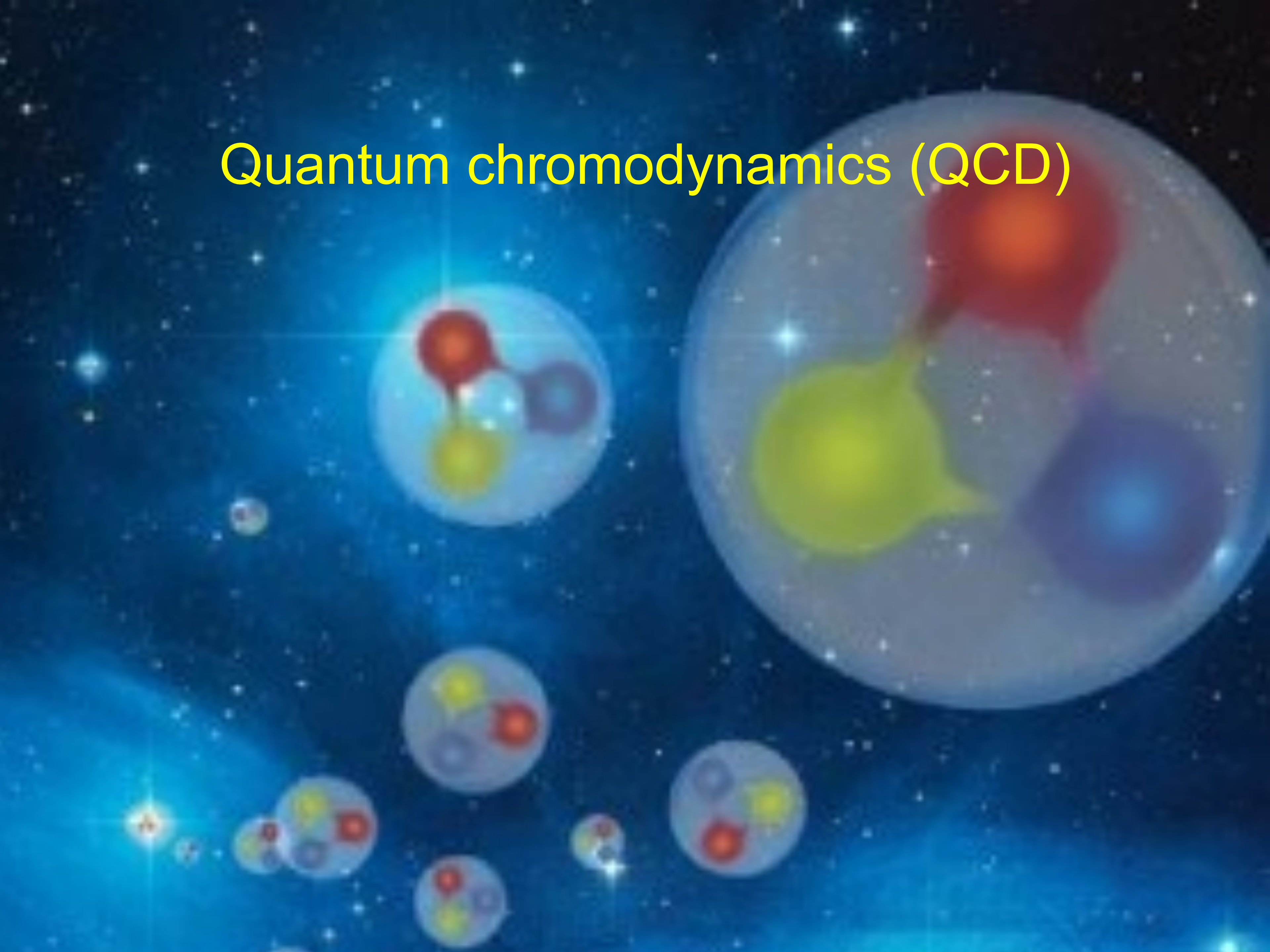
Quantum chromodynamics (QCD)



QCD is the theory that describes the action of the strong force. QCD was constructed in analogy to quantum electrodynamics (QED), the quantum field theory of the electromagnetic force. In QED the electromagnetic interactions of charged particles are described through the emission and subsequent absorption of massless photons (force carriers of QED); such interactions are not possible between uncharged, electrically neutral particles. By analogy with QED, quantum chromodynamics predicts the existence of gluons, which transmit the strong force between particles of matter that carry color, a strong charge.

