## Open quantum systems

## Wikipedia:

## An open quantum system is a quantum system which is found to be in interaction with an external quantum system, the environment. The open quantum system can be viewed as a distinguished part of a larger closed quantum system, the other part being the environment.

## GU^IILOUWUGUf:





## INTERDISCIPLINARY

Small quantum systems, whose properties are profoundly affected by environment, i.e., continuum of scattering and decay channels, are intensely studied in various fields of physics: nuclear physics, atomic and molecular physics, nanoscience, quantum optics, etc.

## Quasistationary States



For the description of a decay, we demand that far from the force center there be only the outgoing wave. The macroscopic equation of decay is

$$
\frac{d N}{d t}=-w N ; \quad N=N_{0} e^{-w t}
$$

$N$ is a number of radioactive nuclei, i.e., number of particles inside of sphere $r=R$ :

$$
N \sim \int\left|\psi^{2}\right| d^{3} r
$$

We should thus seek a solution of the form

$$
\left.\begin{array}{l}
\qquad \psi=\psi(r) e^{-i E_{0} t / \hbar-w t / 2}=\psi(r) e^{-i E t / \hbar} \\
\qquad E=E_{0}-i \frac{\Gamma}{2} ; \quad \Gamma=\hbar w \\
\text { J.J. Thompson, } 1884 \\
\text { G. Gamow, 1928 }
\end{array} \begin{array}{c}
\text { relation between decay width } \\
\text { and decay probability }
\end{array}\right]
$$

The time dependent equation

$$
i \hbar \frac{\partial \psi}{\partial t}=\left[-\frac{\hbar^{2}}{2 m} \Delta+V(r)\right] \psi
$$

can be reduced by the above substitution to the stationary equation

$$
\left[E+\frac{\hbar^{2}}{2 m} \Delta-V(r)\right] \psi(r)=0
$$

$$
E=\frac{\hbar^{2} k^{2}}{2 m}
$$

The boundary condition $\psi(r) \sim e^{i k r}$ for $r \rightarrow \infty \quad \lim _{r \rightarrow \infty} \frac{d \ln \psi(r)}{d r}=i \frac{\sqrt{2 m E}}{\hbar}$
...takes care of the discrete complex values of $E$

In principle, resonances and decaying particles are different entities. Usually, resonance refers to the energy distribution of the outgoing particles in a scattering process, and it is characterized by its energy and width. A decaying state is described in a time dependent setting by its energy and lifetime. Both concepts are related by:

$$
T_{0}=\frac{\hbar}{\Gamma}
$$

This relation has been checked in numerous precision experiments.

See more discussion in
R. de la Madrid, Nucl. Phys. A812, 13 (2008)

TABLE III. Recent theoretical and experimental lifetimes $\tau$ for $\mathrm{NaI} 3 p^{2} P_{1 / 2}$ and ${ }^{2} P_{3 / 2}$ and total line strengths $S(3 s-3 p)$ (uncertainties given in parentheses).

| Ref. | Method | $J$ | $\tau_{J}$ (ns) | $S$ (a.u.) |
| :---: | :---: | :---: | :---: | :---: |
| Theoretical |  |  |  |  |
| [6] | Semiempirical |  |  | 37.03 |
| [7] | Semiempirical |  |  | 37.19 |
| [4] | RMBPT all orders |  |  | 37.38(11) |
| [11] | Coupled clusters |  |  | $37.56{ }^{\text {a }}$ |
| [3] | MCHF-CCP |  |  | $37.30^{\text {a }}$ |
| [5] | $\mathrm{MCHF}+\mathrm{CI}$ |  |  | $37.26^{\text {a }}$ |
| Experimental |  |  |  |  |
| [1] | BGLS | 1/2 | 16.40(3) | 37.04(7) ${ }^{\text {b }}$ |
| [16] | Pulsed laser | 1/2 | 16.35(6) | $37.15(14)^{\text {b }}$ |
| [17] | $C_{3}$ analysis | 1/2 | 16.31(6) | $37.24(12)^{\text {b }}$ |
| [18] | Linewidth | 3/2 | 16.237(35) | $37.30(8){ }^{\text {b }}$ |
| This | BGLS | 1/2 | 16.299(21) | $37.26(5)^{\text {c }}$ |
| work |  | 3/2 | 16.254(22) |  |

${ }^{2}$ Corrected for relativistic effects ( -0.09 a.u.) using the ratio between DF and HF values. The original value of Ref. [3] without relativistic correction is 37.39 a.u.
${ }^{\mathrm{b}}$ A line strength ratio between the two fine-structure components of 0.5 was assumed in the calculation.
${ }^{\text {c }}$ The ratio of the line strengths of the two fine-structure components was determined to $0.50014(44)$. This is in excellent agreement with the nonrelativistic prediction of 0.5 . In the uncertainty estimate for the ratio all those systematical effects were omitted which affect both lifetimes in the same way.
U. Volz et al., Phys. Rev. Lett 76, 2862(1996)

