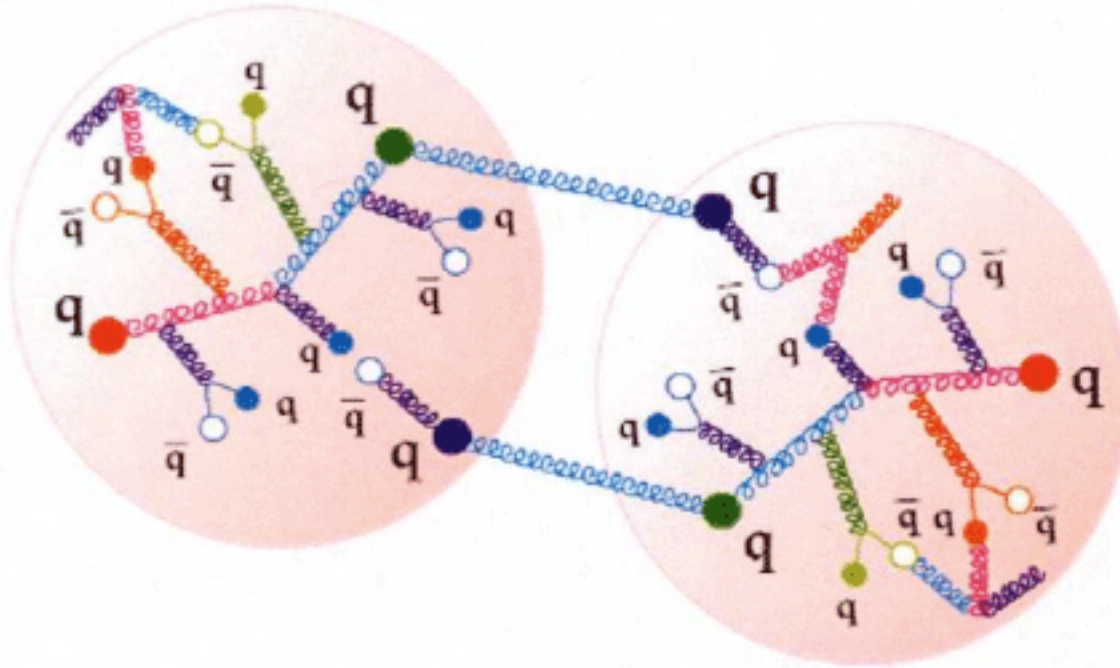


May the Strong Force be with you

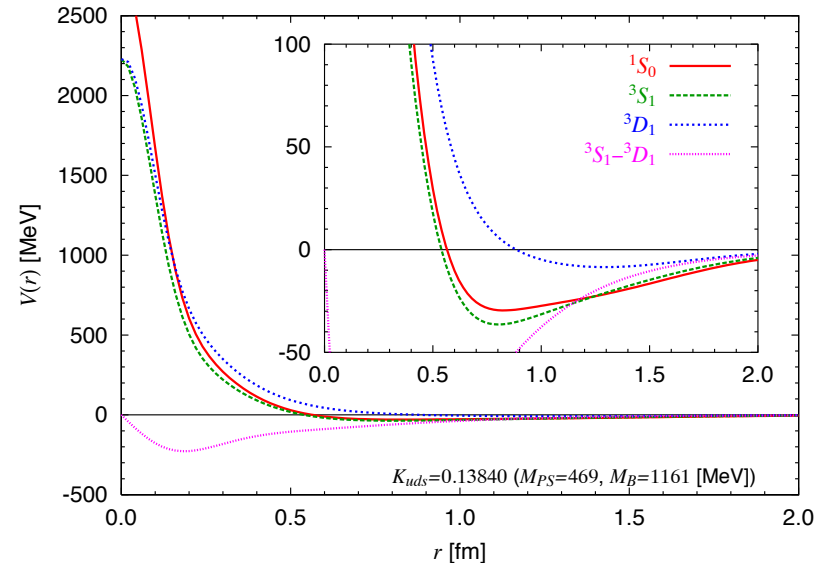
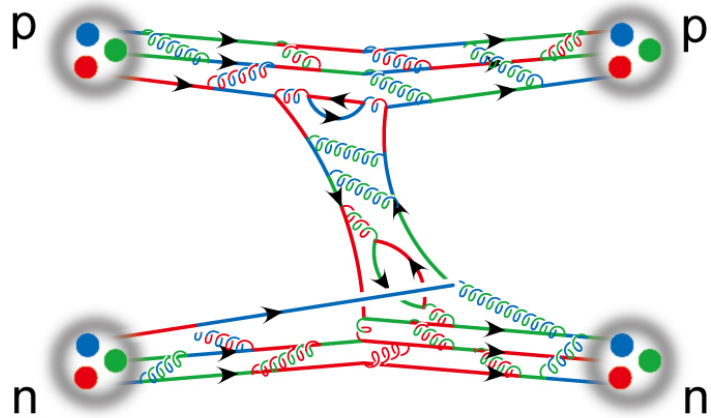


