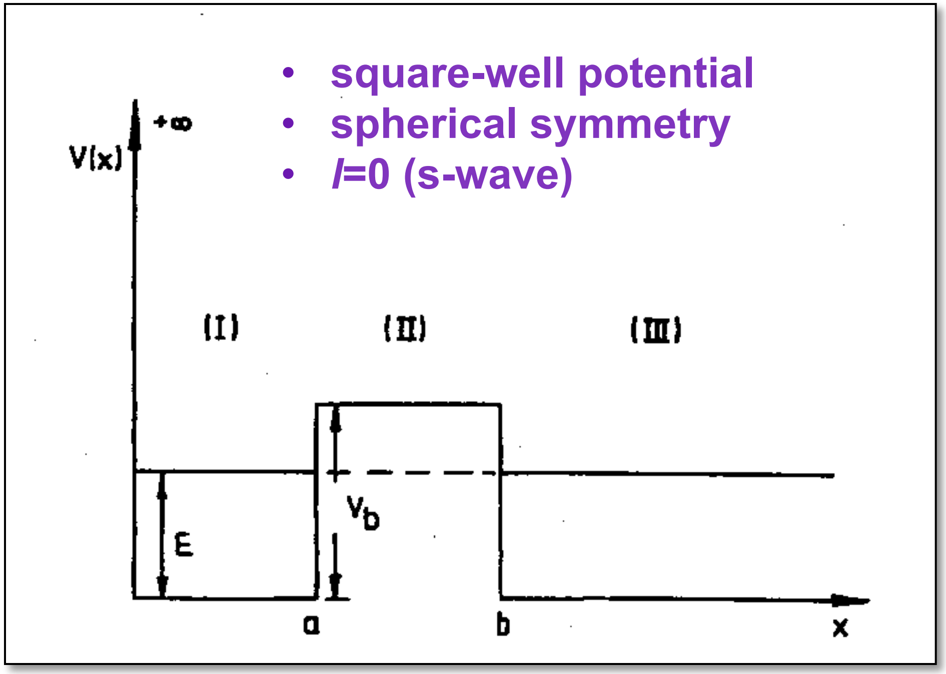
How do resonances appear? A square well example



Radial Schrödinger equation

$$\chi'' + \frac{2M}{\hbar^2}(E - V)\chi = 0 \quad (\chi = \varphi r)$$

Region I:

$$\chi_I = A \sin pr, \quad p^2 = \frac{2ME}{\hbar^2}$$

Region II:

$$\chi_{II} = c_+ e^{q(r-a)} + c_- e^{-q(r-a)}, \quad q^2 = \frac{2M(V_b - E)}{\hbar^2}$$

Region III:

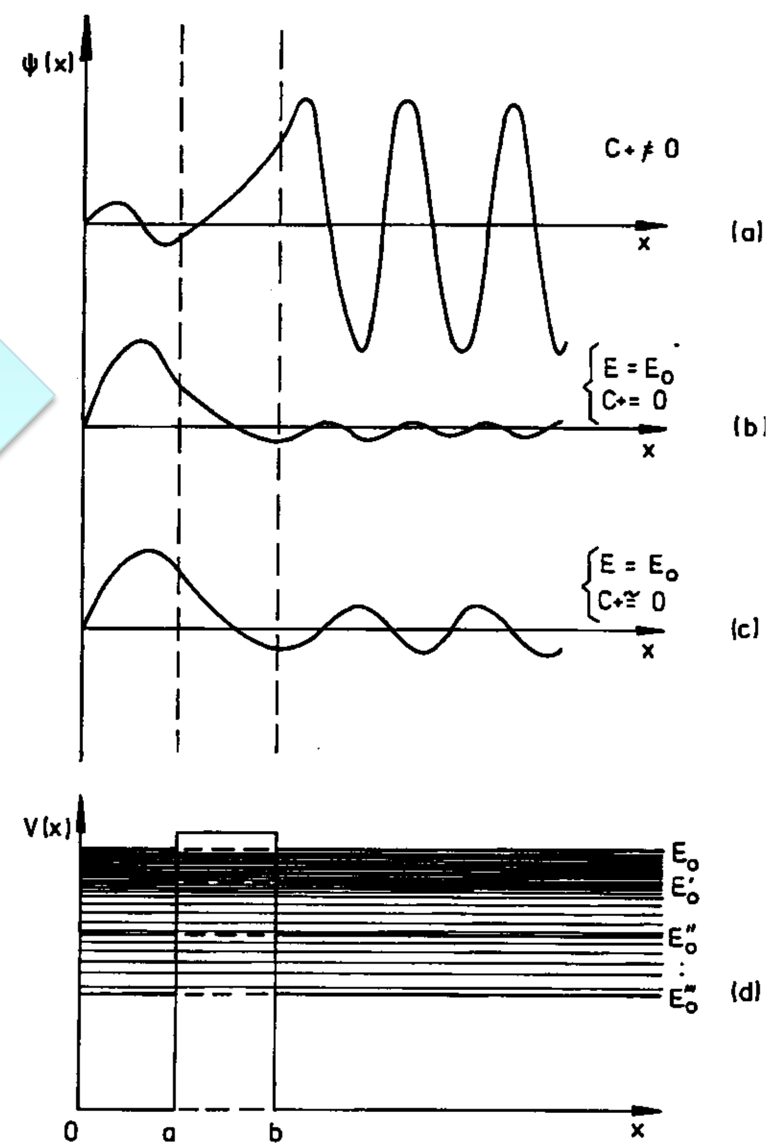
$$\chi_{III} = c_1 e^{ip(r-b)} + c_2 e^{-ip(r-b)}$$

In almost all cases $|\chi_{III}|$ is much larger than $|\chi_I|$. We are now interested in those situations where $|\chi_{III}|$ is as small as possible.

The condition

$$c_+ = 0 \Rightarrow \tan(pa) = -\frac{p}{q}$$

defines "virtual" levels in region I: particle is well localized; very small penetrability through the barrier



When $c_+=0$, the penetrability becomes proportional to

$$\frac{\left|\chi_{III}\right|^2}{\left|\chi_I\right|^2} \propto \exp\left[-\frac{2}{\hbar}\sqrt{2M(V_b-E)}(b-a)\right]$$

This is the semi-classical WKB result:

$$P=\frac{\left|\chi_{III}\right|^2}{\left|\chi_I\right|^2} \propto \exp\left[-2\int_{r_1}^{r_2} k(r)\,dr\right]$$

Width of a narrow resonance

open closed

$$H(t) = H_0 + V(t) \quad (|V| << |H_0|)$$ time-dependent Hamiltonian

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$ …expansion of ψ in the basis of H_o

$$\hat{H}_0 \phi_n = E_n \phi_n \Rightarrow \psi = \sum_n c_n(t)\phi_n e^{-iE_n t/\hbar}$$

$$i\hbar \frac{dc_k}{dt} = \sum_n c_n(t)\langle \phi_k | V | \phi_n \rangle e^{i\omega_{kn}t}, \quad \omega_{kn} = \left(E_k - E_n\right)/\hbar$$

As initial conditions, let us assume that at $t=0$ the system is in the state ϕ_0

$$c_n(0) = \begin{cases} 1 \text{ for n} = 0 \\ 0 \text{ for n} \neq 0 \end{cases}$$

If the perturbation is weak, in the first order, we obtain:

$$i\hbar \frac{dc_k}{dt} = \langle \phi_k | V | \phi_0 \rangle e^{i\omega_{k0}t}$$

Furthermore, if the time variation of V is slow compared with exp$(i\omega_{ko}t)$, we may treat the matrix element of V as a constant. In this approximation:

$$c_k(t) = \frac{\langle \phi_k | V | \phi_0 \rangle}{E_k - E_0}\left(1 - e^{i\omega_{k0}t}\right)$$

