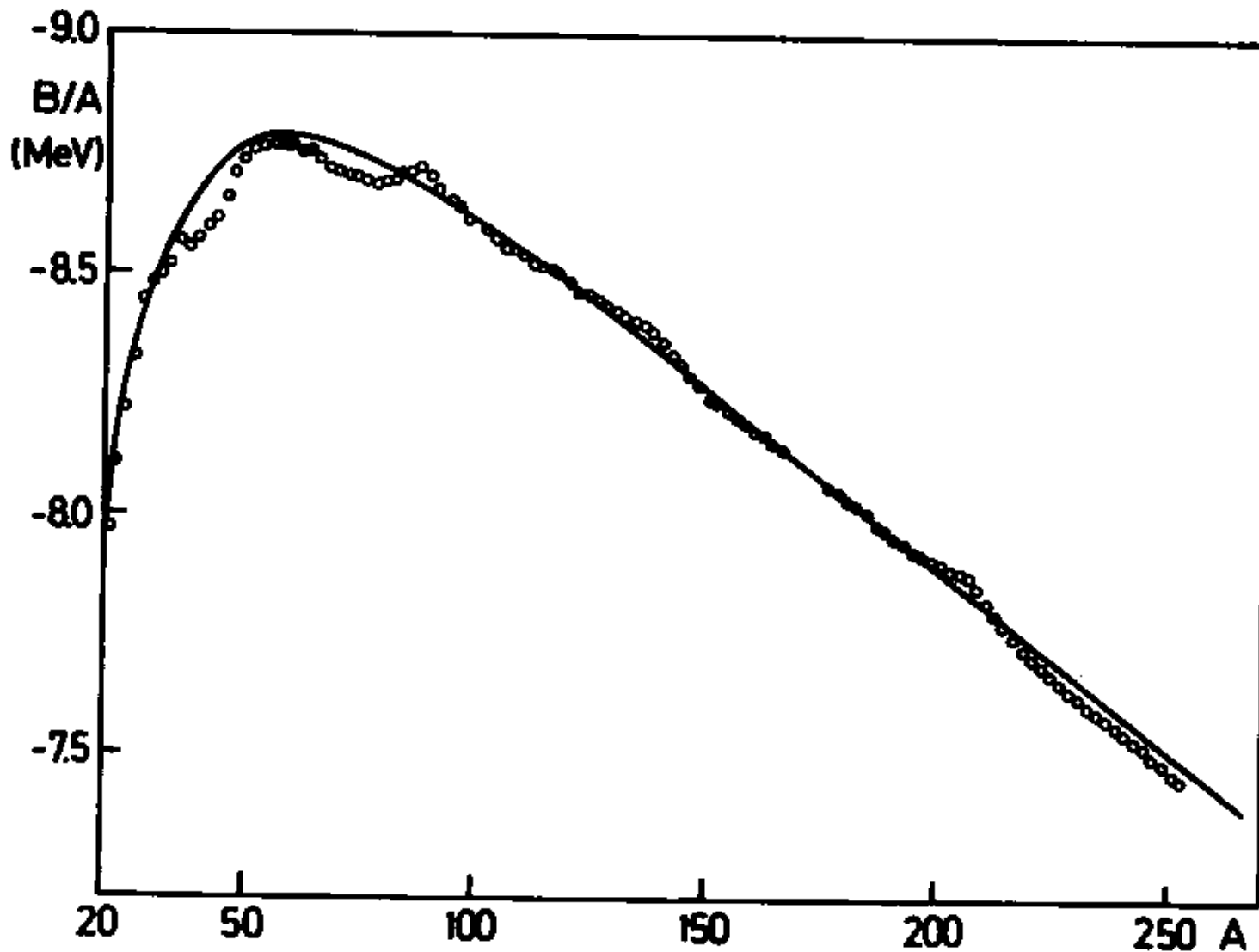