Origin of the nucleon-nucleon force



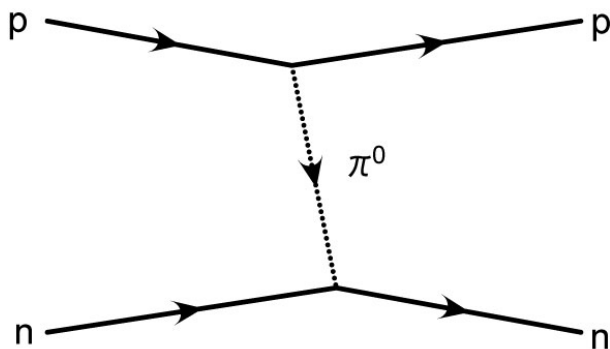
The simplest contributions to the force between nucleons, as viewed from QCD. Here, the exchange of two colored gluons causes two quarks in each nucleon to change their colors (blue changes to green and vice versa in the case illustrated). This process produces a force without violating the overall color neutrality of the nucleons. The strength of the force depends on the separation of the different quark colors within each nucleon. On the other hand, low-energy nuclear physics measurements show clearly that the longest-range part of the force arises from the exchange of a single pi meson between two nucleons, as in In this low-energy view, the internal structure of each nucleon is generally attributed to three pseudo-quarks, which somehow combine the properties of the valence quarks, sea quarks, and gluons predicted by QCD.

The challenge and the prospect: physics of nuclei directly from QCD



Nuclear Force from Lattice QCD

Inoue et al. PRL 111, 112503 (2013); HALQCD/HPCI



The interaction between two nucleons is effected by the exchange of a particle. However, because the nucleon interactions appear to be short-ranged, the particle must have a finite mass. In fact, one can correlate the range and mass roughly by the quantum uncertainty principle, $r \sim 1/m$, therefore, the mass of the quanta exchanged is about $1/\text{fm}$ which is about 200 MeV.

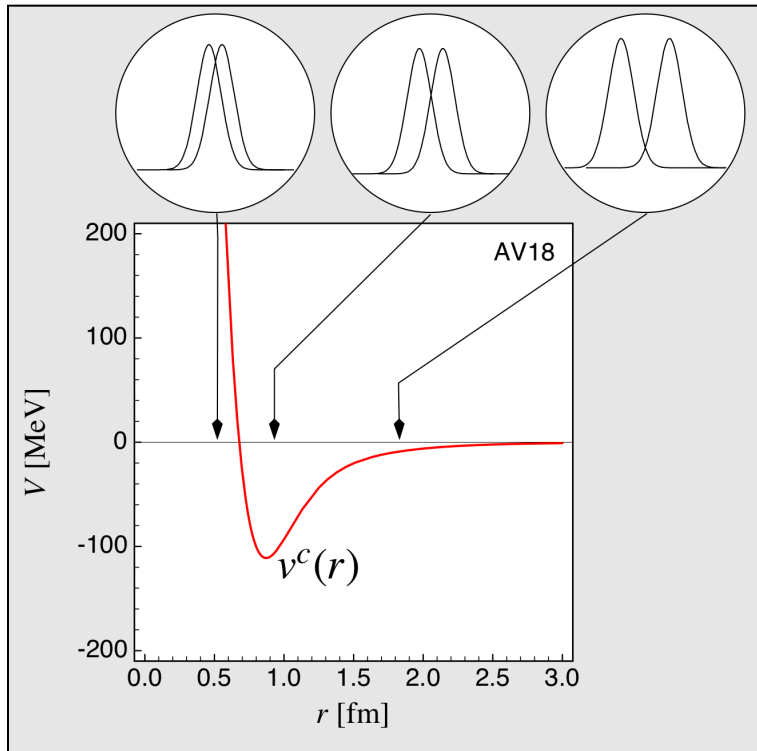


HW: A simple model of nucleon-nucleon (NN) potential is based on the exchange of virtual light mesons, such as the virtual pions, as well as two types of vector mesons, the ρ mesons and the ω mesons. Considering the dependence of the Compton wavelength of the boson on its mass (Jan. 31 lecture), find the range of the NN force attributed to π, ρ , and ω meson exchange.

