Strong coupling constant $\alpha_s = \frac{g^2}{4\pi}$

In quantum field theory, the coupling constant is an effective constant, which depends on four-momentum Q^2 transferred. For strong interactions, the Q^2 dependence is very strong (gluons - as the field quanta - carry color and they can couple to other gluons). A first-order perturbative QCD calculation (valid at very large Q^2) gives:

running coupling constant!

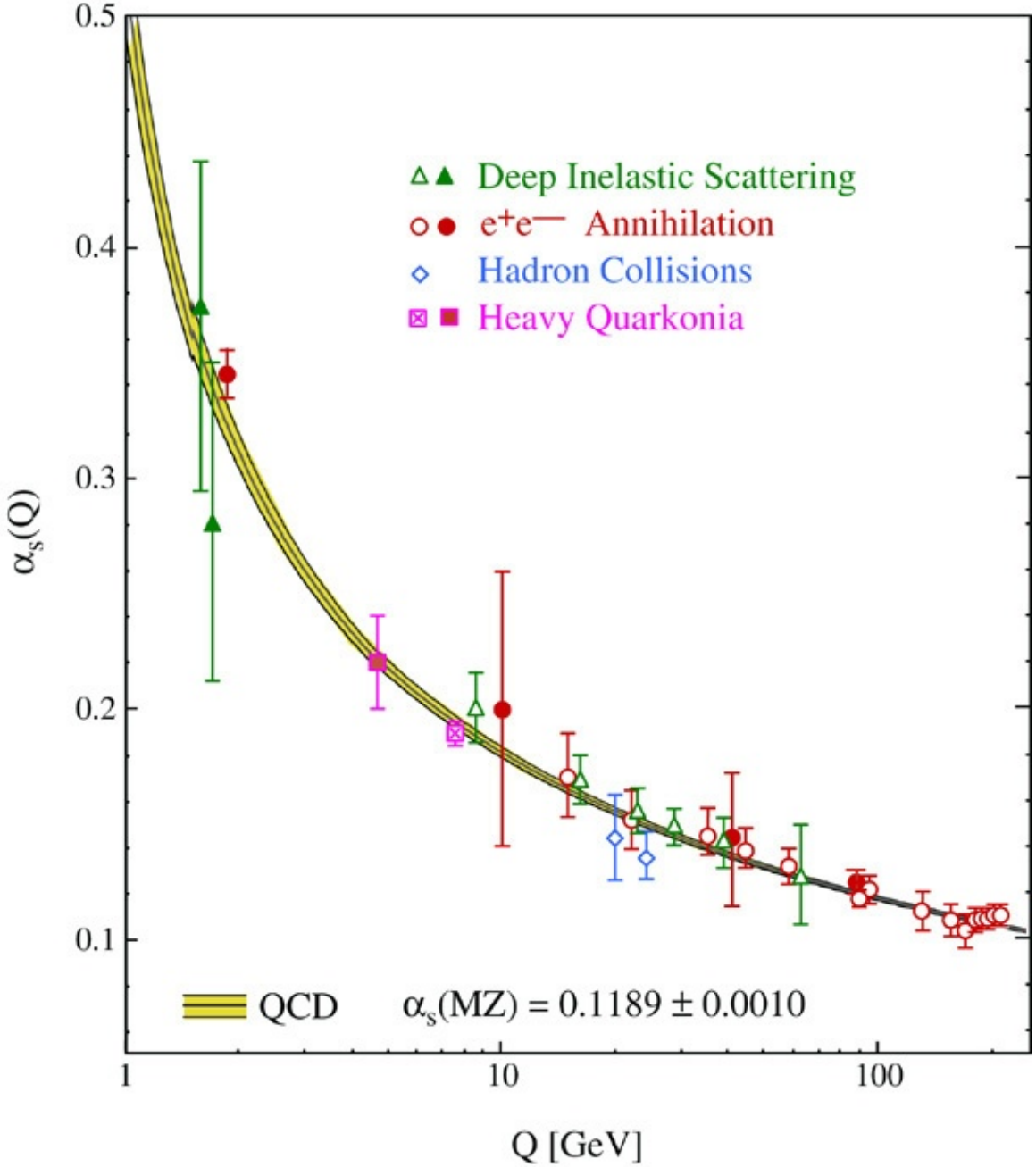
$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(22 - 2n_f\right)\cdot \ln\left(Q^2 / \Lambda_{\mathrm{QCD}}^2\right)}$$

