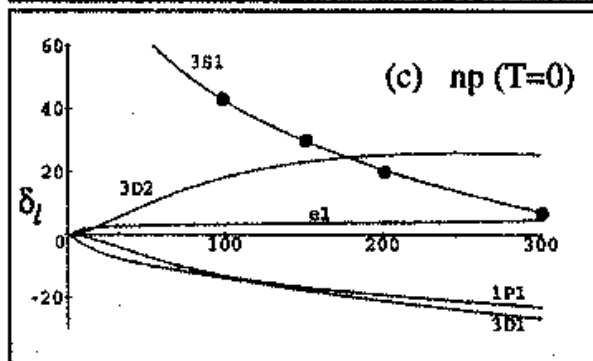
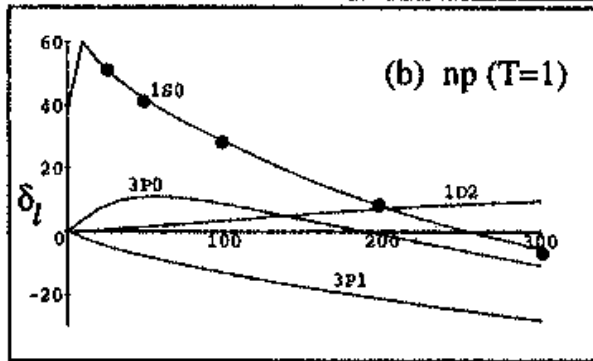
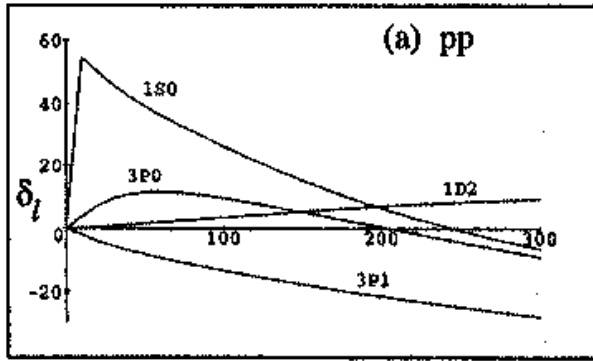
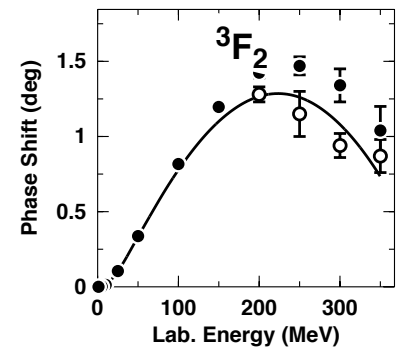
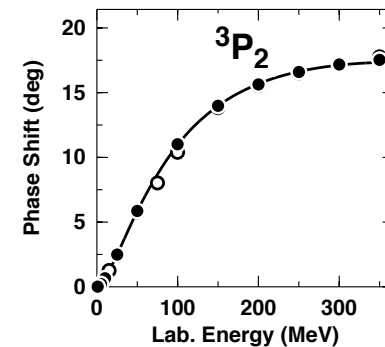
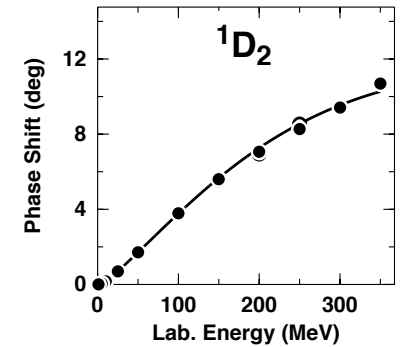
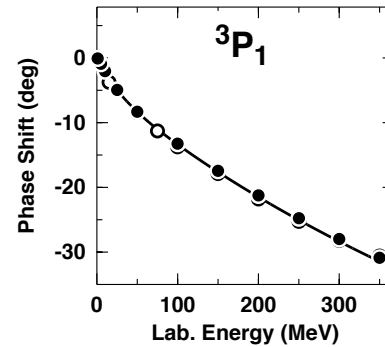
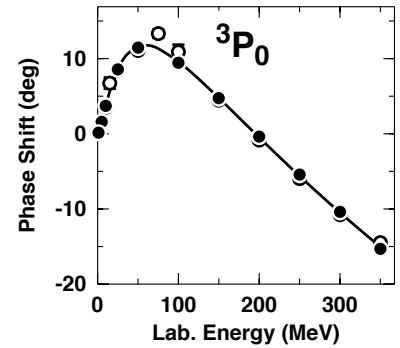
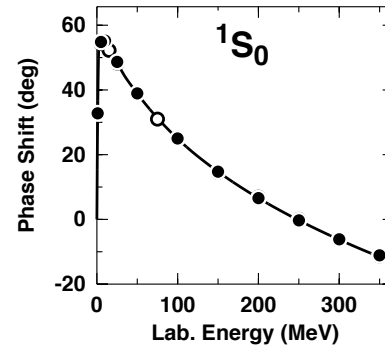


