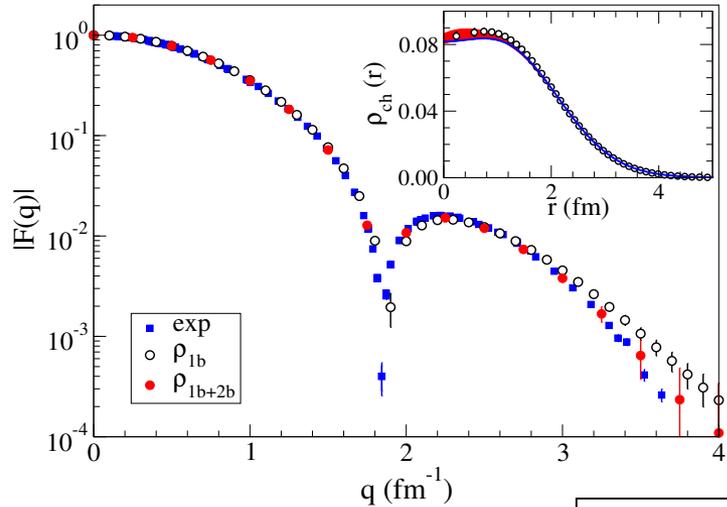
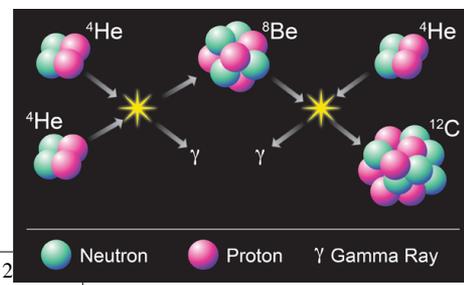


GFMC: S. Pieper, ANL

1-2% calculations of  $A = 6 - 12$  nuclear energies are possible  
excited states with the same quantum numbers computed

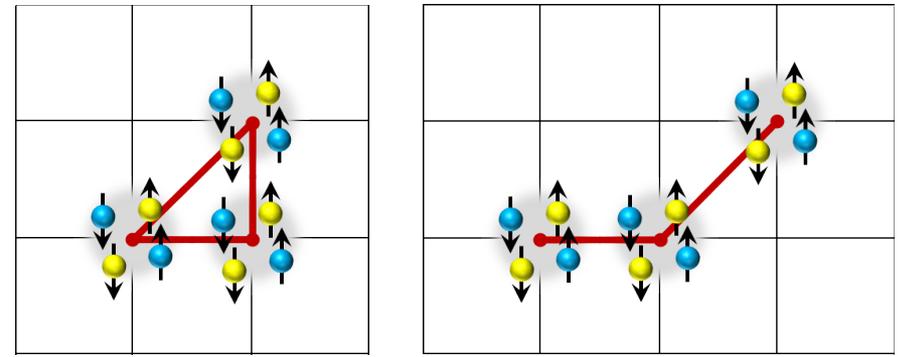
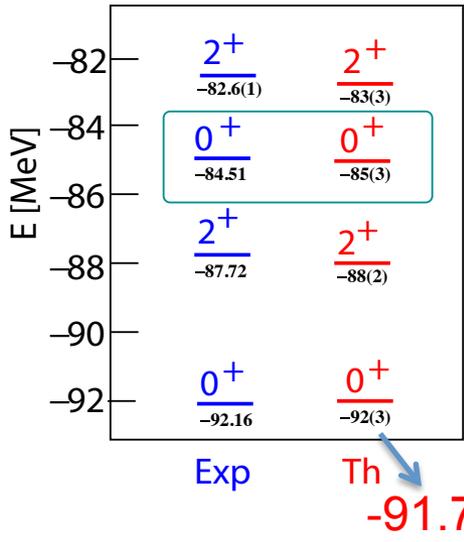
# $^{12}\text{C}$ : ground state and Hoyle state state-of-the-art computing

Wiringa et al. Phys. Rev. C 89, 024305 (2014); A. Lovato et al., Phys. Rev. Lett. 112, 182502 (2014)



The ADLB (Asynchronous Dynamic Load-Balancing) version of GFMC was used to make calculations of  $^{12}\text{C}$  with a complete Hamiltonian (two- and three-nucleon potential AV18+IL7) on **32,000 processors** of the Argonne BGP. The **computed binding energy is 93.5(6) MeV** compared to the experimental value of 92.16 MeV and the point rms radius is 2.35 fm vs 2.33 from experiment.