Nuclear force

A realistic nuclear force force: schematic view

- Nucleon r.m.s. radius ~ 0.86 fm
- Comparable with interaction range
- Half-density overlap at max. attraction
- V_{NN} not fundamental (more like inter-molecular van der Waals interaction)
- Since nucleons are composite objects, three-and higher-body forces are expected.



$$v_{\pi}(\mathbf{r}) = \frac{f_{\pi NN}^2 m_{\pi}}{4\pi} \frac{m_{\pi}}{3} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[T_{\pi}(r) S_{12} + [Y_{\pi}(r) - \frac{4\pi}{m_{\pi}^3} \delta(\mathbf{r})] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right],$$

where $Y_{\pi}(r)$ and $T_{\pi}(r)$ are dimensionless functions of $m_{\pi}r$ defined as

$$Y_{\pi}(r) = \frac{e^{-m_{\pi}r}}{m_{\pi}r}, \quad \text{Yukawa force}$$

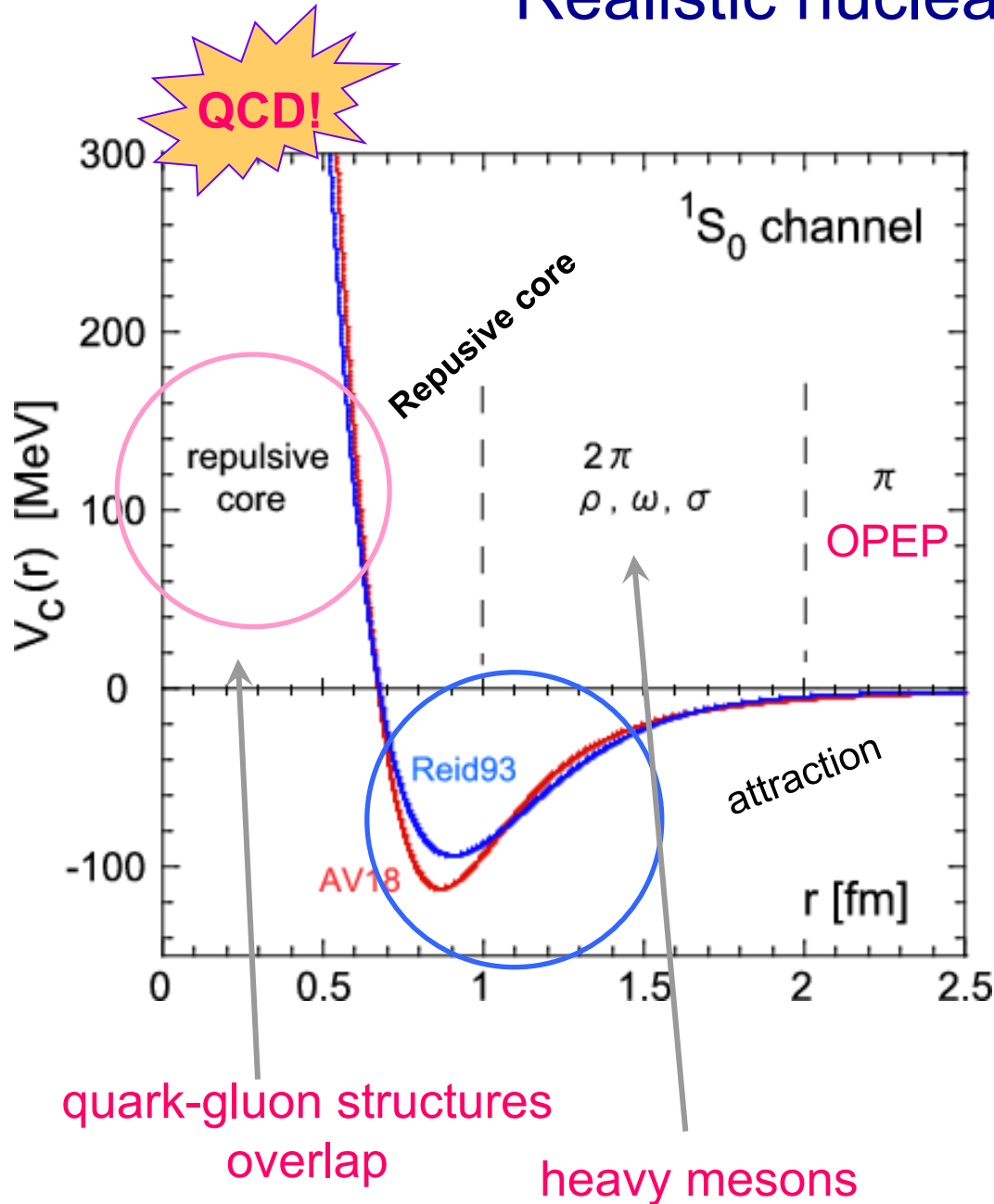
$$T_{\pi}(r) = \left(1 + \frac{3}{m_{\pi}r} + \frac{3}{m_{\pi}^2 r^2} \right) Y_{\pi}(r),$$