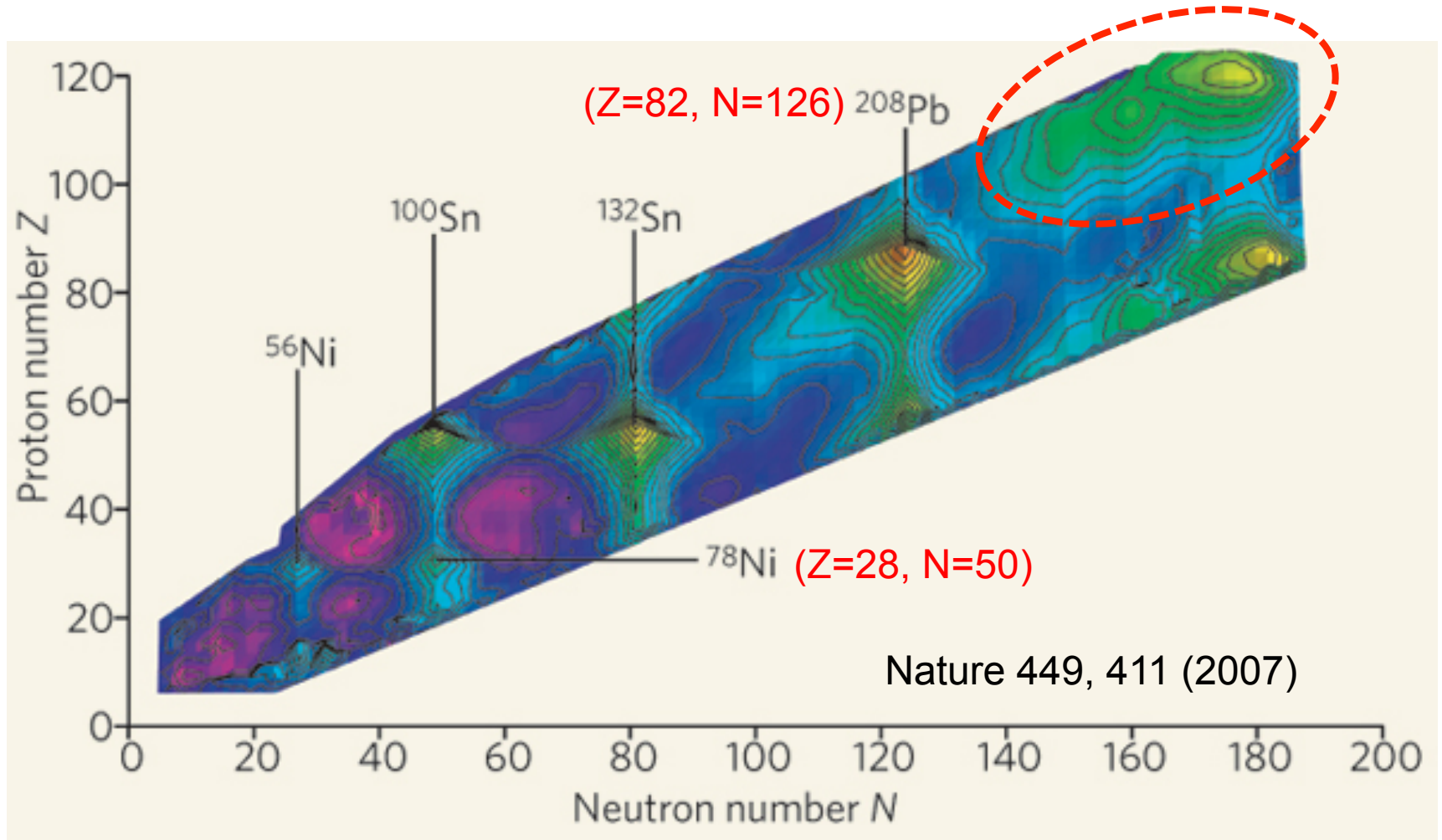