NN potentials from partial wave analysis



Laboratory energy in MeV

Paris potential



CD-Bonn potential

Scattering length and effective range

At very low energies, the $\ell=0$ total cross section remains finite for NN scattering

$$\lim_{E \rightarrow 0} \sigma = 4\pi a^2$$

S-wave scattering length

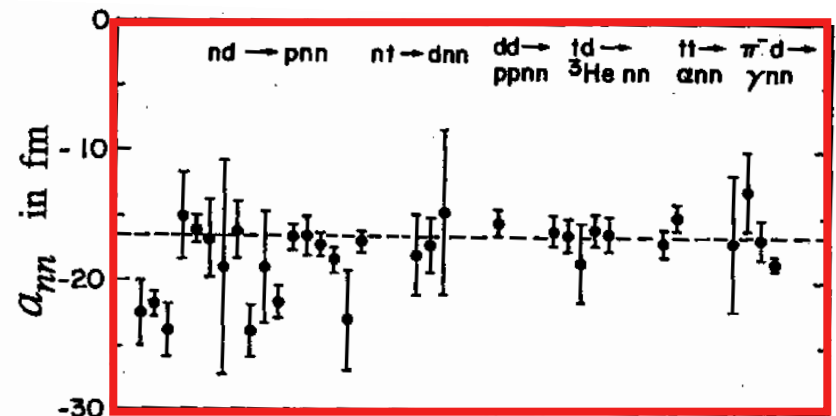
$$a = \lim_{k \rightarrow 0} \Re \left\{ -\frac{1}{k} e^{i\delta_0} \sin \delta_0 \right\}$$

is positive if there is a bound state

$$k \cot \delta_0 = -\frac{1}{a} + -\frac{1}{2} r_e k^2$$

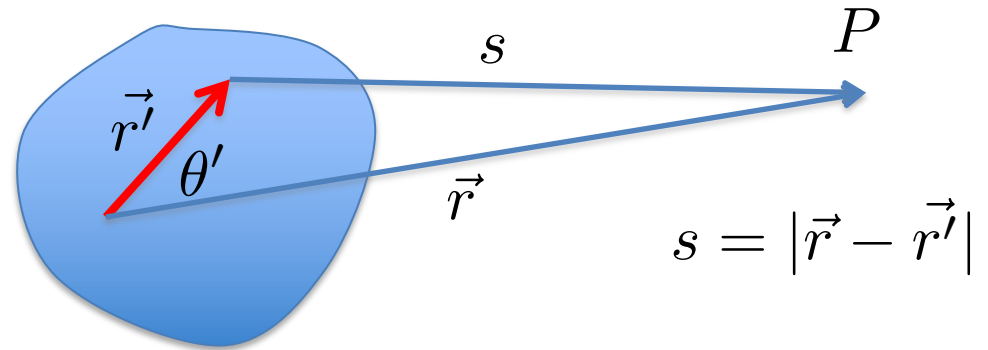
effective range

		$S = 0, T = 1$ (fm)	$S = 1, T = 0$ (fm)
pp	a	-17.1 ± 0.2	—
	r_e	2.794 ± 0.015	—
nn	a	-16.6 ± 0.6	—
	r_e	2.84 ± 0.03	—
np	a	-23.715 ± 0.015	5.423 ± 0.005
	r_e	2.73 ± 0.03	1.73 ± 0.02