n_f =6 – number of quark flavors
Λ_{QCD} – a parameter in QCD (~0.22 GeV), an infrared cutoff

The spatial separation between quarks goes as $\lambda\!\!\!^{-} = \frac{\hbar}{\sqrt{Q^2}}$

Therefore, for very small distances and high values of Q^2, the inter-quark coupling decreases, vanishing asymptotically. In the limit of very large Q^2, quarks can be considered to be "free" (asymptotic freedom). On the other hand, at large distances, the inter-quark coupling increases so it is impossible to detach individual quarks from hadrons (confinement).

Asymptotic freedom was described in 1973 by Gross, Wilczek, and Politzer (Nobel Prize 2004).



It is customary to quote α_s at the 91 GeV energy scale (the mass of the Z boson)

Chiral symmetry

For massless quarks, QCD Lagrangian preserves helicity. Indeed, since a massless quark travels at the speed of light, the handedness or chirality of the quark is independent of any Lorentz frame from which the observation is made.

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}(\psi_L) + \mathcal{L}_{QCD}(\psi_R)$$

the QCD interaction does not couple the left and right-handed quarks

The mass term explicitly breaks the chiral symmetry as:

$$m_q \bar{\psi}_q \psi_q = m_q \bar{\psi}_{qL} \psi_{qR} + m_q \bar{\psi}_{qR} \psi_{qL}$$

The main origin of the chiral symmetry breaking, however, may be described in terms of the fermion condensate (vacuum condensate of bilinear expressions involving the quarks in the QCD vacuum) formed through nonperturbative action of QCD gluons.

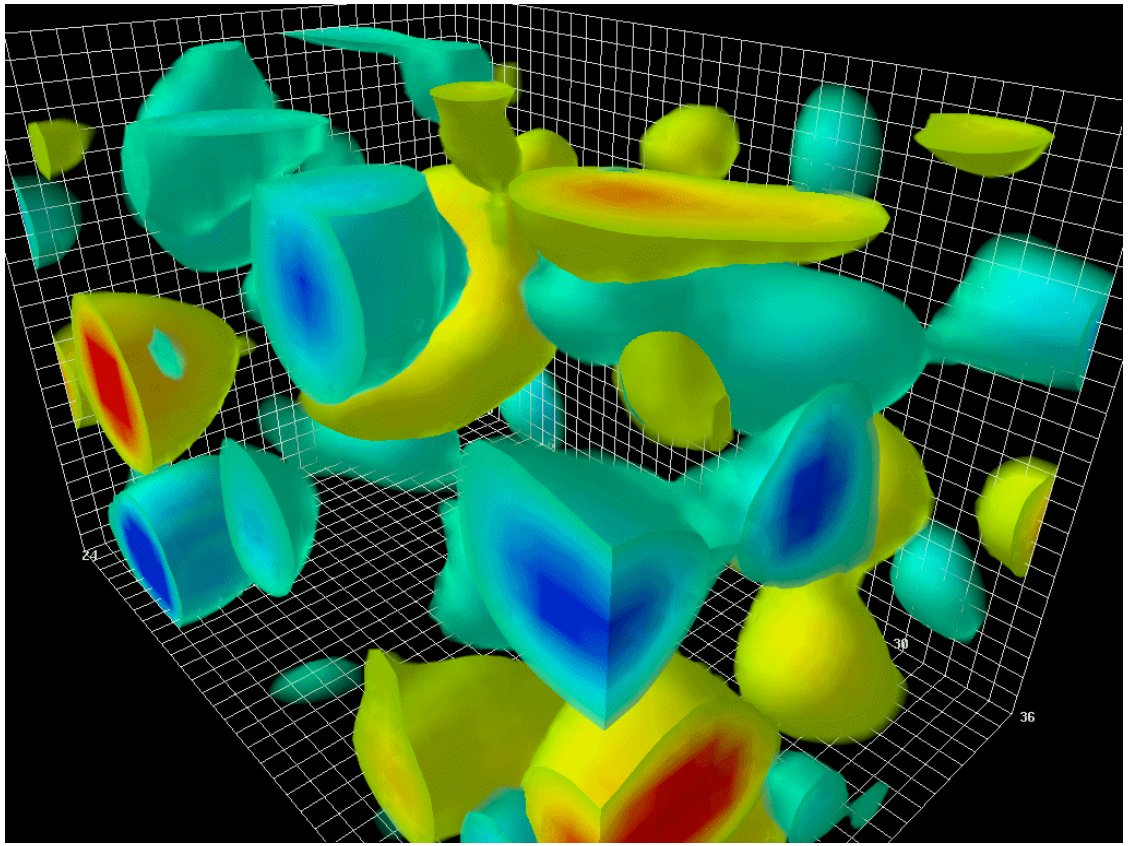
Spontaneous symmetry breaking due to the strong low-energy QCD dynamics, which rearranges the QCD vacuum:

$$\left\langle \bar{\psi}_{qL} \psi_{qR} \right\rangle \propto \Lambda_{QCD}^3 \neq 0$$

QCD vacuum

In QED vacuum polarization effects are extremely weak, because the electron has a small charge and a non-zero rest mass. On the other hand, the QCD gluons are massless, and their strong interaction is not damped by a small parameter. As a result, the QCD vacuum polarization effect is extremely strong, and the empty space is not empty at all - it must contain a soup of spontaneously appearing, interacting, and disappearing gluons. Moreover, in the soup there also must be pairs of virtual quark-antiquark pairs that are also color-charged, and emit and absorb more virtual gluons. It turns out that the QCD ground state of an "empty" space is extremely complicated. At present, we do not have any glimpse of a possibility to find the vacuum wave function analytically. Some ideas of what happens are provided by the QCD lattice calculations, in which the gluon and quark fields are discretized on a four-dimensional lattice of space-time points, and the differential field equations are transformed into finite-difference equations solvable on a computer.

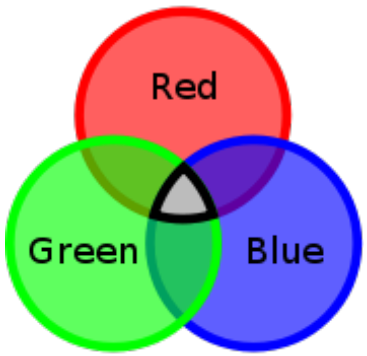
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/index.html

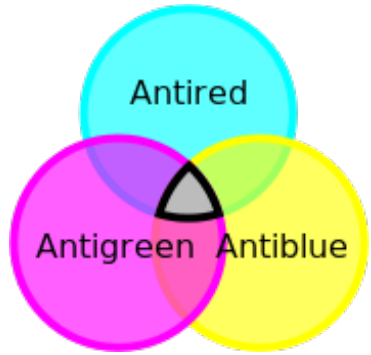


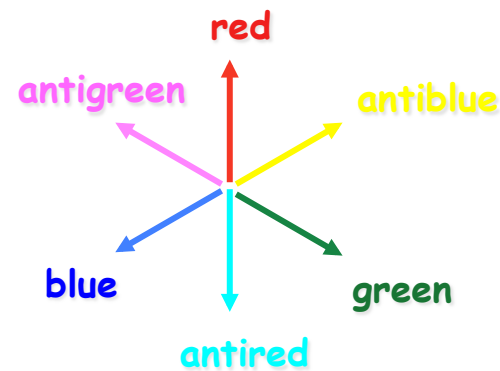
The typical four-dimensional structure of gluon-field configurations averaged over in describing the vacuum properties of QCD. The volume of the box is 2.4 by 2.4 by 3.6 fm, big enough to hold a couple of protons.