# Nucleonic Shells

The semi-empirical mass formula, based **on the liquid drop model**, compared to the data

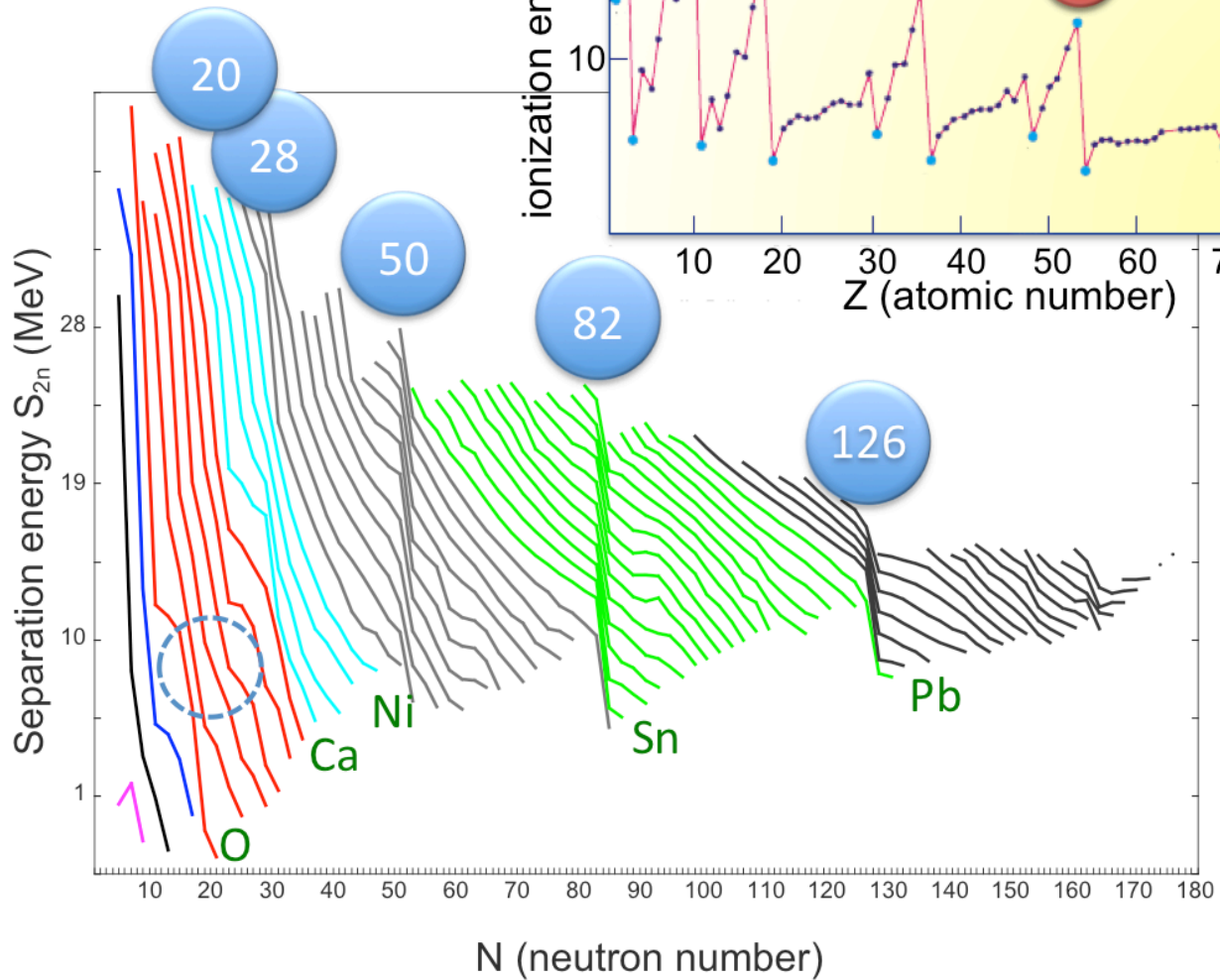


$$E_{\text{shell}} = E_{\text{total}} - E_{\text{LD}}$$

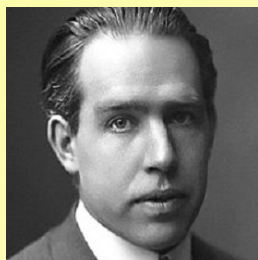


**Magic numbers** at Z or N= 2, 8, 20, 28, 50, 82, 126

# Regularities and periodicities in atoms and nuclei



1912

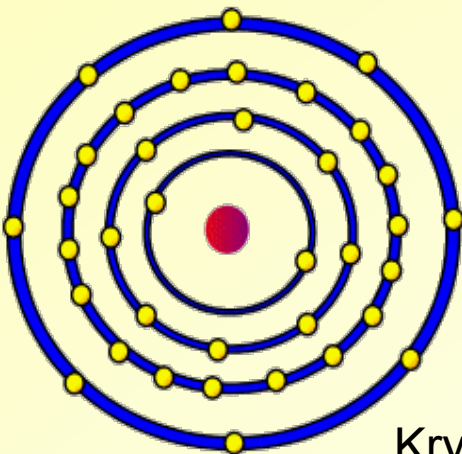


electronic shells of the atom

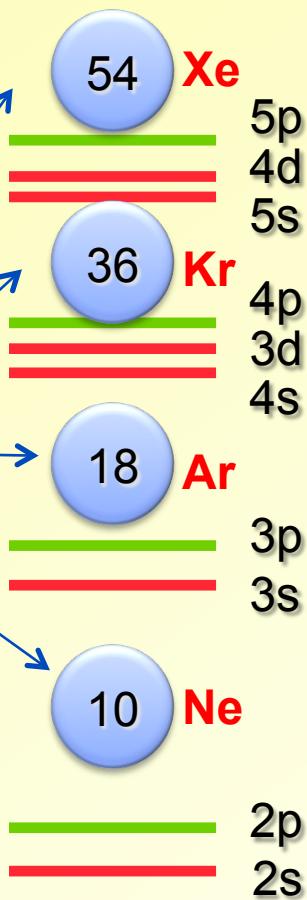
Nobel Prize 1922

Bohr's picture still serves as an elucidation of the physical and chemical properties of the elements.

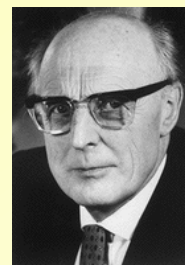
noble gases (closed shells)



Krypton Atom



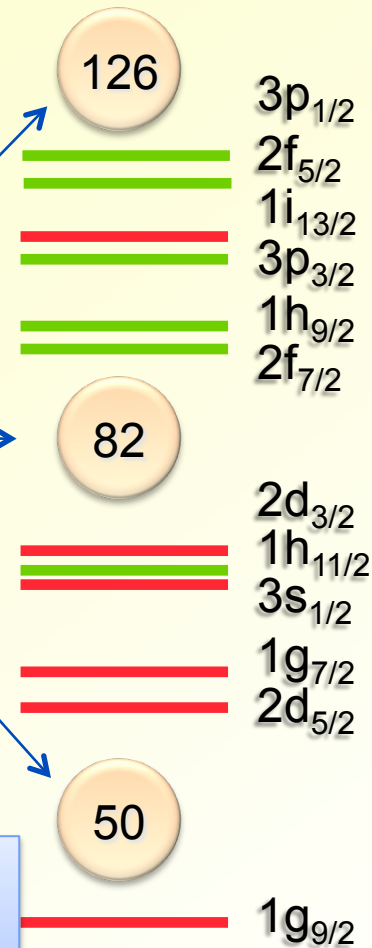
1949



nucleonic shells of the nucleus

Nobel Prize 1963

magic nuclei (closed shells)



We know now that this picture is **very** incomplete...

# Spherical Harmonic Oscillator

$$\hat{h} = \hat{t} + \frac{m\omega_0^2 r^2}{2} \Rightarrow \epsilon_N = \left(N + \frac{3}{2}\right) \hbar\omega_0$$