Resolution and Effective Field Theory

multipole expansion
of electrostatic potential



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') d^3 r'$$

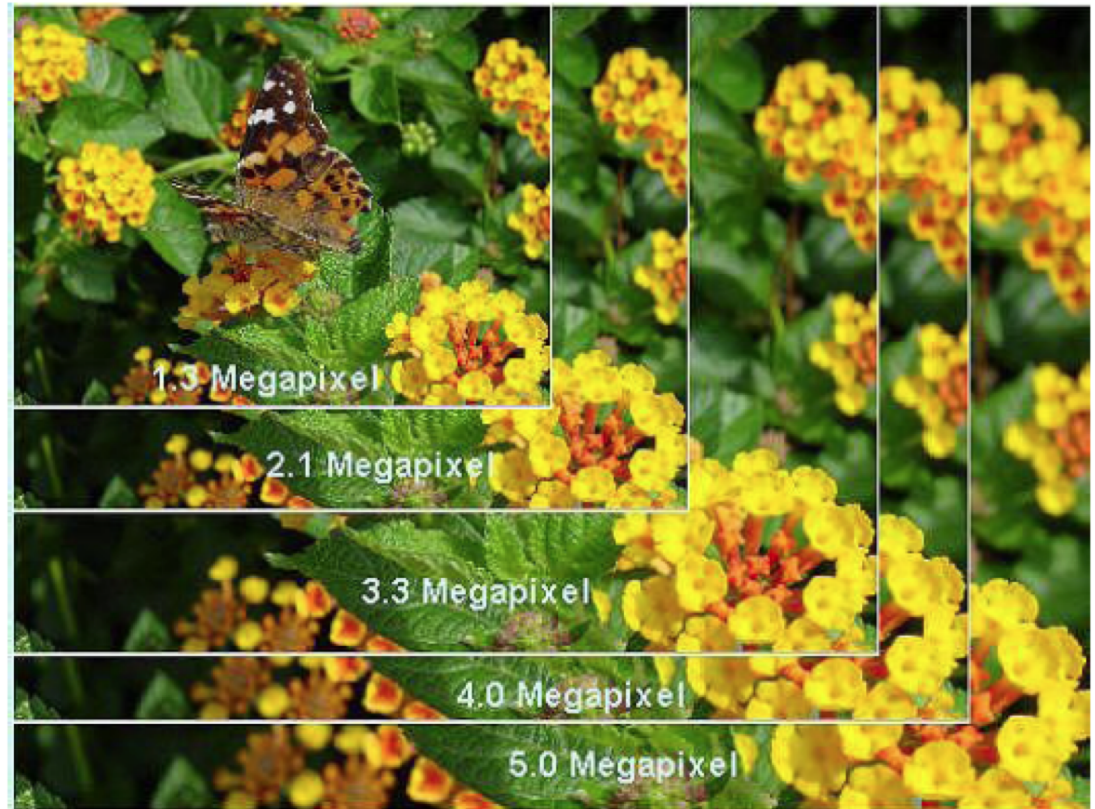
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \dots \right)$$

$$\vec{p} = \int \vec{r}' \rho(\vec{r}') d^3 r' = \sum_{k=1}^N q_k \vec{r}'_k$$

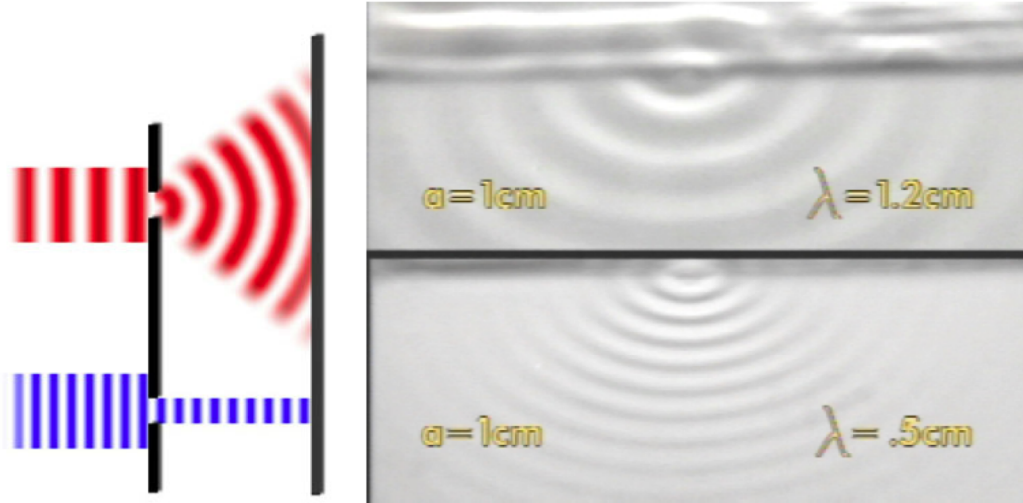
$$Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\vec{r}') d^3 r'$$

Digital Resolution: Higher Resolution is Better (?)

- Computer screens, printers, digital cameras, TV's ...
- Higher resolution \Rightarrow more pixels
- Pixel size \ll characteristic scale \Rightarrow greater detail
- Greater resolution \Rightarrow more \$\$\$