and S_{12} is the tensor operator,

$$S_{12} = 3 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2.$$

OPEP: One-pion-exchange potential (at large distances)

Realistic nuclear force



There are infinitely many equivalent nuclear potentials!

$$\hat{H}\Psi = E\Psi$$

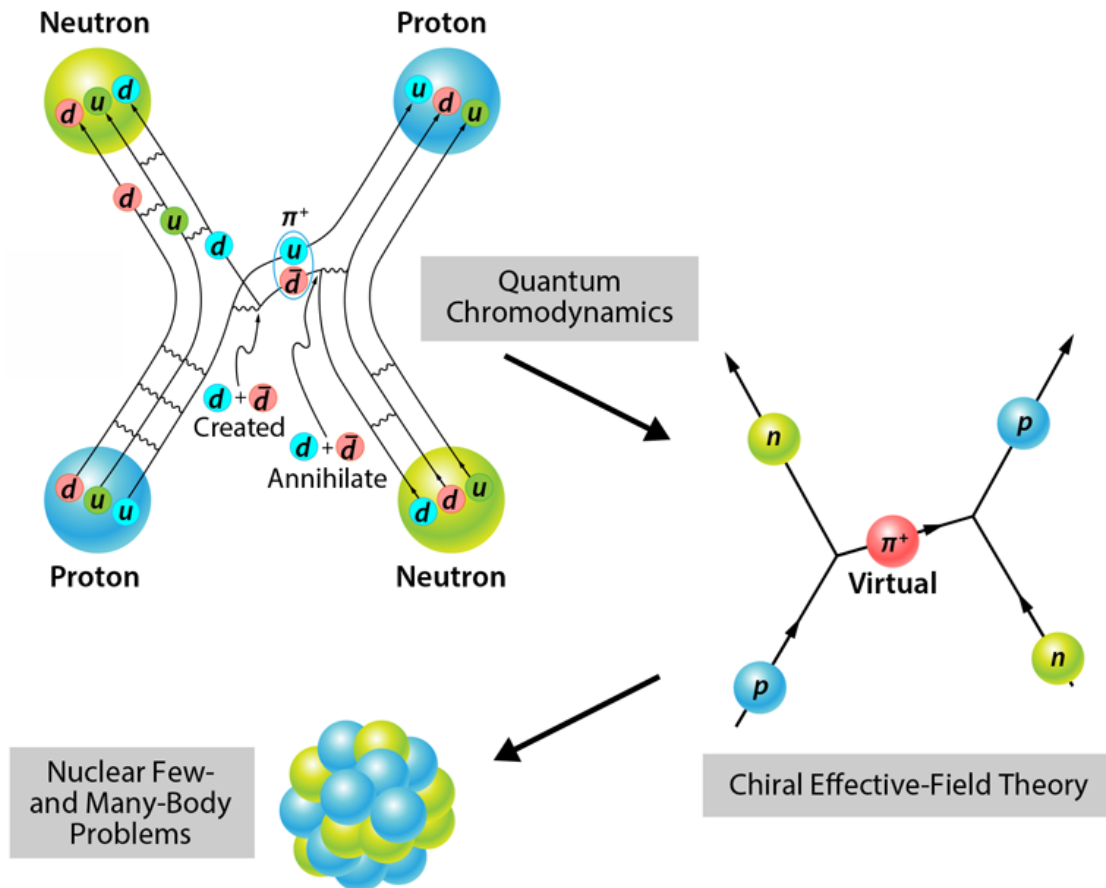
$$(\hat{U}\hat{H}\hat{U}^{-1})\hat{U}\Psi = E\hat{U}\Psi$$

Reid93 is from
V.G.J.Stoks et al., PRC49, 2950 (1994).