Color, Gluons

Gluons are the exchange particles which couple to the color charge . They carry simultaneously color and anticolor.







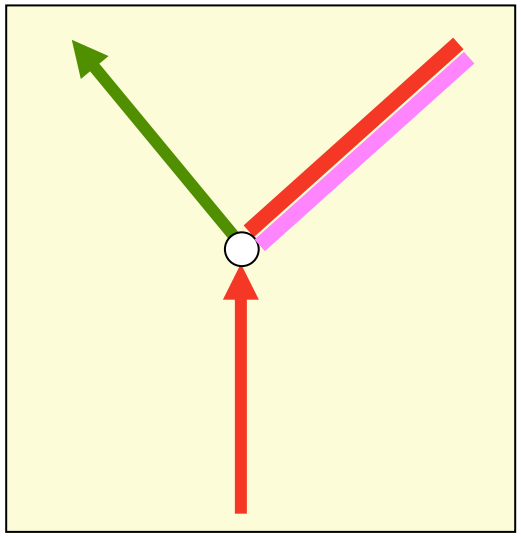
What is the total number of gluons? According to SU_3, 3x3 color combinations form a singlet and an octet. The octet states form a basis from which all other color states can be constructed. The way in which these eight states are constructed from colors and anticolors is a matter of convention. One possible choice is:

$$\left|R\bar{G}\right\rangle,\ \left|R\bar{B}\right\rangle,\ \left|G\bar{B}\right\rangle,\ \left|G\bar{R}\right\rangle,\ \left|B\bar{R}\right\rangle,\ \left|B\bar{G}\right\rangle,$$
$$\sqrt{1/2}\left(\left|R\bar{R}\right\rangle-\left|G\bar{G}\right\rangle\right),\ \sqrt{1/6}\left(\left|R\bar{R}\right\rangle+\left|G\bar{G}\right\rangle-2\left|B\bar{B}\right\rangle\right)$$

The color singlet:

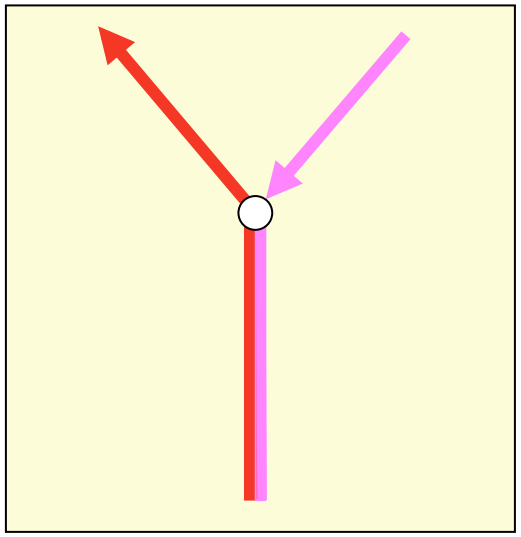
$$\sqrt{1/3}\left(\left|R\bar{R}\right\rangle+\left|G\bar{G}\right\rangle+\left|B\bar{B}\right\rangle\right)$$

is invariant with respect of a re-definition of the color names (rotation in color space). Therefore, it has no effect in color space and cannot be exchanged between color charges.