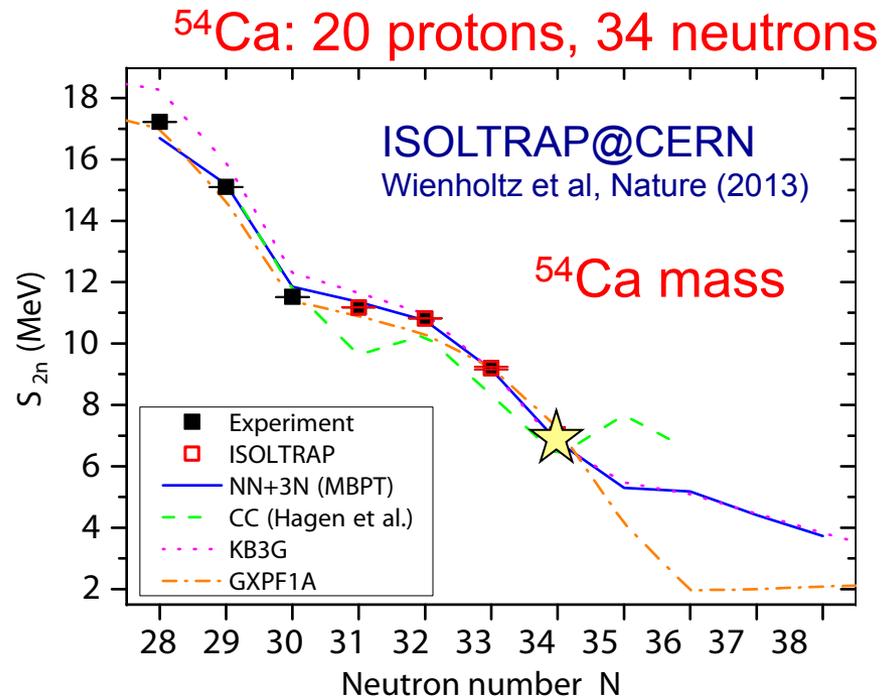
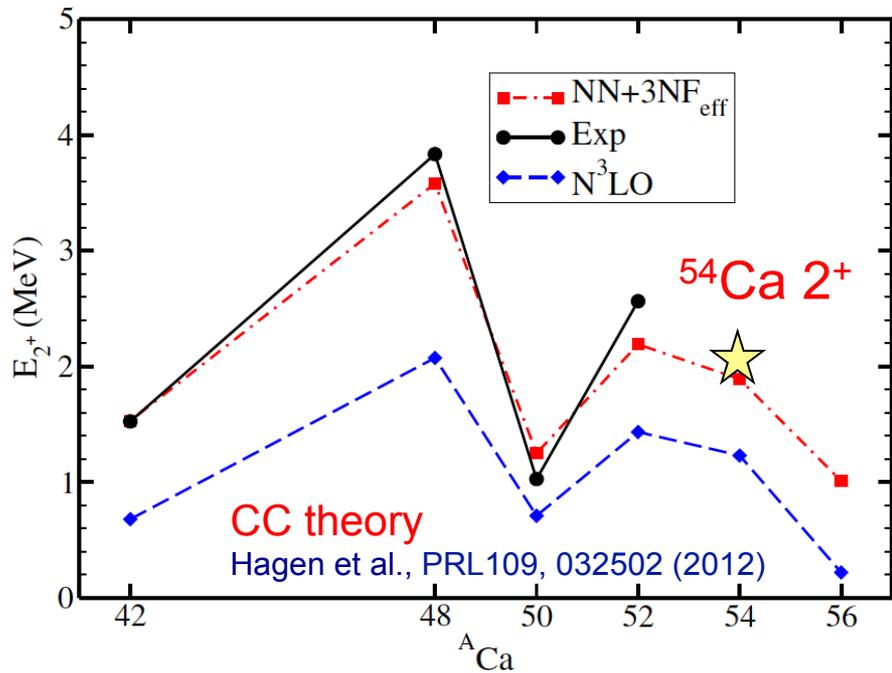
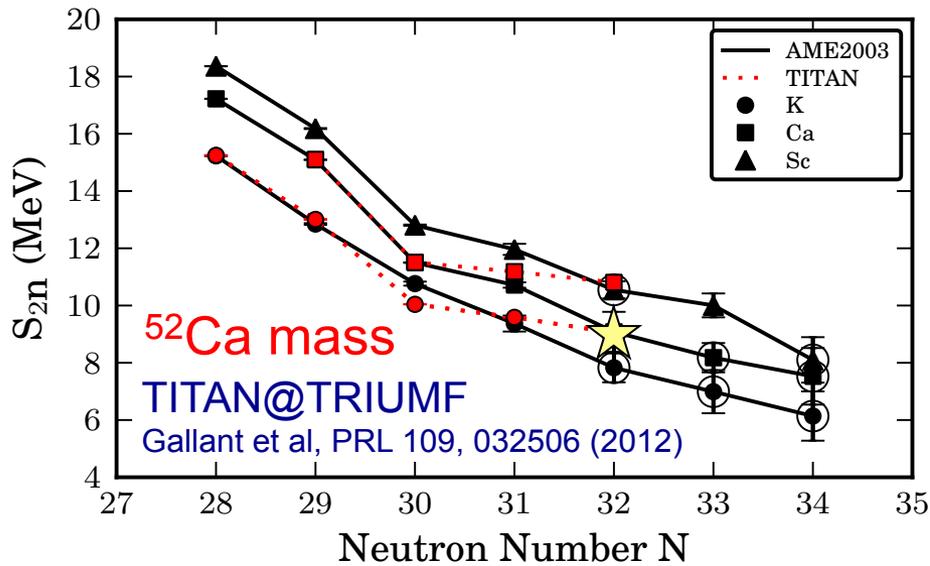
Pieper et al.