For each shell, the allowed orbital angular momenta are:

$$\ell = N, N-2, \dots, 1, \text{ or } 0, \quad j = \ell \pm \frac{1}{2}$$

Since each nucleon has an intrinsic spin  $s=1/2$ , the maximum number of nucleons in a HO shell is:

$$D_N = \sum_{\ell} 2(2\ell + 1) = (N+1)(N+2) \approx \left(N + \frac{3}{2}\right)^2$$

The total number of states is:

$$\sum_{N'=0}^N (N'+1)(N'+2) = \frac{1}{3}(N+1)(N+2)(N+3)$$

$$\approx \frac{1}{3}(N+2)^3$$

N	l	DEGEN.	TOTAL
5	1,3,5	42	112
4	0,2,4	30	70
3	1,3	20	40
2	0,2	12	20
1	1	6	8
0	0	2	2

Dimension of orbits:

$$\langle r^2 \rangle_{N\ell} = \frac{\hbar}{m\omega_0} \left(N + \frac{3}{2}\right)$$

$$\Rightarrow \hbar\omega_0 \approx \frac{41}{A^{1/3}} \text{ (MeV)}$$

# Spin-orbit potential

# Flat bottom

## Nuclear Configurations in the Spin-Orbit Coupling Model. I. Empirical Evidence

MARIA GOEPPERT MAYER  
*Argonne National Laboratory, Chicago, Illinois*  
 (Received December 7, 1949)