The color charge was introduced in 1964 by Greenberg to resolve spin-statistics contradictions in hadron spectroscopy. In 1965 Nambu and Han introduced the octet of gluons. In 1972, Gell-Mann and Fritzsch, coined the term quantum chromodynamics as the gauge theory of the strong interaction. In particular, they employed the general field theory developed in the 1950s by Yang and Mills, in which the carrier particles of a force can themselves radiate further carrier particles. (This is different from QED, where the photons that carry the electromagnetic force do not radiate further photons.)

First, QED Lagrangian…

$$\mathcal{L}_{QED} = \bar{\psi}_e i\gamma^\mu \left[\partial_\mu + ieA_\mu\right]\psi_e - m_e\bar{\psi}_e\psi_e - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ EM field tensor
- A^μ four potential of the photon field
- γ^μ Dirac 4x4 matrices
- ψ_e Dirac four-spinor of the electron field
- $e = \sqrt{4\pi\alpha}, \quad 1/\alpha \approx 137, \quad \hbar = c = 1$

Einstein notation:
when an index variable appears twice in a single term, it implies summation of that term over all the values of the index

electromagnetic four-current density of the electron: $J^\mu = e\,\bar{\psi}_e\gamma^\mu\psi_e$

From the Euler–Lagrange equation of motion for a field, we get:

$$(i\gamma^\mu\partial_\mu - m_e)\psi_e = e\gamma^\mu A_\mu\psi_e$$

Dirac equation for the electron

$$\Box A^\mu = e\,\bar{\psi}_e\gamma^\mu\psi_e$$

Maxwell equations for the EM fields in the Lorentz gauge

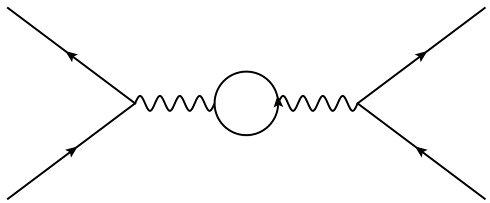
$\Box = \partial^\mu\partial_\mu$ d'Alembert operator

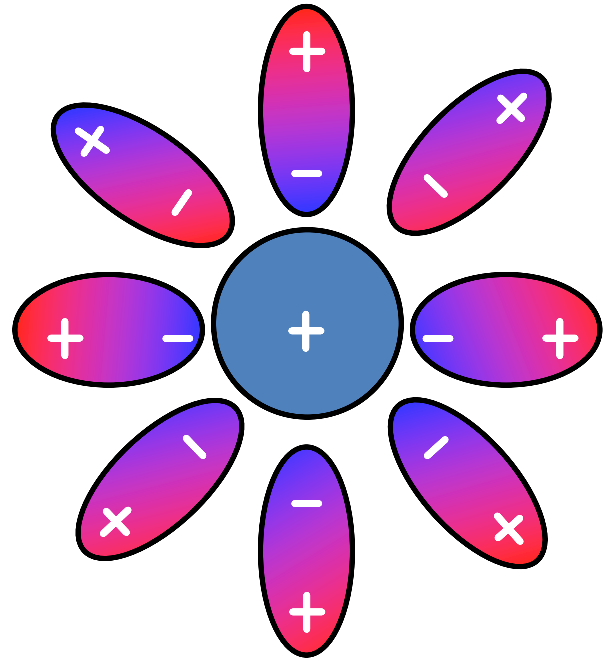
Vacuum polarization in QED

force between two electrons
(in natural units)

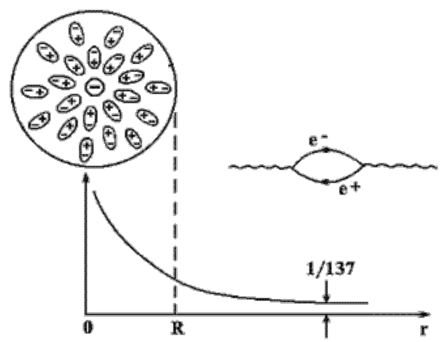
$$F = \frac{1}{4\pi}\frac{e^2}{r^2} = \frac{\alpha}{r^2}$$