Consequences

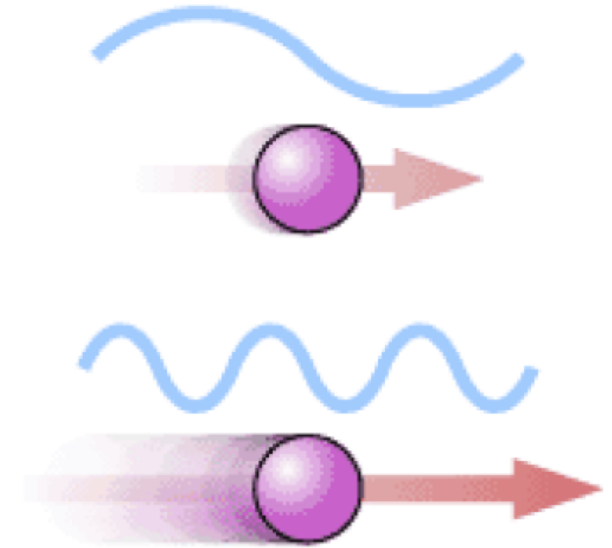


For a slit of width a ,

$$\theta_{\min} = \sin^{-1}(\lambda/a)$$

so if $\lambda \geq a$, you don't learn anything about the details of the slit

• de Broglie relation: $\lambda = h/p$



• $\lambda \approx 10^{-10} \text{ m} \implies$ probe atoms

• $\lambda \approx 10^{-14} \text{ m} \implies$ probe nucleus

• $\lambda \approx 10^{-18} \text{ m} \implies$ probe quarks

- If system is probed at low energies, fine details not resolved
- Use low-energy variables for low-energy processes
- Short-distance structure can be replaced by something simpler without distorting low-energy observables
- Physics interpretation can change with resolution!

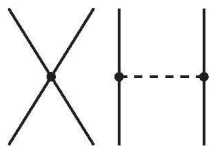
2N forces

3N forces

4N forces

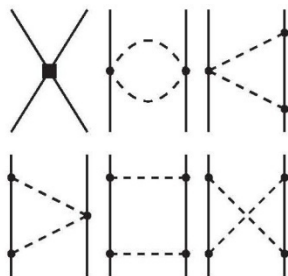
Leading
Order

Q^0
LO



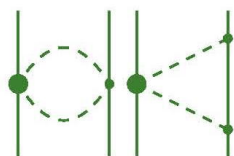
Next-to
Leading
Order

Q^2
NLO



Next-to-
Next-to
Leading
Order

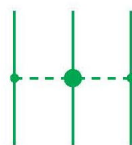
Q^3
 N^2LO



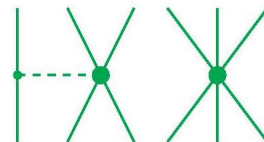
+...

derived 2000

The Hierarchy of Nuclear Forces in Chiral-EFT



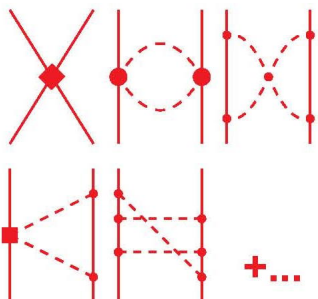
derived 2002



optimized 2014

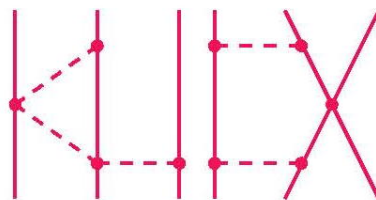
Next-to-
Next-to-
Next-to
Leading
Order

Q^4
 N^3LO



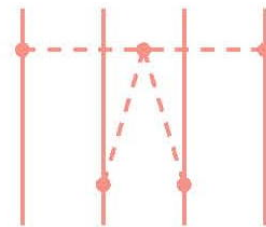
+...

derived 2003



+...