The probability for finding the system in state k at time t if it started from state 0 at time t=0 is:

$$\left|c_k(t)\right|^2 = 2\frac{\left|\langle \phi_k | V | \phi_0 \rangle\right|^2}{\left(E_k - E_0\right)^2}\left(1 - \cos\omega_{k0}t\right)$$

The total probability to decay to a group of states within some interval labeled by f equals:

$$\sum_{k\in f}\left|c_k(t)\right|^2 = \frac{2}{\hbar^2}\int \frac{\left|\langle \phi_k | V | \phi_0 \rangle\right|^2}{\omega_{k0}^2}\left(1 - \cos\omega_{k0}t\right)\rho\left(E_k\right)dE_k$$

The transition probability per unit time is

$$\mathcal{W} = \frac{d}{dt}\sum_{k\in f}\left|c_k(t)\right|^2 = \frac{2}{\hbar^2}\int\left|\langle\phi_k|V|\phi_0\rangle\right|^2\frac{\sin\omega_{k0}t}{\omega_{k0}}\rho(E_k)\,dE_k$$

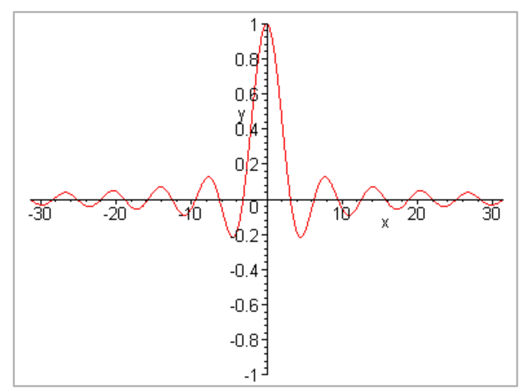
Since the function sin*(x)/x* oscillates very quickly except for *x*~0, only small region around E_0 can contribute to this integral. In this small energy region we may regard the matrix element and the state density to be constant. This finally gives:

$$\mathcal{W}_{0\to f} = \frac{2\pi}{\hbar}\left|\langle\phi_f|V|\phi_0\rangle\right|^2\rho(E_f)$$

Fermi's golden rule

Although named after Fermi, most of the work leading to the Golden Rule was done by Dirac, who formulated an almost identical equation 1927. It is given its name because Fermi called it "Golden Rule No. 2." in 1950.

$$\int_{-\infty}^{\infty}\frac{\sin x}{x}dx = \pi$$



$$E = E_0 - i\frac{\Gamma}{2}; \quad \Gamma = \hbar w \qquad N = N_0 e^{-wt}$$

mean lifetime $$T_0 = \frac{\hbar}{\Gamma}$$

half-life $$T_{1/2} = \ln 2 \, \frac{\hbar}{\Gamma} = \ln 2 \, T_0 \qquad \frac{N_0}{2} = N_0 e^{-wT_{1/2}}$$

transition probability $$\mathcal{W}_{0\to f} = \frac{1}{T_{0\to f}} = \frac{\Gamma_{0\to f}}{\hbar}$$

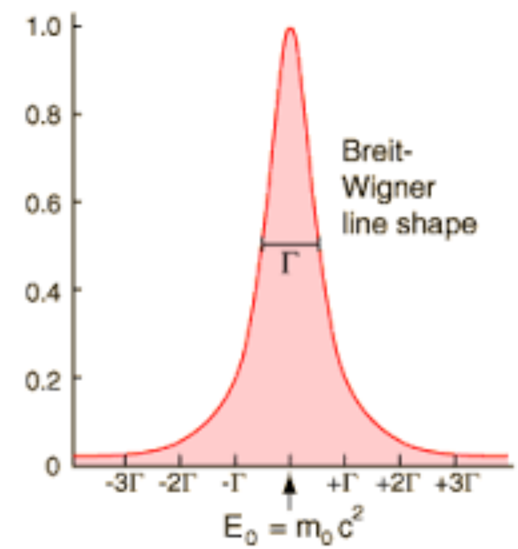
Fermi's golden rule $$\Gamma_{0\to f} = 2\pi \left|\langle \phi_f | V | \phi_0 \rangle\right|^2 \rho(E_f)$$