An extreme one particle model of the nucleus is proposed. The model is based on the succession of energy levels of a single particle in a potential between that of a three-dimensional harmonic oscillator and a square well. (1) Strong spin orbit coupling leading to inverted doublets is assumed. (2) An even number of identical nucleons are assumed to couple to zero angular momentum, and, (3) an odd number to the angular momentum of the single odd particle. (4) A (negative) pairing energy, increasing with the  $j$  value of the orbit is assumed. With these four assumptions all but 2 of the 64 known spins of odd nuclei are satisfactorily explained, and all but 1 of the 46 known magnetic moments. The two spin discrepancies are probably due to failure of rule (3). The magnetic moments of the five known odd-odd nuclei are also in agreement with the model. The existence, and region in the periodic table, of nuclear isomerism is correctly predicted.

$$V_{SO} \approx \kappa \vec{l} \cdot \vec{s}$$

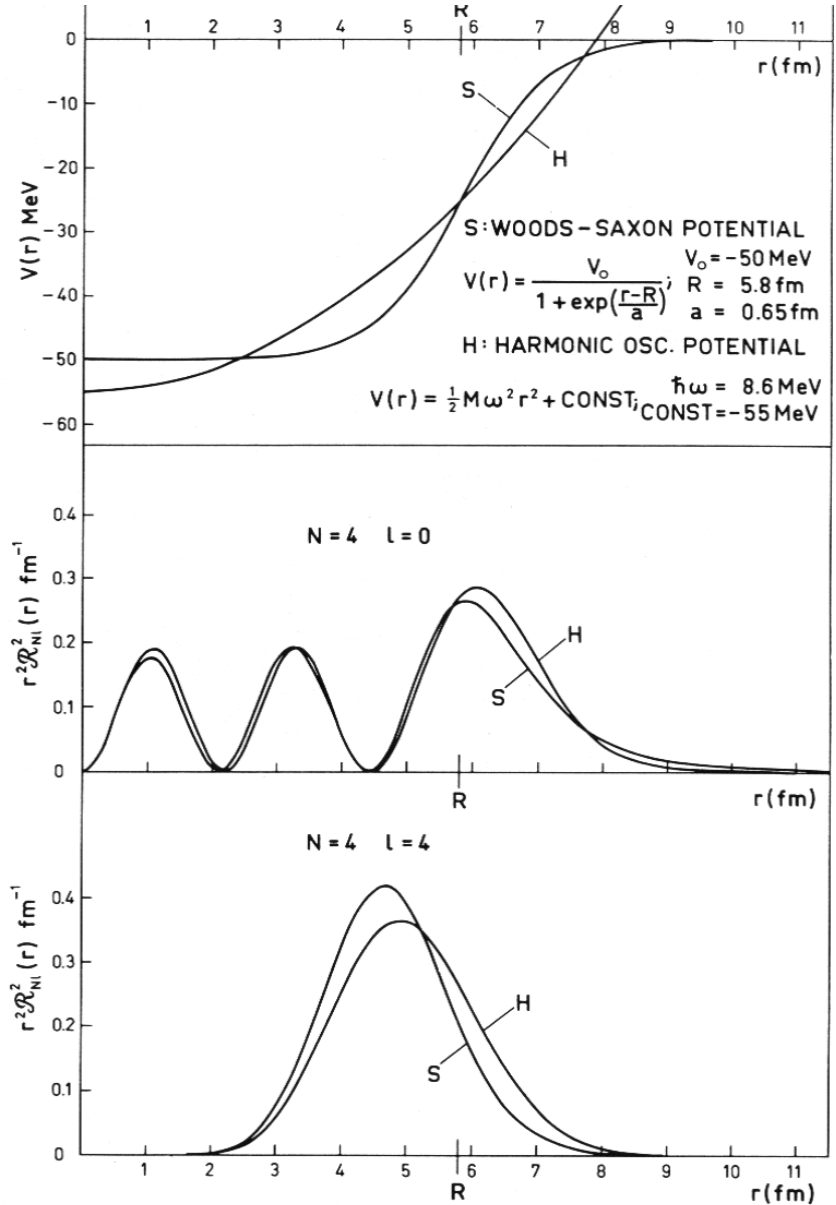
$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{\hbar^2}{2} [j(j+1) - l(l+1) - s(s+1)]$$

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{\hbar^2}{2} \begin{cases} l & \text{for } j = l + \frac{1}{2} \\ -(l+1) & \text{for } j = l - \frac{1}{2} \end{cases}$$

$\kappa$  is negative!