derived 2011

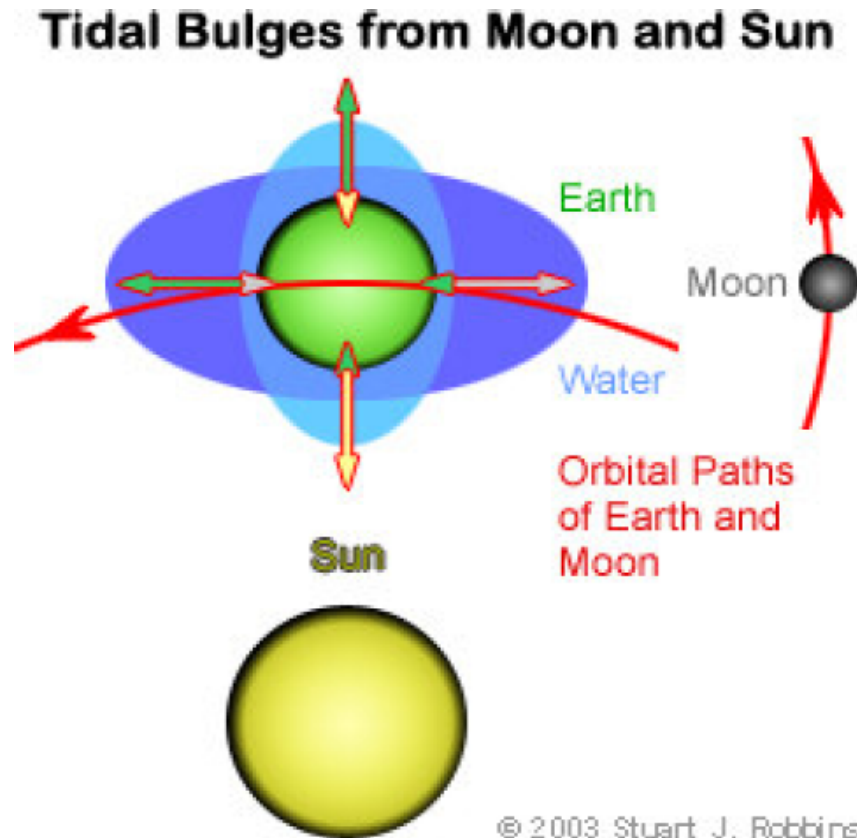


+...

derived 2007

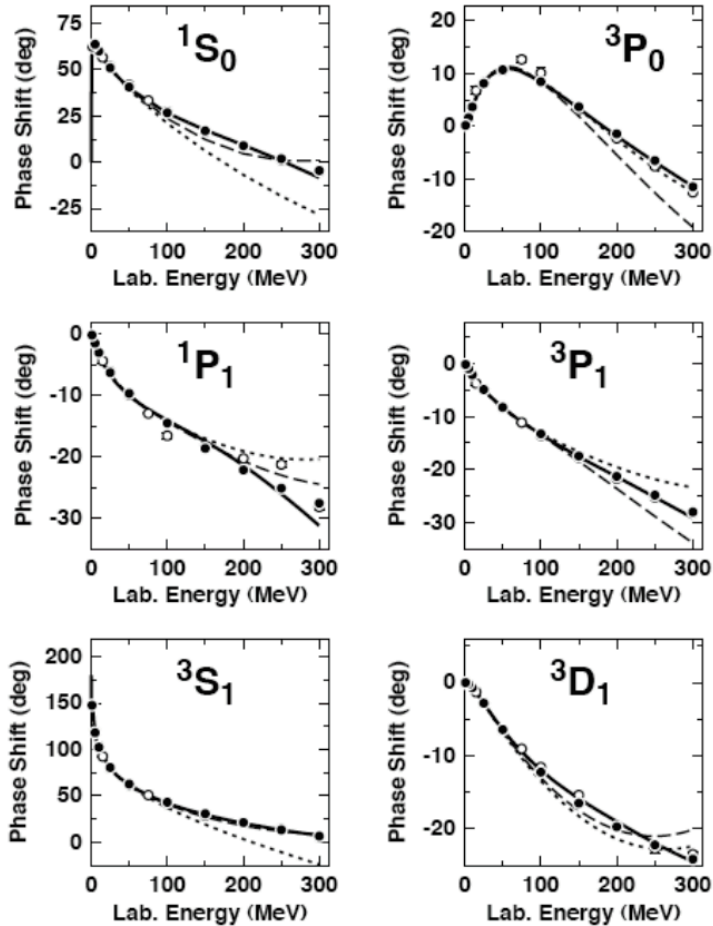
three-nucleon interactions

Three-body forces between protons and neutrons are analogous to tidal forces: the gravitational force on the Earth is *not* just the sum of Earth-Moon and Earth-Sun forces (if one employs point masses for Earth, Moon, Sun)



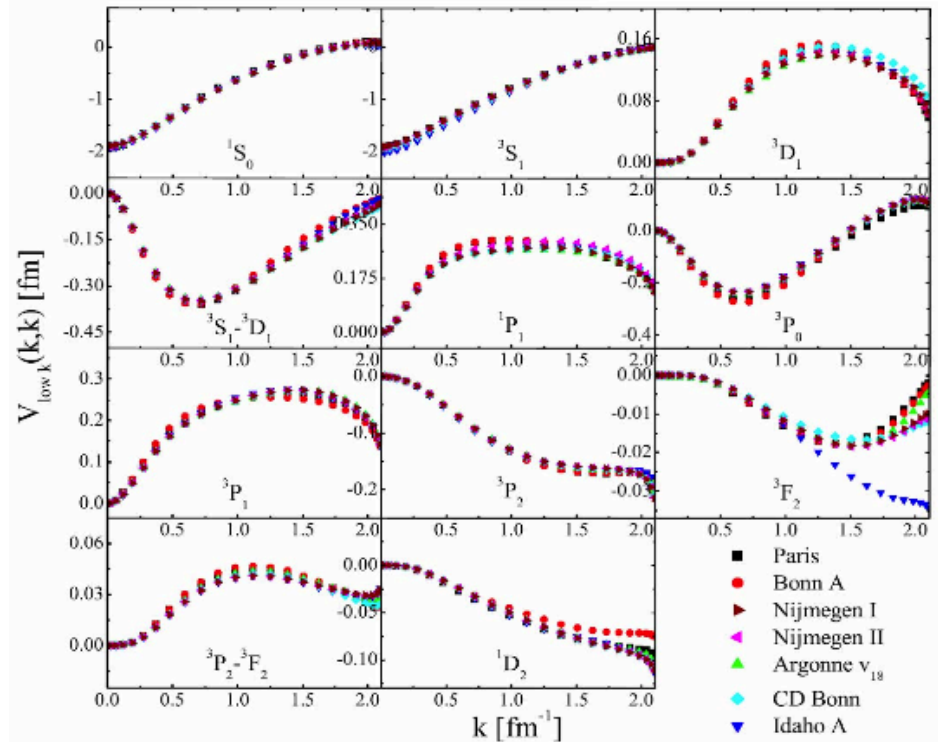
nucleon-nucleon interactions from chiral EFT