$$\psi(t) = \psi(0) \int c(E) e^{-iEt\hbar} dE/2\pi$$

$$c(E) = \int_0^\infty e^{i(E-E_0+i\Gamma/2)t\hbar} dt = \frac{i\hbar}{E - E_0 + i\Gamma/2}$$

normalized amplitude $$|c(E)|^2 = \frac{1}{\pi} \frac{\Gamma/2}{(E-E_0)^2 + (\Gamma/2)^2}$$

uncertainty principle $$\Gamma T_0 = \hbar$$



When can we talk about “existence” of an unbound nuclear system?

$$T_{1/2} = \ln 2 \frac{\hbar}{\Gamma}, \qquad \hbar = 6.58 \cdot 10^{-22}\ \mathrm{MeV} \cdot \mathrm{sec}$$

$$T_{s.p.} \approx 3 \cdot 10^{-22}\ \mathrm{sec} = 3\,\mathrm{baby\,sec}$$

A typical time associated with the s.p. nucleonic motion

$$T_{1/2} >> T_{s.p.}$$

$$\Rightarrow \quad \Gamma << 1\,\mathrm{MeV}$$

Compute the half-life of:

- ^{141}Ho proton emitter (Γ=$2 \cdot 10^{-20}$ MeV)
- 3^- state in ^{10}Be at E=10.16 MeV (Γ=296 keV)
- First 2^+ state in ^{6}He at E=1.797MeV (Γ=113 keV)
- Hoyle state in ^{12}C at E=7.654 MeV (Γ=8.5 eV)
- ^{8}Be ground state (Γ=5.57 eV)
- Baryon N(1440)$1/2^+$ (Γ=350 MeV)

Discuss the result.