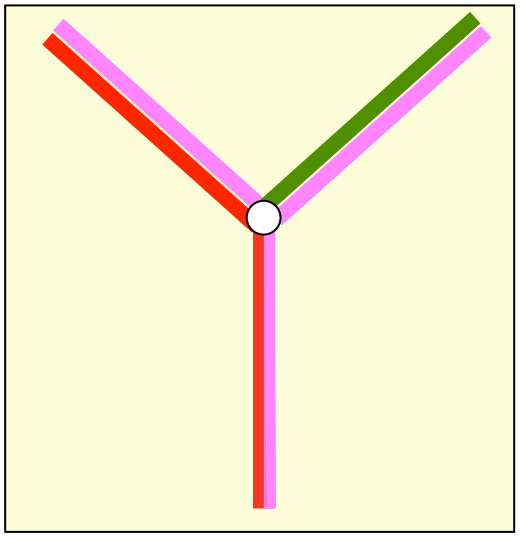
emission of a gluon
by a quark

$$q \rightarrow q + g$$



splitting of a gluon
into a quark-antiquark pair

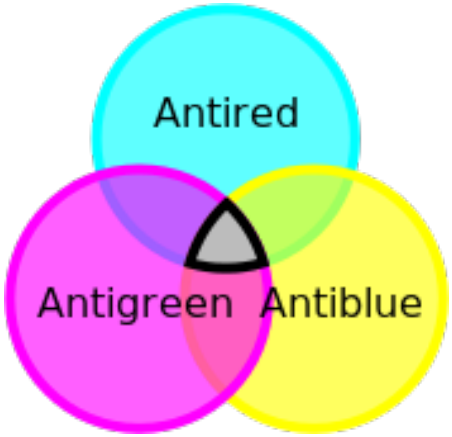
$$g \rightarrow q + \bar{q}$$

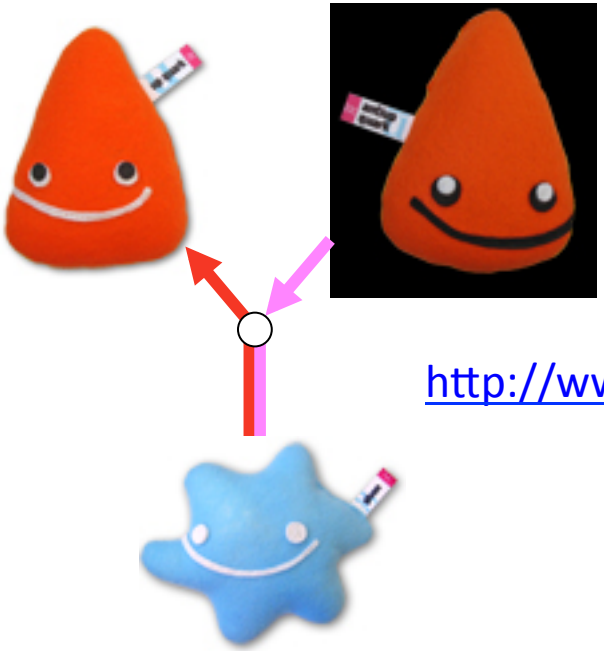


self-coupling of gluons

$$g \rightarrow g + g$$

$$g + g \rightarrow g + g$$





http://www.particlezoo.net/shop.html

Quarks

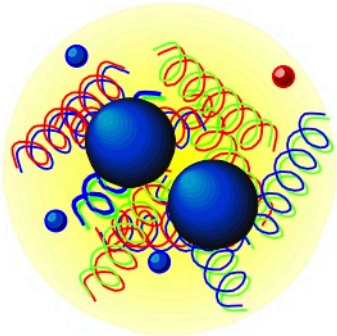
In 1968, **deep inelastic scattering** experiments at the Stanford Linear Accelerator Center showed that the proton contained much smaller, point-like objects and was therefore not an elementary particle