Effective-field theory potentials



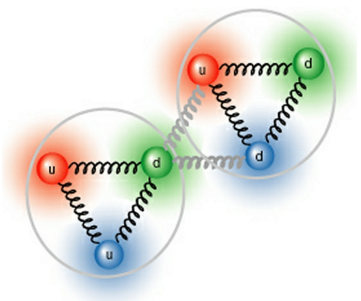
Renormalization group (RG) evolved nuclear potentials

$V_{\text{low-k}}$ unifies NN interactions at low energy



$N^3\text{LO}$: Entem et al., PRC68, 041001 (2003)
Epelbaum, Meissner, et al.

Bogner, Kuo, Schwenk, Phys. Rep. 386, 1 (2003)

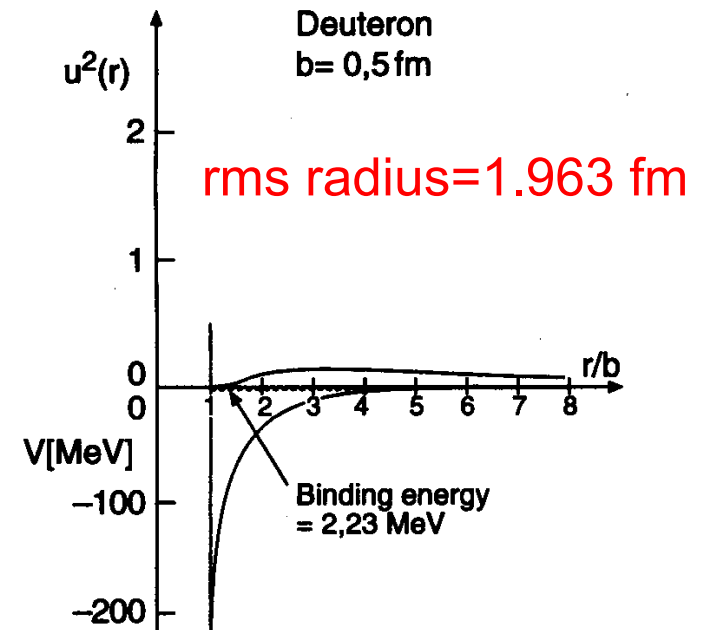
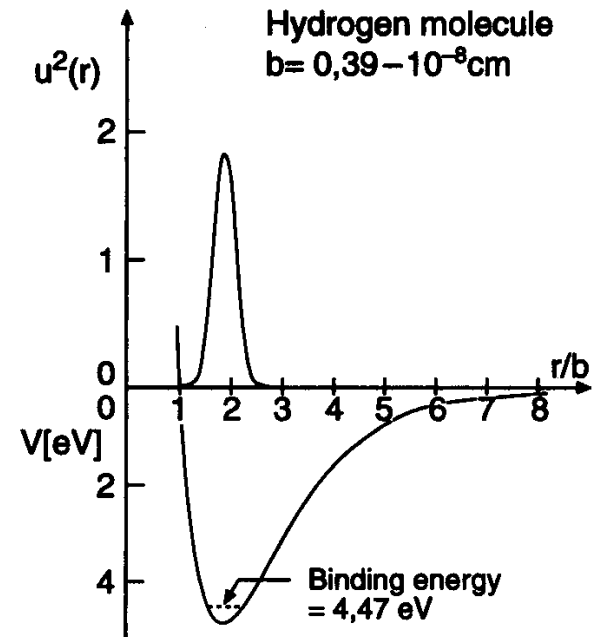
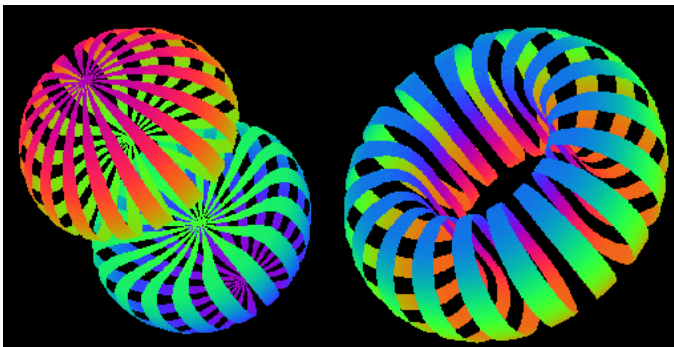