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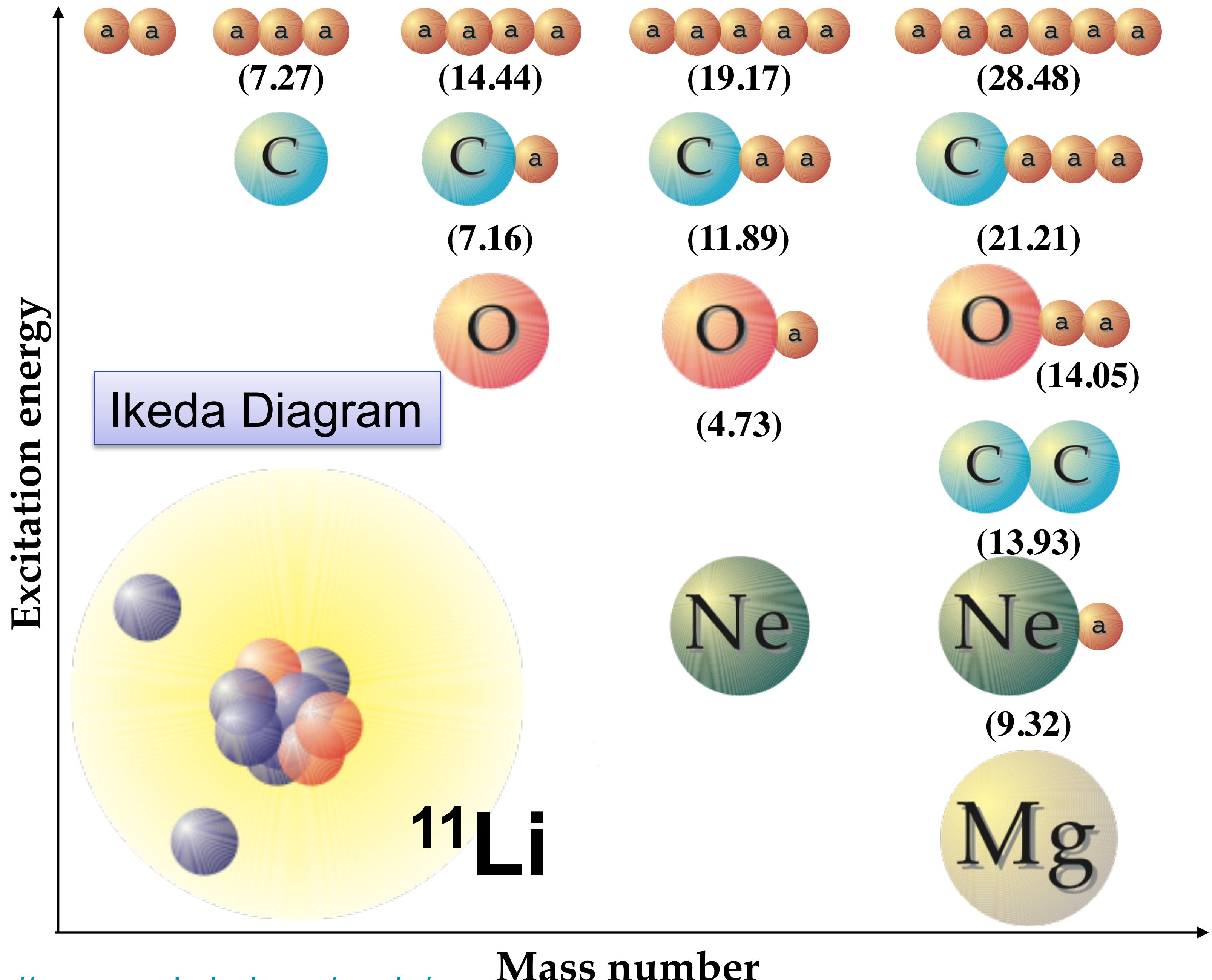
as such' [22]. This statement was supported later by Cerny and Hardy [23]: '...lifetimes longer than 10^{-12} s, a possible lower limit for the process to be called radioactivity'.

This definition would be more restrictive than the definition of an element and thus is inappropriate. The International Union of Pure and Applied Chemistry (IUPAC) has published guidelines for the discovery of a chemical element [24]. In addition to other criteria they state that 'the discovery of a chemical element is the experimental demonstration, beyond reasonable doubt, of the existence of a nuclide with an atomic number Z not identified before, existing for at least 10^{-14} s'. The justification for this limit is also given: 'This lifetime is chosen as a reasonable estimate of the time it takes a nucleus to acquire its outer electrons. It is not considered self-evident that talking about an 'element' makes sense if no outer electrons, bearers of the chemical properties, are present'.

Similarly the definition of a nucleus should be related to the typical timescales of nuclear motion. Nuclear rotation and vibration times are of the order of 10^{-22} s which can be considered a characteristic nuclear timescale [22]. The above mentioned definitions of the driplines by Mueller and Sherrill [10] and the Chart of Nuclei [19] can be used as the definition of the existence of a nucleus. If a nucleus lives long compared to 10^{-22} s it should be considered a nucleus. Unfortunately this is no sharp clear limit. The most recent editions of the chart of nuclei include unbound nuclei with lifetimes that are of the order of 10^{-22} s [19, 25].