The value of spin-orbit strength  $\kappa$  **cannot** be derived from a simple Thomas precession, as incorrectly stated in Jackson (next slide)

$$V_{ls} = \frac{g}{2m^2c^2} \frac{1}{r} \frac{dV}{dr} \vec{l} \cdot \vec{s}$$



orbits with higher angular momentum shifted down!

In atomic nuclei the accelerations due to the specifically nuclear forces are comparatively weak. In an approximation we treat the nucleons as moving separately in a short-range attractive, potential well,  $V_N(r)$ . Then each nucleon has in addition a spin-orbit interaction given by the contribution  $U'$  omitted:



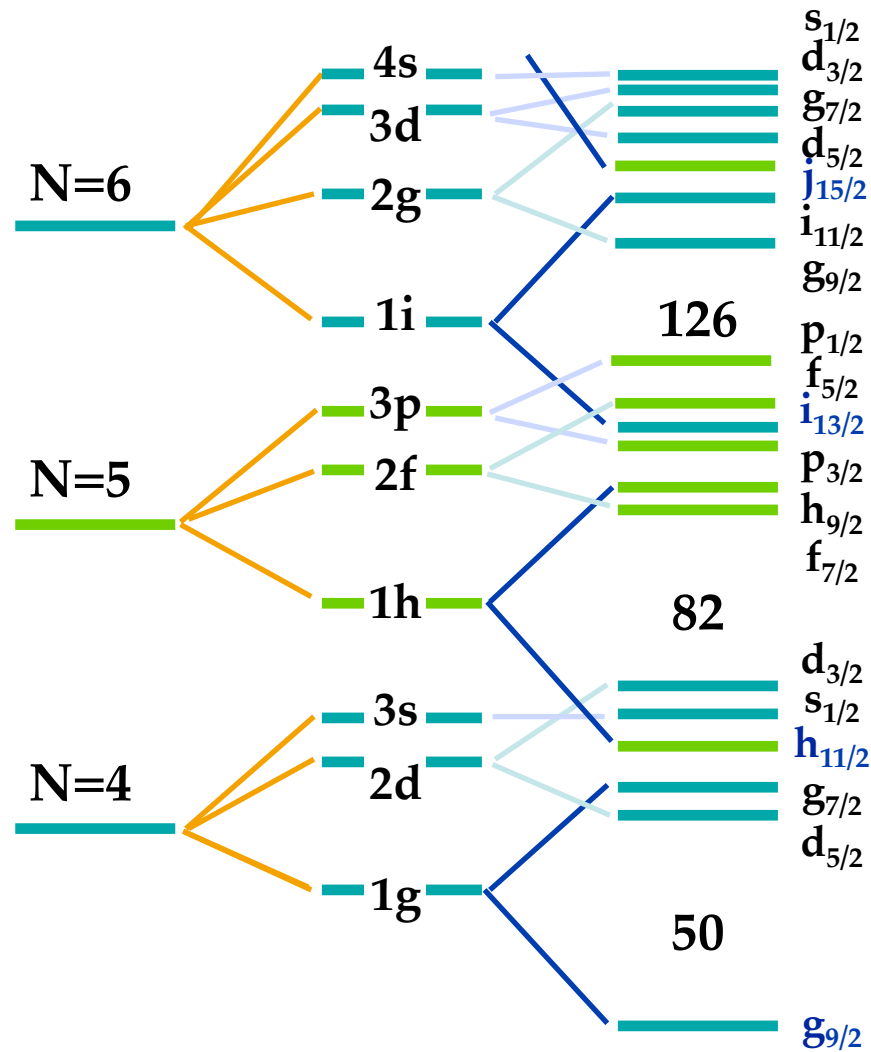
$$U_N = -\mathbf{S} \cdot \boldsymbol{\omega}_T \quad (11.57)$$

where the acceleration in  $\boldsymbol{\omega}_T$  is determined by  $V_N(r)$ . The form of  $\boldsymbol{\omega}_T$  is the same as (11.55) with  $V$  replaced by  $V_N$ . Thus the nuclear spin-orbit interaction is approximately