$$e = \sqrt{4\pi\alpha}$$





$$F = \frac{\alpha_{\rm em}(r)}{r^2}$$



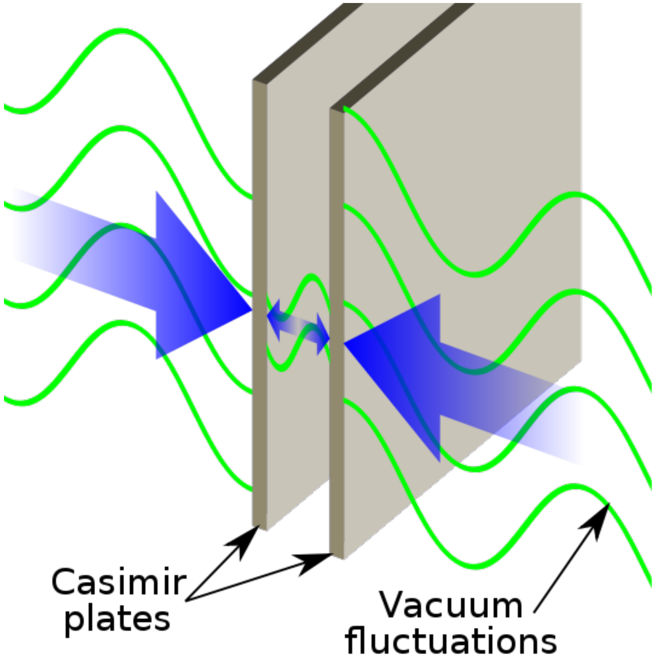
The interaction strength of the two electrons gets stronger as the distance between them becomes smaller

Electric charge is screened;
interaction becomes weak at large distances

QED vacuum

Let us consider empty space. In a quantum field theory, we cannot just say that the ground state of the empty space is the state with no quanta - we have to solve the proper field equations, with proper boundary conditions, and determine what is the state of the field. Such a state may or may not contain quanta. In particular, whenever the space has a boundary, the ground state of the field does contain quanta - this fact is called the vacuum polarization effect.

In QED, this is a very well known, and experimentally verified effect. For example, two conducting parallel plates attract each other, even if they are not charged and placed in otherwise empty space (this is called the Casimir effect). Here, the vacuum fluctuations of the electron field may create in an empty space virtual electron-positron pairs. These charged particles induce virtual polarization charges in the conducting plates (it means virtual photons are created, travel to plates, and reflect from them). Hence, the plates become virtually charged, and attract one another during a short time when the existence of the virtual charges, and virtual photons, is allowed by the Heisenberg principle. All in all, a net attractive force between plates appears.



$$\frac{F_c}{A} = -\frac{\hbar c \pi^2}{240 d^4}$$

d – distance between the plates

Also: Lamb shift

QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_q \left(\bar{\psi}_{qi} i\gamma^\mu \left[\delta_{ij}\partial_\mu + ig\left(G^\alpha_\mu t_\alpha\right)_{ij} \right] \psi_{qj} - m_q \bar{\psi}_{qi}\psi_{qi} \right) - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$$

$$\mathcal{L}_{QED} = \bar{\psi}_e i\gamma^\mu \left[\partial_\mu + ieA_\mu \right] \psi_e - m_e \bar{\psi}_e \psi_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- $G^{\mu\nu}_\alpha = \partial^\mu G^\nu_\alpha - \partial^\nu G^\mu_\alpha - g f^{\alpha\beta\gamma} G^\mu_\beta G^\nu_\gamma$ color fields tensor
- G^μ_α four potential of the gluon fields (α=1,..8)
- t_α 3x3 Gell-Mann matrices; generators of the SU(3) color group
- $f^{\alpha\beta\gamma}$ structure constants of the SU(3) color group
- ψ_i Dirac spinor of the quark field (*i* represents color)
- $g = \sqrt{4\pi\alpha_s}$ $(\hbar = c = 1)$ color charge (strong coupling constant)

- The quarks have three basic color-charge states, which can be labeled as *i*=red, green, and blue. Three color states form a basis in a 3-dimensional vector space. A general color state of a quark is then a vector in this space. The color state can be rotated by 3 × 3 unitary matrices. All such unitary transformations with unit determinant form a Lie group SU(3).
- A crucial difference between the QED and QCD is that the gluon field tensors contain the additional term representing interaction between color-charged gluons.
- While sources of the electromagnetic field depend on currents that involve a small parameter, gluons are sources of the color field without any small parameter. Gluons are not only color-charged, but they also produce very strong color fields.