Flavor	$\mathcal{A}$	t	t_z	$\mathcal{S}$	$\mathcal{C}$	$\mathcal{B}$	$\mathcal{T}$	$Q(e)$	Mc^2 (GeV)
u (up)	$1/3$	$1/2$	$-1/2$	0	0	0	0	$+2/3$	$0.002-0.003$
d (down)	$1/3$	$1/2$	$+1/2$	0	0	0	0	$-1/3$	$0.004-0.006$
s (strange)	$1/3$	0	0	-1	0	0	0	$-1/3$	$0.08-0.13$
c (charm)	$1/3$	0	0	0	1	0	0	$+2/3$	$1.2-1.3$
b (bottom)	$1/3$	0	0	0	0	-1	0	$-1/3$	$4.1-4.3$
t (top)	$1/3$	0	0	0	0	0	1	$+2/3$	173 ± 1

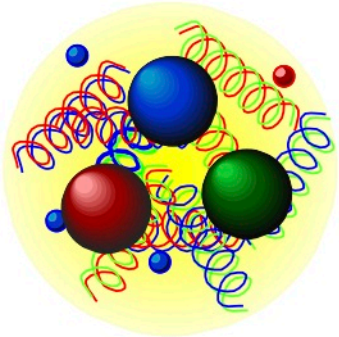
- The least massive are *u*- and *d*-quarks (hence the lightest baryons and mesons are made exclusively of these two quarks)
- Each quark has baryon number $\mathcal{A}$=1/3.
- Strange quark carries a quantum number called strangeness $\mathcal{S}$. Strange particles (such as kaons) carry this quark
- Six antiquarks complement the list
- Quarks are all fermions; they carry half-integer spins
- *d*- and *u*-quarks form an isospin doublet

$$\tau_+|u\rangle = |d\rangle \qquad \tau_-|d\rangle = |u\rangle$$

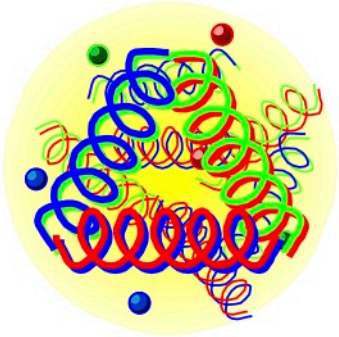
- Strong interactions conserve the total number of each type of quarks. However, quarks can be transformed from one flavor to another through weak interactions (CKM matrix!).



meson

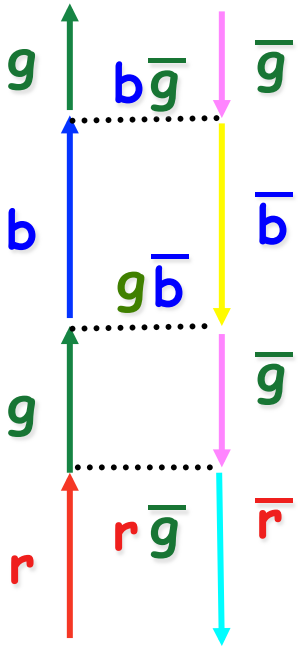


baryon



glueball?

$$\left|\pi^{+}\right\rangle = \begin{cases} \left|u_R \bar{d}_R\right\rangle \\ \left|u_B \bar{d}_B\right\rangle \\ \left|u_G \bar{d}_G\right\rangle \end{cases}$$