AV16 is from
R.B.Wiringa et al., PRC51, 38 (1995).

Viewpoint: Scattering Experiments Tease Out the Strong Force

<https://physics.aps.org/articles/v10/72>



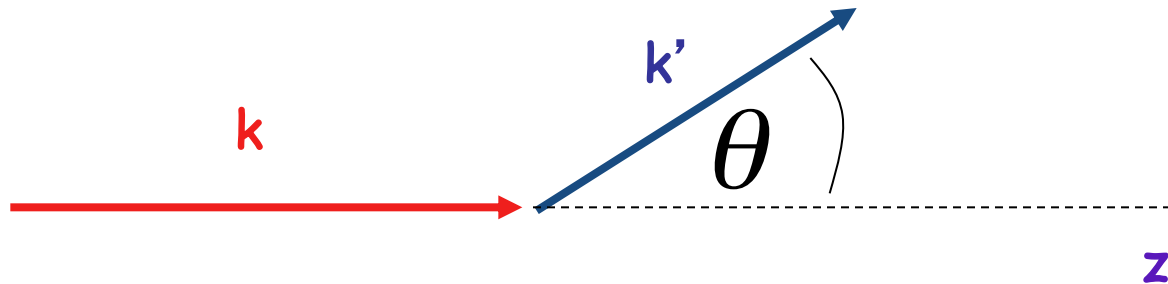
Nucleon-nucleon scattering

pp, np, nn scattering in T=0, 1 channels

- pp scattering easy
- neutron beams (np,nn) and neutron targets (pn,nn) difficult

${}^7\text{Li}(p,n)$, reactors

short neutron lifetime ($\sim 10\text{min}$); deuteron - a substitute (pp and 3N effects have to be subtracted)



$$\psi(r, \theta) \xrightarrow{r \rightarrow \infty} e^{ikz} + f(\theta) \frac{e^{ikr}}{r}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta, \varphi)|^2 \quad k = \frac{1}{\lambda} = \frac{|\vec{p}|}{\hbar} = \frac{\sqrt{2\mu E}}{\hbar}$$

Elastic scattering (the same value of k) valid for short-ranged potentials ($V=0$ at large r); Coulomb contribution must be removed

Partial-wave decomposition

For scattering off a short-ranged potential, it is useful to carry out a partial-wave decomposition.

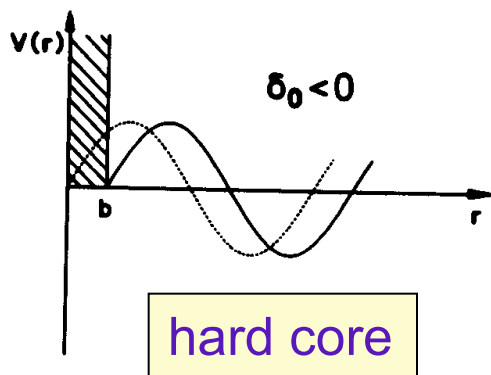
$$f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell + 1) e^{i\delta_{\ell}} \sin \delta_{\ell} P_{\ell}(\cos \theta)$$

$$R_{\ell}(kr) \xrightarrow{r \rightarrow \infty} \frac{1}{k} \sin \left(kr - \frac{\ell\pi}{2} + \delta_{\ell} \right) \quad \leftarrow \text{phase shift}$$

The probability current density in each partial wave is conserved - unitarity (valid only for elastic scattering!). The partial wave decomposition is very convenient at low energies since only a few terms enter the expansion.

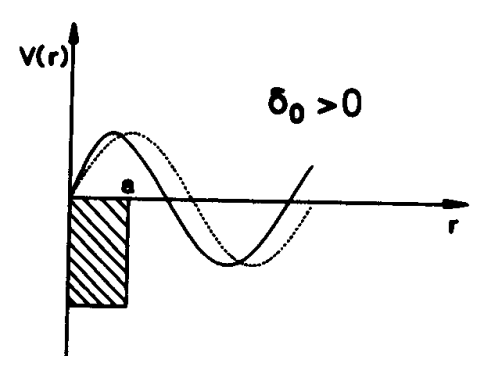
$$\ell \leq \frac{|p|a}{\hbar} \quad \leftarrow \text{potential range}$$