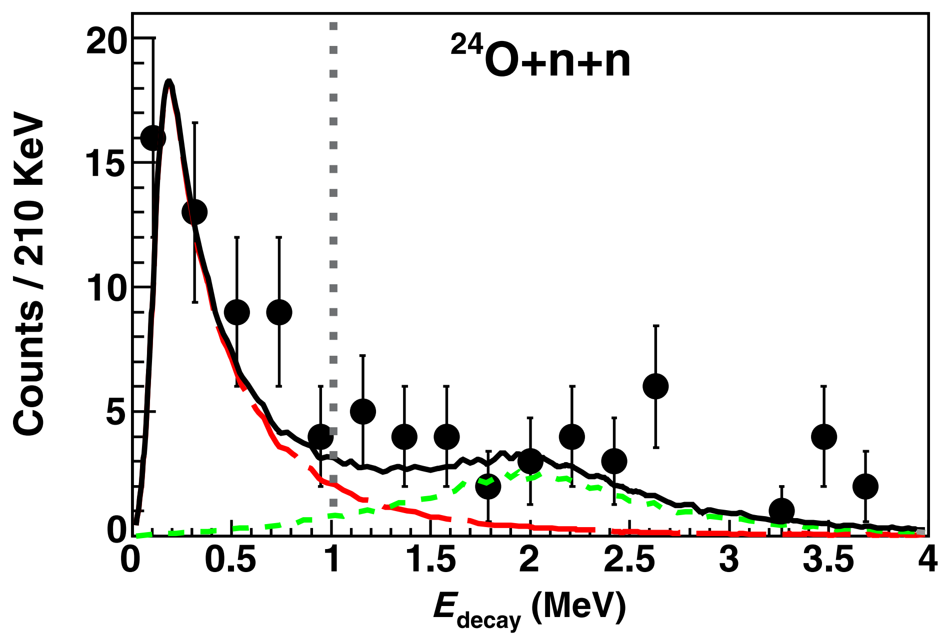


http://www.nndc.bnl.gov/qcalc/

sequence of reaction channels

Beyond the Neutron Drip-Line

http://www.tandfonline.com/doi/pdf/10.1080/10619127.2014.882735



http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.110.152501

A new technique was developed to measure the lifetimes of neutron unbound nuclei in the picosecond range. The decay of $^{26}O \rightarrow {}^{24}O + n + n$ was examined as it had been predicted to have an appreciable lifetime due to the unique structure of the neutron-rich oxygen isotopes. The half-life of ^{26}O was extracted as $4.5^{+1.1}_{-1.5}$(stat) $\pm$ 3(syst) ps. This corresponds to ^{26}O having a finite lifetime at an 82% confidence level and, thus, suggests the possibility of two-neutron radioactivity.

Tetraneutron???