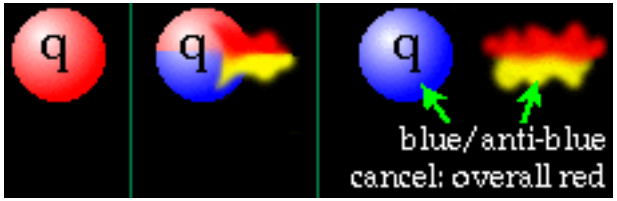


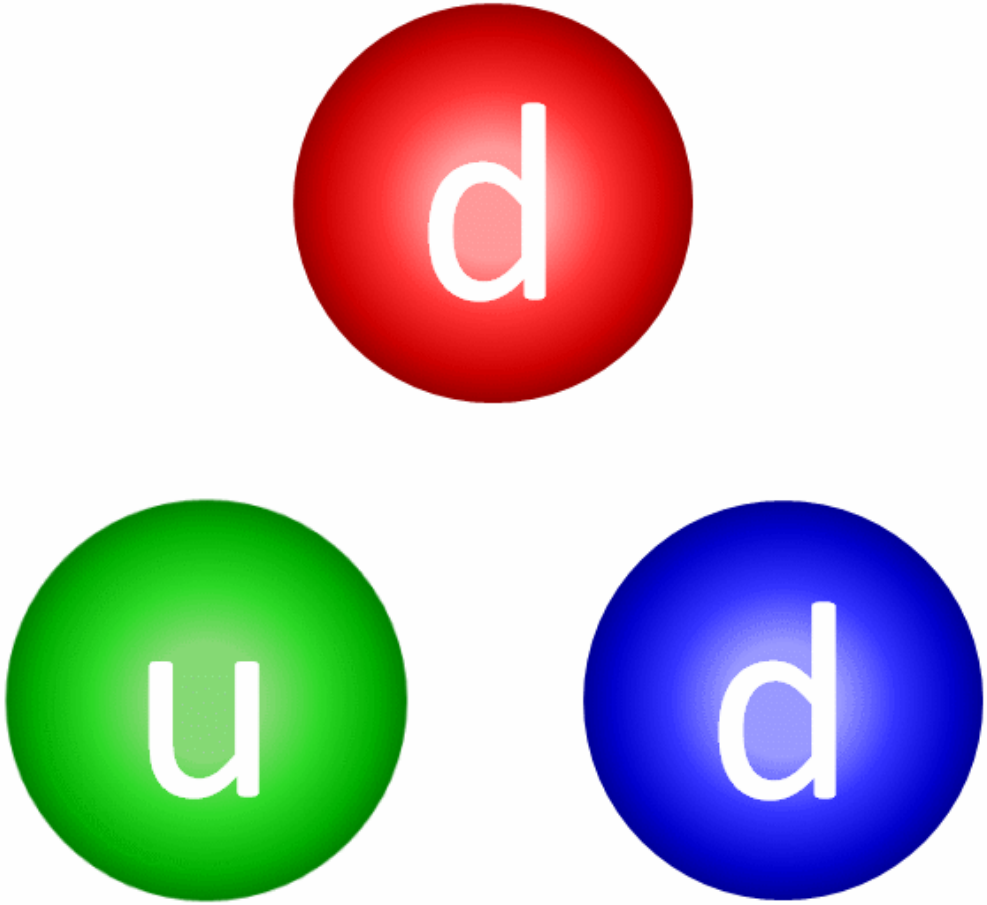
Meson can exist in three different color combinations. The actual pion is a **mixture** of these color states. By exchange of gluons, the color combination continuously changes.

Can
a) 4-quark
b) 5-quark
systems exist according to QCD?