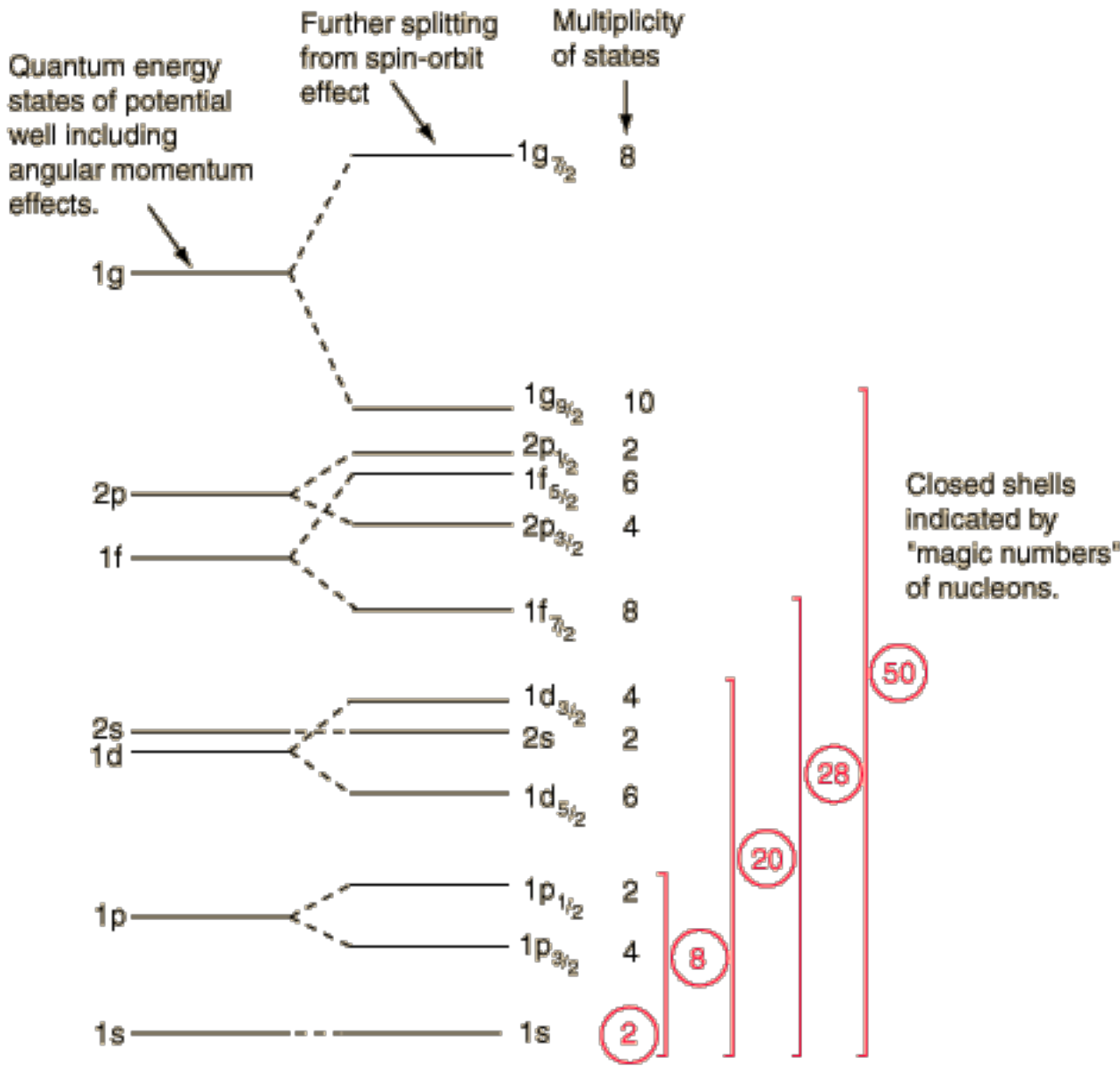
$$U_N \simeq -\frac{1}{2M^2c^2} \mathbf{S} \cdot \mathbf{L} \frac{1}{r} \frac{dV_N}{dr} \quad (11.58)$$

In comparing (11.58) with atomic formula (11.56) we note that both  $V$  and  $V_N$  are attractive (although  $V_N$  is much larger), so that the signs of the spin-orbit energies are opposite. This means that in nuclei the single particle levels form “inverted” doublets. With a reasonable form for  $V_N$ , (11.58) is in qualitative agreement with the observed spin-orbit splittings in nuclei.





Harmonic +flat bottom +spin-orbit  
oscillator



HW: Extend the nuclear shell model scheme beyond Z=82, N=126. What should be the next neutron and proton magic numbers in superheavy/hyperheavy nuclei?