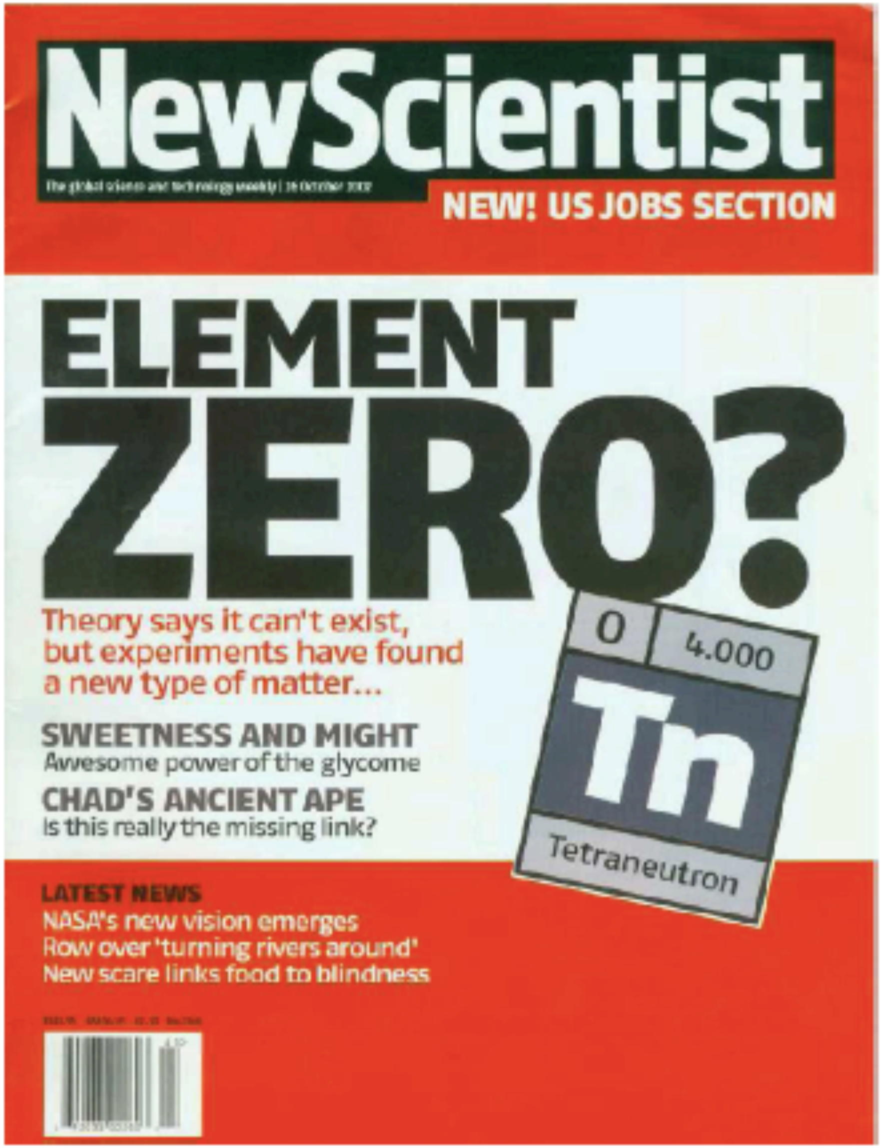
PHYSICAL REVIEW C, VOLUME 65, 044006 (2002)

Detection of neutron clusters

A new approach to the production and detection of bound neutron clusters is presented. The technique is based on the breakup of beams of very neutron-rich nuclei and the subsequent detection of the recoiling proton in a liquid scintillator. The method has been tested in the breakup of intermediate energy (30–50 MeV/nucleon) ^{11}Li, ^{14}Be, and ^{15}B beams. Some six events were observed that exhibit the characteristics of a multineutron cluster liberated in the breakup of ^{14}Be, most probably in the channel $^{10}Be + {}^{4}n$. The various backgrounds that may mimic such a signal are discussed in detail.

http://www.cnrs.fr/cw/en/pres/compress/noyau.htm

http://www.gamefaqs.com/pc/944906-mass-effect-2/answers/157357-where-is-the-best-planet-to-find-element-zero-resources



nothing is known [4,5]. The discovery of such neutral systems as bound states would have far-reaching implications for many facets of nuclear physics. In the present paper, the production and detection of free neutron clusters is discussed.

The question as to whether neutral nuclei may exist has a long and checkered history that may be traced back to the early 1960s [5]. Forty years later, the only clear evidence in this respect is that the dineutron is particle unstable. Although ^{3}n is the simplest multineutron candidate, the effects of pairing observed on the neutron drip line suggest that $^{4,6,8}n$ could exhibit bound states [6]. Concerning the tetraneutron, an upper limit on the binding energy of 3.1 MeV is provided by the particle stability of ^{8}He, which does not decay into $\alpha + {}^{4}n$. Furthermore, if ^{4}n was bound by more than 1 MeV, $\alpha + {}^{4}n$ would be the first particle threshold in ^{8}He. As the breakup of ^{8}He is dominated by the ^{6}He channel [7], the tetraneutron, if bound, should be so by less than 1 MeV.

