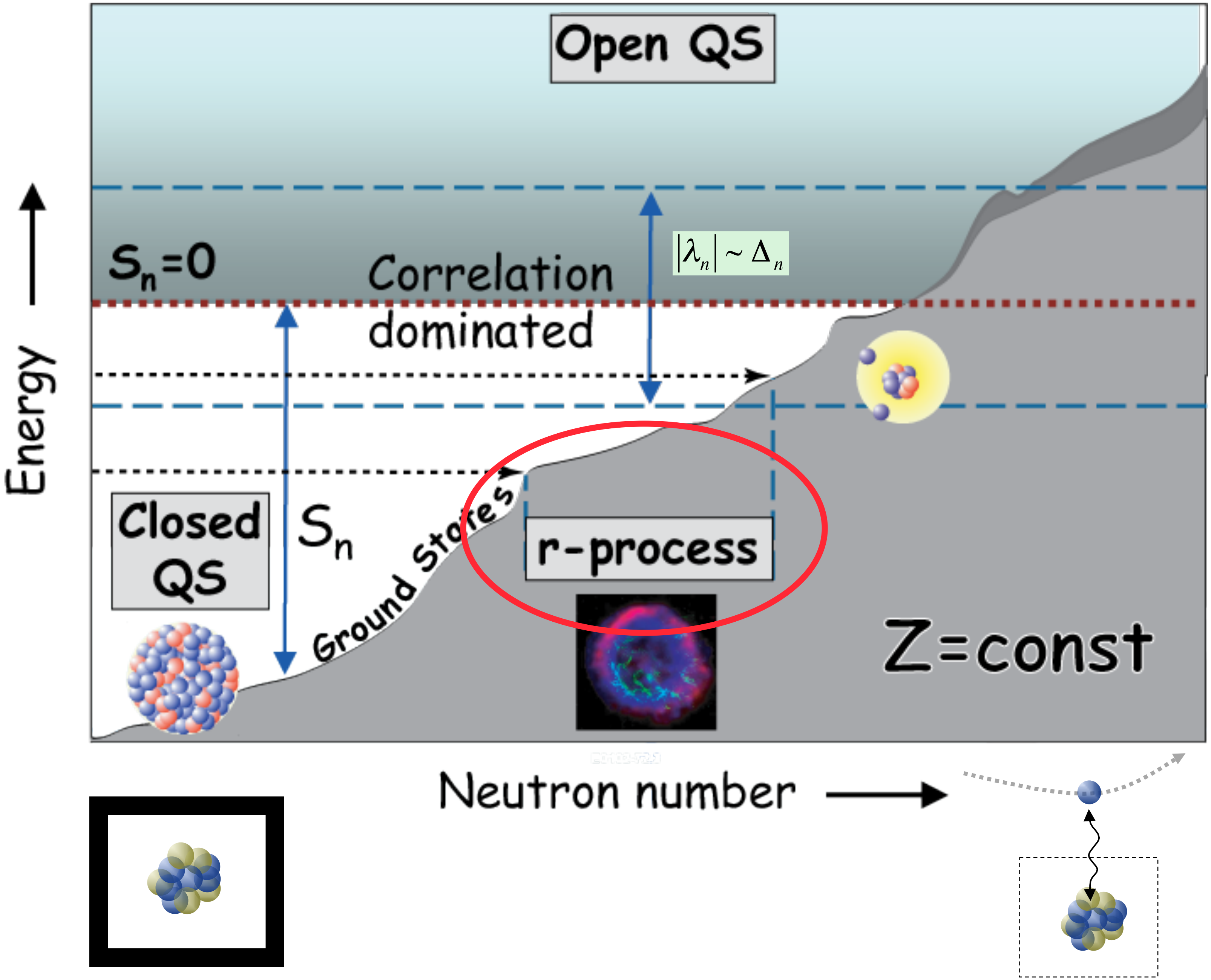
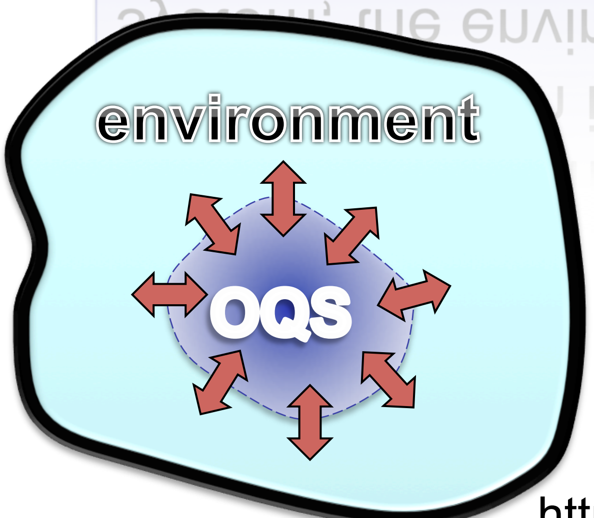
Open quantum systems

Prog. Part. Nucl. Phys. 59, 432 (2007)



Wikipedia:

An open quantum system is a quantum system which is found to be in interaction with an external quantum system, the environment. The open quantum system can be viewed as a distinguished part of a larger closed quantum system, the other part being the environment.