Deuteron

Binding energy	2.225 MeV
Spin, parity	1^+
Isospin	0
Magnetic moment	$\mu=0.857 \mu_N$
Electric quadrupole moment	$Q=0.282 e \text{ fm}^2$

$$|\psi_d\rangle = 0.98 |^3S_1\rangle + 0.20 |^3D_1\rangle$$

D-wave produced by tensor force!



Deuteron Shapes with V_{18}



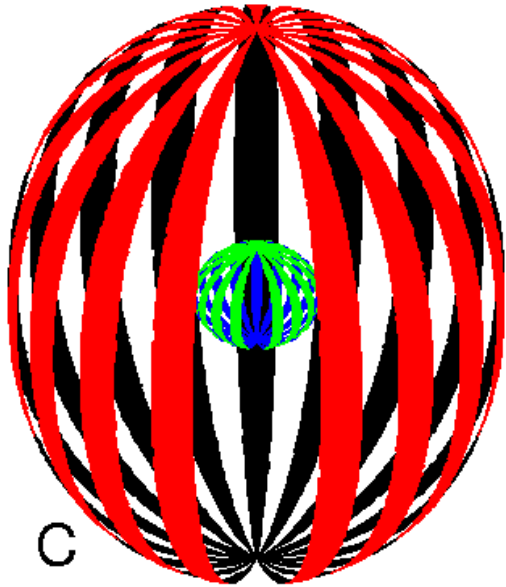
A

$M_d = \pm 1$

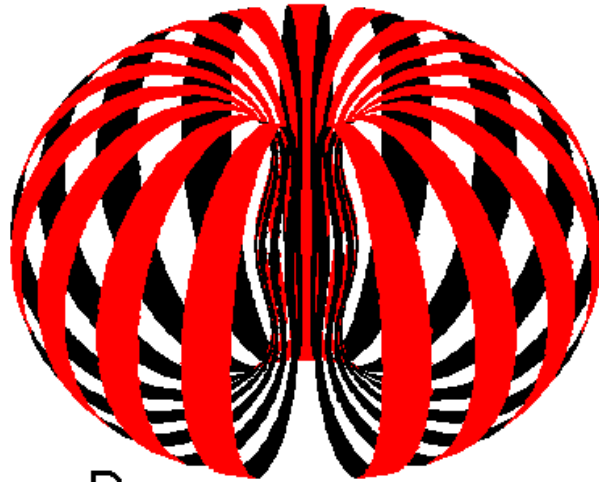


B

$M_d = 0$

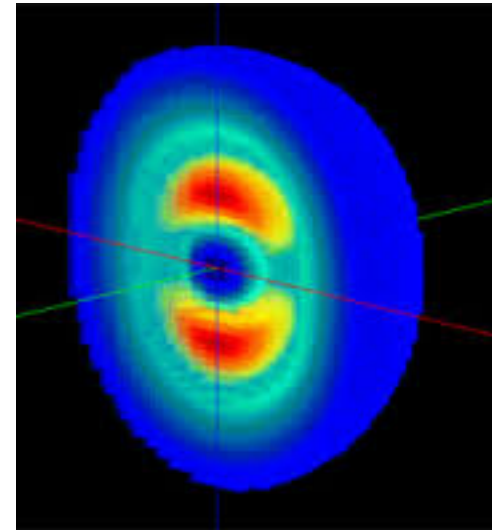


C



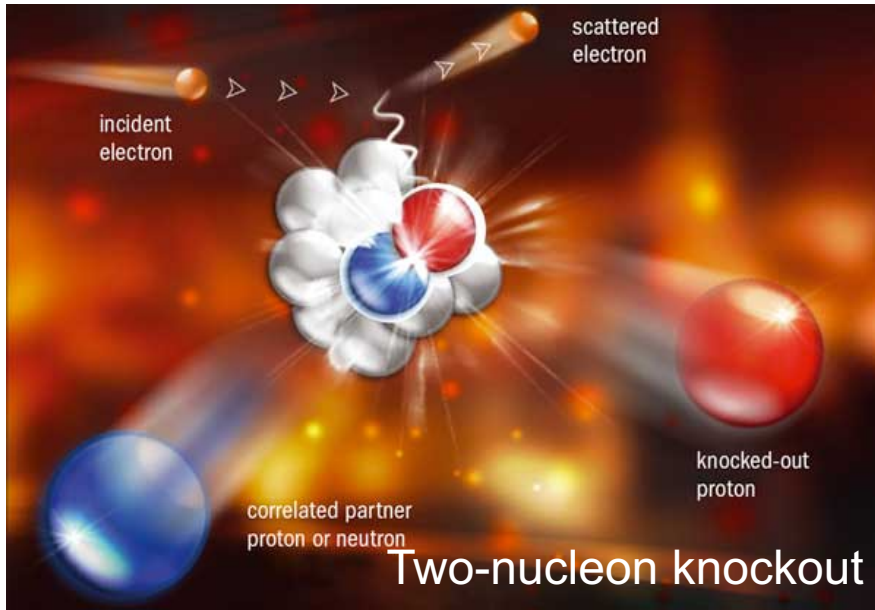
D

Jlab data