The majority of the calculations performed to date suggest that multineutron systems are unbound [4]. Interestingly, it was also found that subtle changes in the N-N potentials that do not affect the phase shift analyses may generate bound neutron clusters [5]. In addition to the complexity of such *ab initio* calculations, which include the uncertainties in many-body forces, the n-n interaction is the most poorly known N-N interaction, as demonstrated by the controversy regarding the determination of the scattering length a_{nn} [8]. The

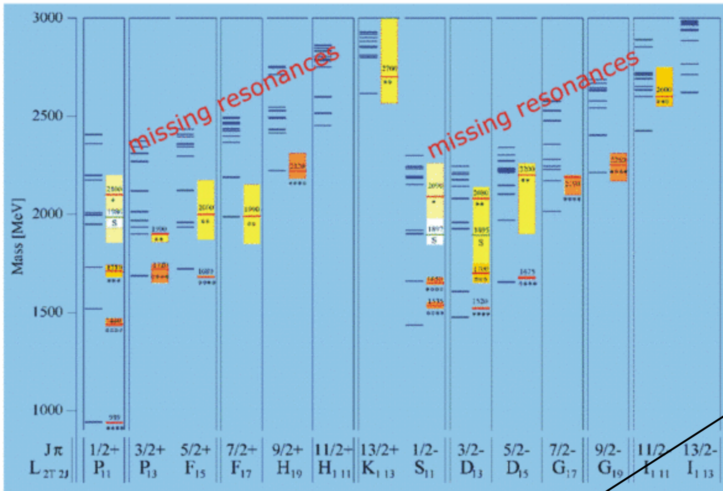
Can Modern Nuclear Hamiltonians Tolerate a Bound Tetraneutron?

http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.90.252501

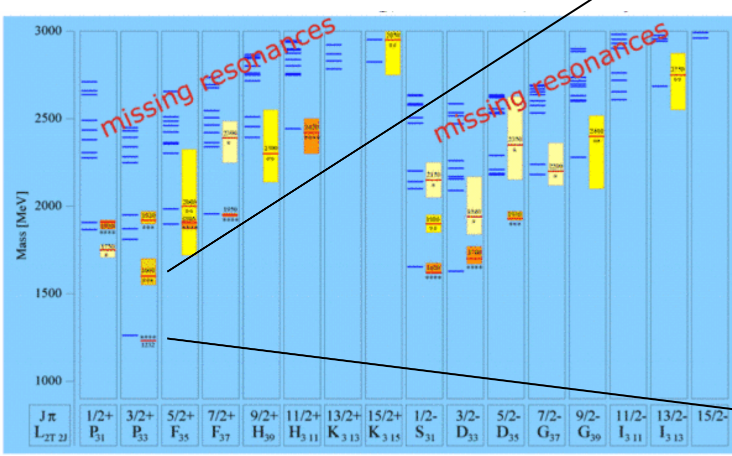
Baryon and meson resonances

Lots of unbound states!

N*



Δ*



$\Delta(1600)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	10–25 %
Γ_2	ΣK	
Γ_3	$N\pi\pi$	75–90 %
Γ_4	$\Delta\pi$	40–70 %
Γ_5	$\Delta(1232)\pi$, P-wave	
Γ_6	$\Delta(1232)\pi$, F-wave	
Γ_7	$N\rho$	<25 %
Γ_8	$N\rho$, S=1/2, P-wave	
Γ_9	$N\rho$, S=3/2, P-wave	
Γ_{10}	$N\rho$, S=3/2, F-wave	
Γ_{11}	$N(1440)\pi$	10–35 %
Γ_{12}	$N(1440)\pi$, P-wave	
Γ_{13}	$N\gamma$	0.001–0.035 %
Γ_{14}	$N\gamma$, helicity=1/2	0.0–0.02 %
Γ_{15}	$N\gamma$, helicity=3/2	0.001–0.015 %

$\Delta(1232)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	100 %
Γ_2	$N\gamma$	0.55–0.65 %
Γ_3	$N\gamma$, helicity=1/2	0.11–0.13 %
Γ_4	$N\gamma$, helicity=3/2	0.44–0.52 %