Phase shift

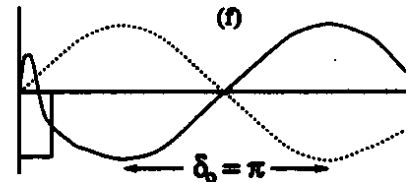
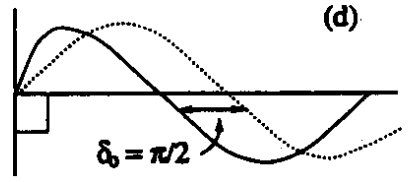
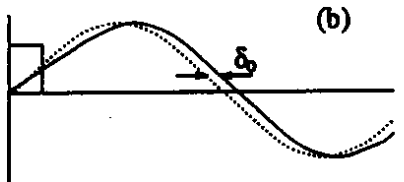
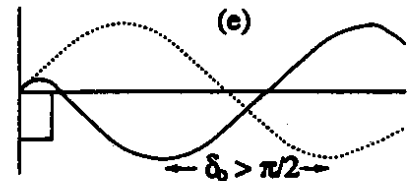
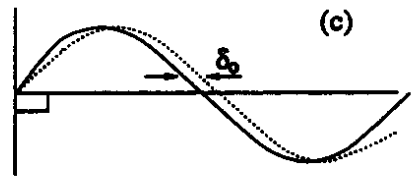
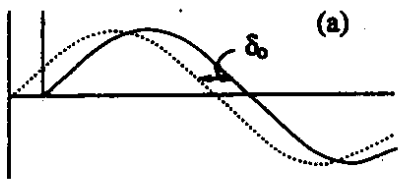


hard core

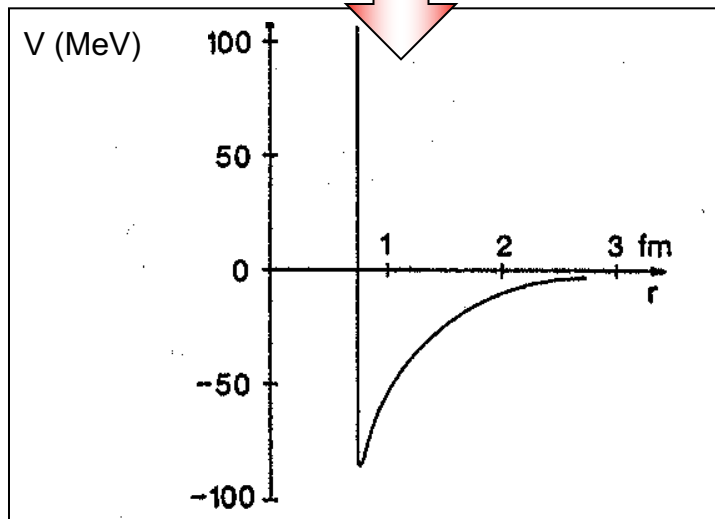
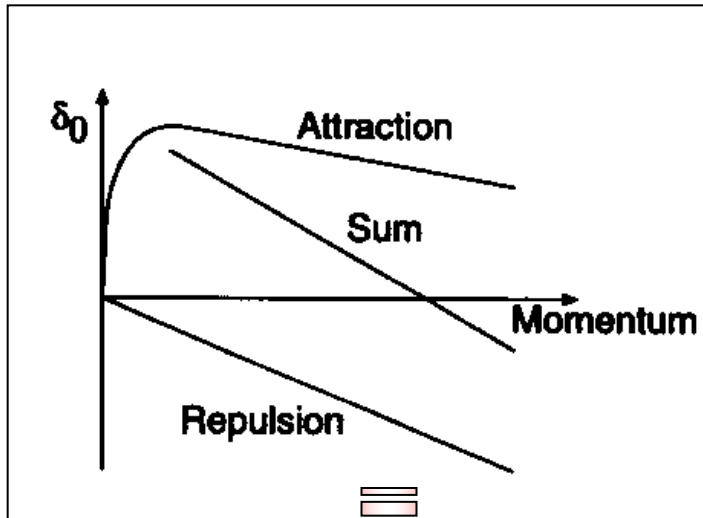
$$\delta_0 = -kb < 0$$



attractive square well



Partial wave analysis



For pn scattering (potential range 2fm) and low momenta (<400 MeV/c), the s-waves dominate. The phase shift δ_0 is decisive for nuclear binding.

For momenta > 400 MeV/c the phase shift is negative. This indicates that the nuclear force is repulsive at short distances!