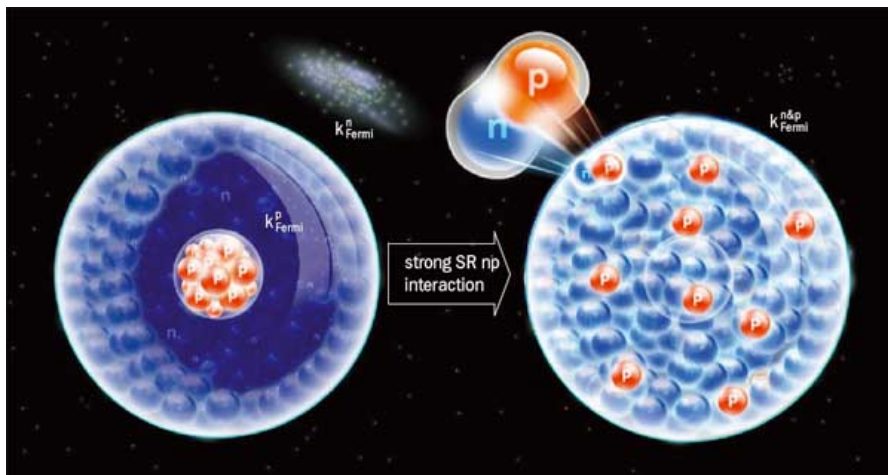


<http://www.phy.anl.gov/theory/movie-run.html>

Short Range Correlations and Tensor Force

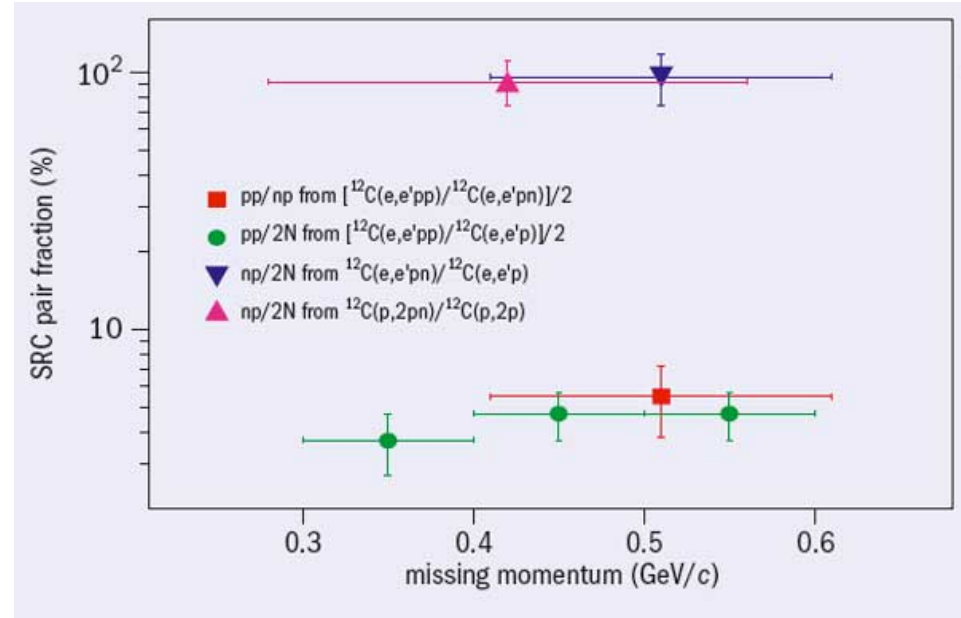


<http://cerncourier.com/cws/article/cern/34919>
<http://cerncourier.com/cws/article/cern/37330>



Dominance of proton–neutron pairs at intermediate range (c.o.m. momenta around 400 MeV/c)

Science 320, 1475 (2008)



Theory explains the pn pair dominance in terms of the tensor force:

Phys. Rev. Lett. 98, 132501 (2007)