Epelbaum et al., Phys. Rev. Lett. 109, 252501 (2012). Lattice EFT  
Lahde et al., Phys. Lett. B 732, 110 (2014).

# The frontier: neutron-rich calcium isotopes

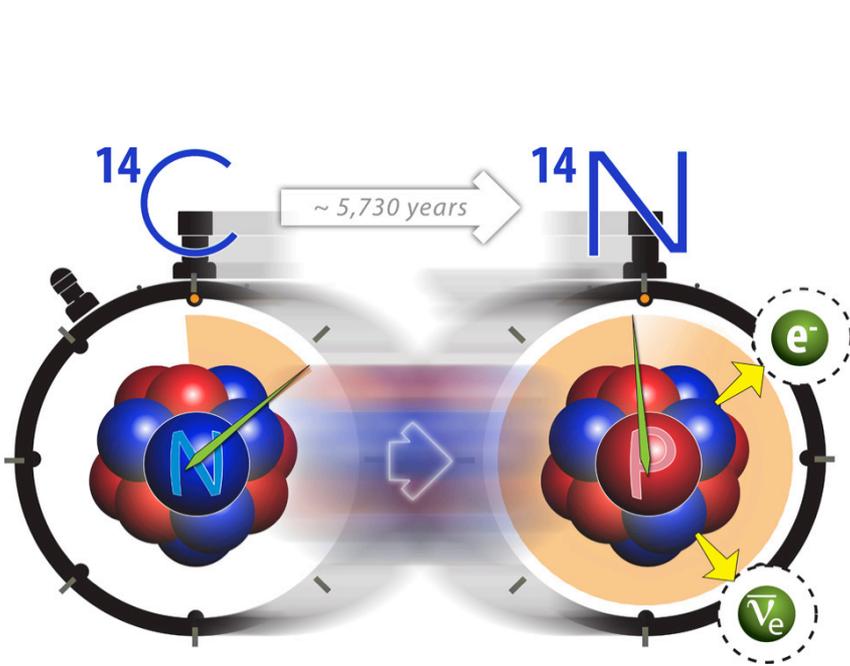
probing nuclear forces and shell structure in a neutron-rich medium



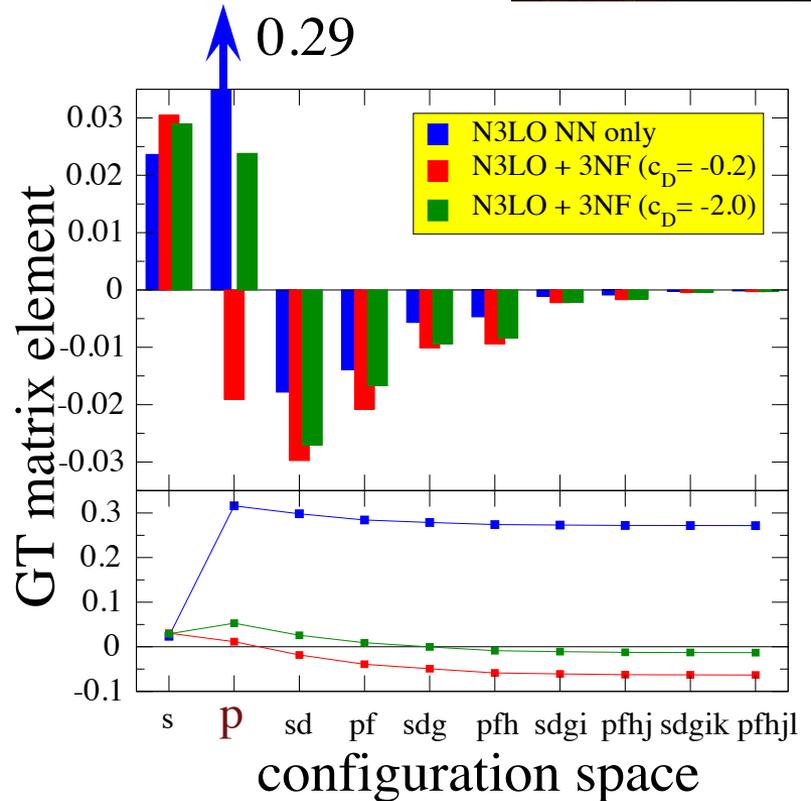
RIBF@RIKEN  
Steppenbeck et al  
Nature (2013)

# Anomalous Long Lifetime of $^{14}\text{C}$

Determine the microscopic origin of the suppressed  $\beta$ -decay rate: 3N force



Maris et al., PRL 106, 202502 (2011)



Dimension of matrix solved for 8 lowest states  $\sim 10^9$   
Solution took  $\sim 6$  hours on 215,000 cores on Cray XT5  
Jaguar at ORNL