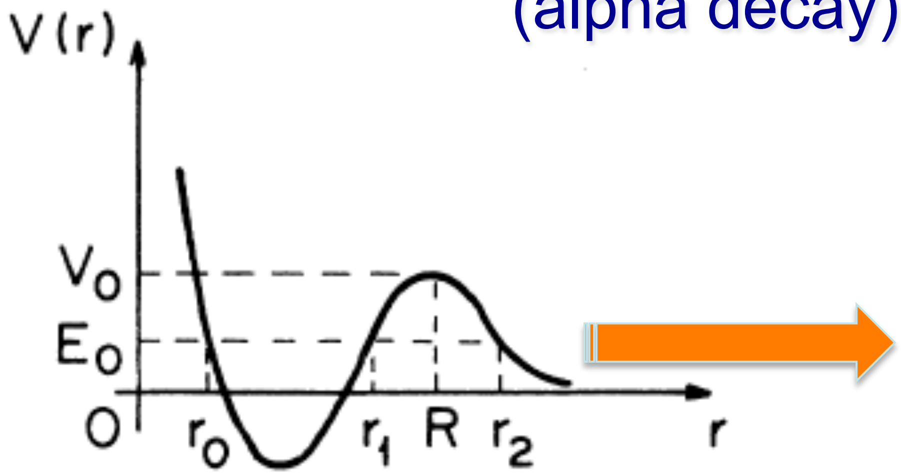


INTERDISCIPLINARY

Small quantum systems, whose properties are profoundly affected by environment, i.e., continuum of scattering and decay channels, are intensely studied in various fields of physics: nuclear physics, atomic and molecular physics, nanoscience, quantum optics, etc.

http://www.phy.ornl.gov/theory/MBOQS/Manifesto_09.html

Quasistationary States (alpha decay)



For the description of a decay, we demand that far from the force center there be only the outgoing wave. The macroscopic equation of decay is

$$\frac{dN}{dt} = -wN; \quad N = N_0 e^{-wt}$$

N is a number of radioactive nuclei, i.e., number of particles inside of sphere *r=R:*

$$N \sim \int |\psi^2| d^3r$$

We should thus seek a solution of the form

$$\psi = \psi(r) e^{-iE_0 t/\hbar - wt/2} = \psi(r) e^{-iEt/\hbar}$$

$$E = E_0 - i\frac{\Gamma}{2}; \quad \Gamma = \hbar w$$

J.J. Thompson, 1884
G. Gamow, 1928

relation between decay width and decay probability

The time dependent equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\Delta + V(r)\right]\psi$$

can be reduced by the above substitution to the stationary equation

$$\left[E + \frac{\hbar^2}{2m}\Delta - V(r)\right]\psi(r) = 0$$

The boundary condition $$\lim_{r\to\infty}\frac{d\ln\psi(r)}{dr} = i\frac{\sqrt{2mE}}{\hbar}$$

takes care of the discrete complex values of *E*

In principle, resonances and decaying particles are different entities. Usually, resonance refers to the energy distribution of the outgoing particles in a scattering process, and it is characterized by its energy and width. A decaying state is described in a time dependent setting by its energy and lifetime. Both concepts are related by:

$$T_0 = \frac{\hbar}{\Gamma}$$

This relation has been checked in numerous precision experiments.

See more discussion in
R. de la Madrid,
Nucl. Phys. A812, 13 (2008)

TABLE III. Recent theoretical and experimental lifetimes τ for NaI $3p\ ^2P_{1/2}$ and $^2P_{3/2}$ and total line strengths $S(3s\text{-}3p)$ (uncertainties given in parentheses).

Ref.	Method	J	τ_J (ns)	S (a.u.)
	Theoretical			
[6]	Semiempirical			37.03
[7]	Semiempirical			37.19
[4]	RMBPT all orders			37.38(11)
[11]	Coupled clusters			37.56[a]
[3]	MCHF-CCP			37.30[a]
[5]	MCHF+CI			37.26[a]
	Experimental			
[1]	BGLS	1/2	16.40(3)	37.04(7)[b]
[16]	Pulsed laser	1/2	16.35(6)	37.15(14)[b]
[17]	C_3 analysis	1/2	16.31(6)	37.24(12)[b]
[18]	Linewidth	3/2	16.237(35)	37.30(8)[b]
This	BGLS	1/2	16.299(21)	37.26(5)[c]
work		3/2	16.254(22)	

[a]Corrected for relativistic effects (−0.09 a.u.) using the ratio between DF and HF values. The original value of Ref. [3] without relativistic correction is 37.39 a.u.
[b]A line strength ratio between the two fine-structure components of 0.5 was assumed in the calculation.
[c]The ratio of the line strengths of the two fine-structure components was determined to 0.50014(44). This is in excellent agreement with the nonrelativistic prediction of 0.5. In the uncertainty estimate for the ratio all those systematical effects were omitted which affect both lifetimes in the same way.

U. Volz et al., Phys. Rev. Lett 76, 2862(1996)

Basic Equations

Time Dependent (many body) Schödinger Equation

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

+ boundary conditions

Often impractical/impossible to solve but an excellent starting point

Time Independent (many body) Schödinger Equation

$$\hat{H}\psi = E\psi$$

$$\left.\begin{array}{l}\text{Box boundary conditions (w.f. vanishes at large distances)}\\ \text{Decaying boundary conditions}\\ \text{Incoming or capturing boundary conditions}\\ \text{Scattering boundary conditions}\\ \\ \text{Absorbing boundary conditions}\end{array}\right\}\ \text{choice depends on physics case}$$

A comment on the time scale…

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$ TDSE

$$T_{1/2} = \ln 2\frac{\hbar}{\Gamma}, \qquad \hbar = 6.58\cdot 10^{-22}\ \mathrm{MeV}\cdot\mathrm{sec}$$

Can one calculate Γ with sufficient accuracy using TDSE?

$$T_{s.p.} \approx 3\cdot 10^{-22}\,\mathrm{sec} = 3\,\mathrm{baby\,sec}$$

^{238}U: $T_{1/2}=10^{16}$ years
^{256}Fm: $T_{1/2}=3$ hours

For very narrow resonances, explicit time propagation impossible!