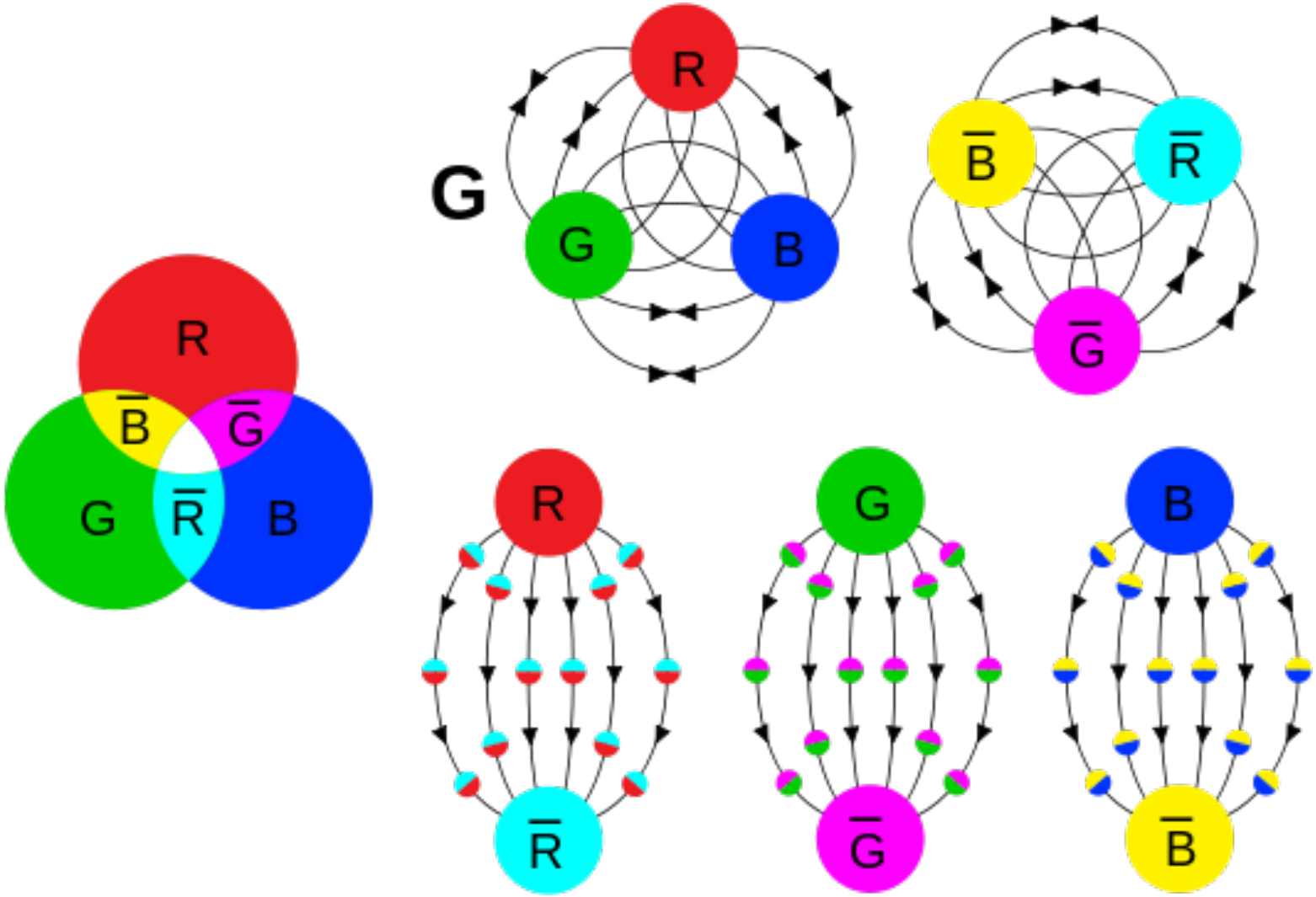
The Quark Song

https://www.youtube.com/watch?v=xYZkj2FPeoc&spfreload=10





http://commons.wikimedia.org/wiki/File:Neutron_QCD_Animation.gif


R
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Hadrons

baryons

fermions
made up of three quarks
must have half-integer spins

mesons

bosons
made up of quark-antiquarks*
must have integer spins

* Bosons can be annihilated; strong interactions conserve the # of quarks

$$p + \bar{p} \rightarrow \gamma + \gamma$$

particle-antiparticle annihilation

- particle and antiparticle have opposite charges, baryon numbers, etc.
- they must have opposite intrinsic parities
- they must have opposite isospins

charge number

$$Q = -t_0 + \frac{A}{2}$$

baryon number

third component of isospin

$$\tau_{\pm}(\text{hadron}) \rightarrow \sum_{i=1}^{A} \tau_{\pm}(q_i)$$

$$|u\rangle \xrightarrow[C]{} |\bar{u}\rangle \qquad |d\rangle \xrightarrow[C]{} -|\bar{d}\rangle$$