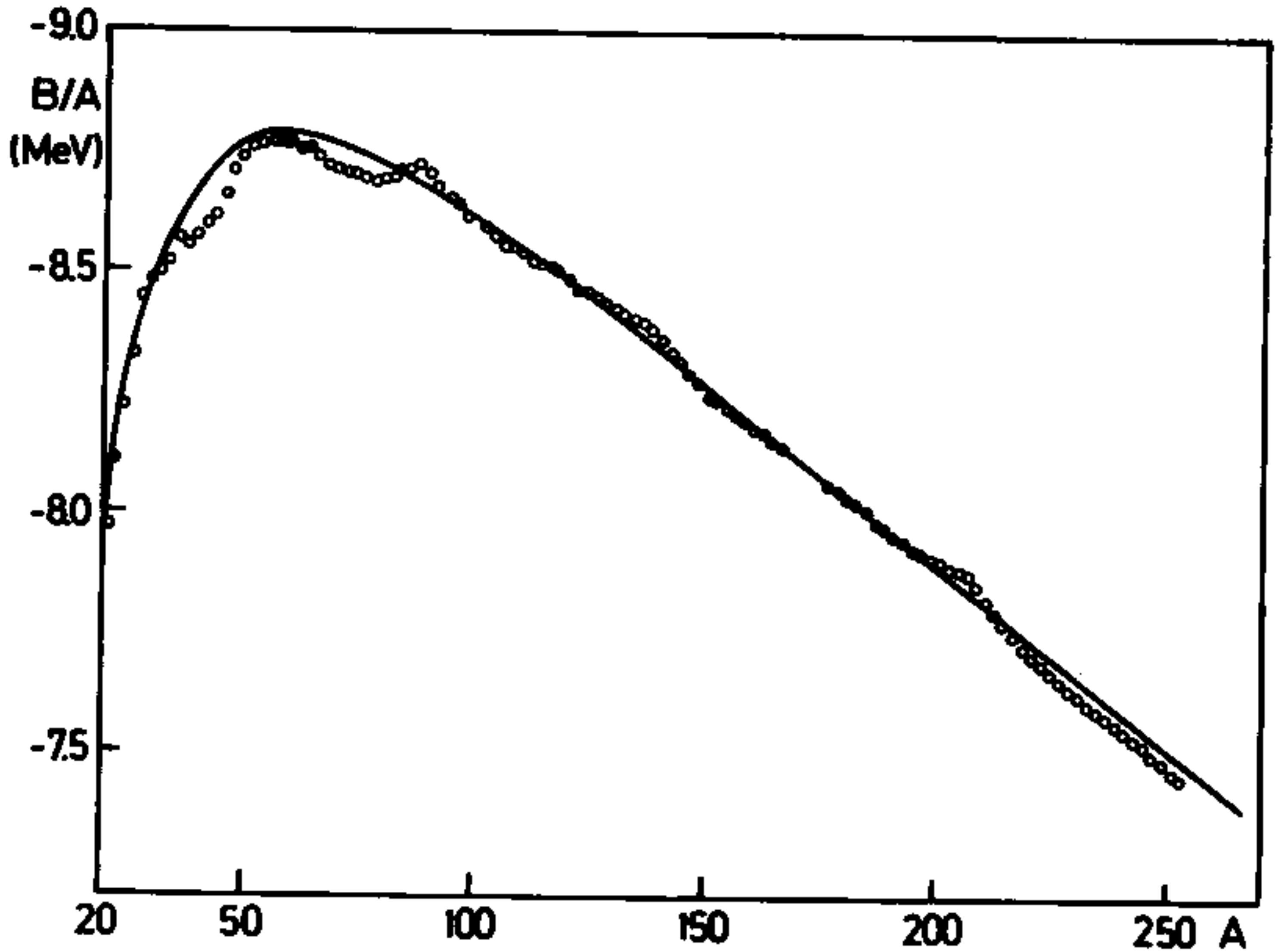
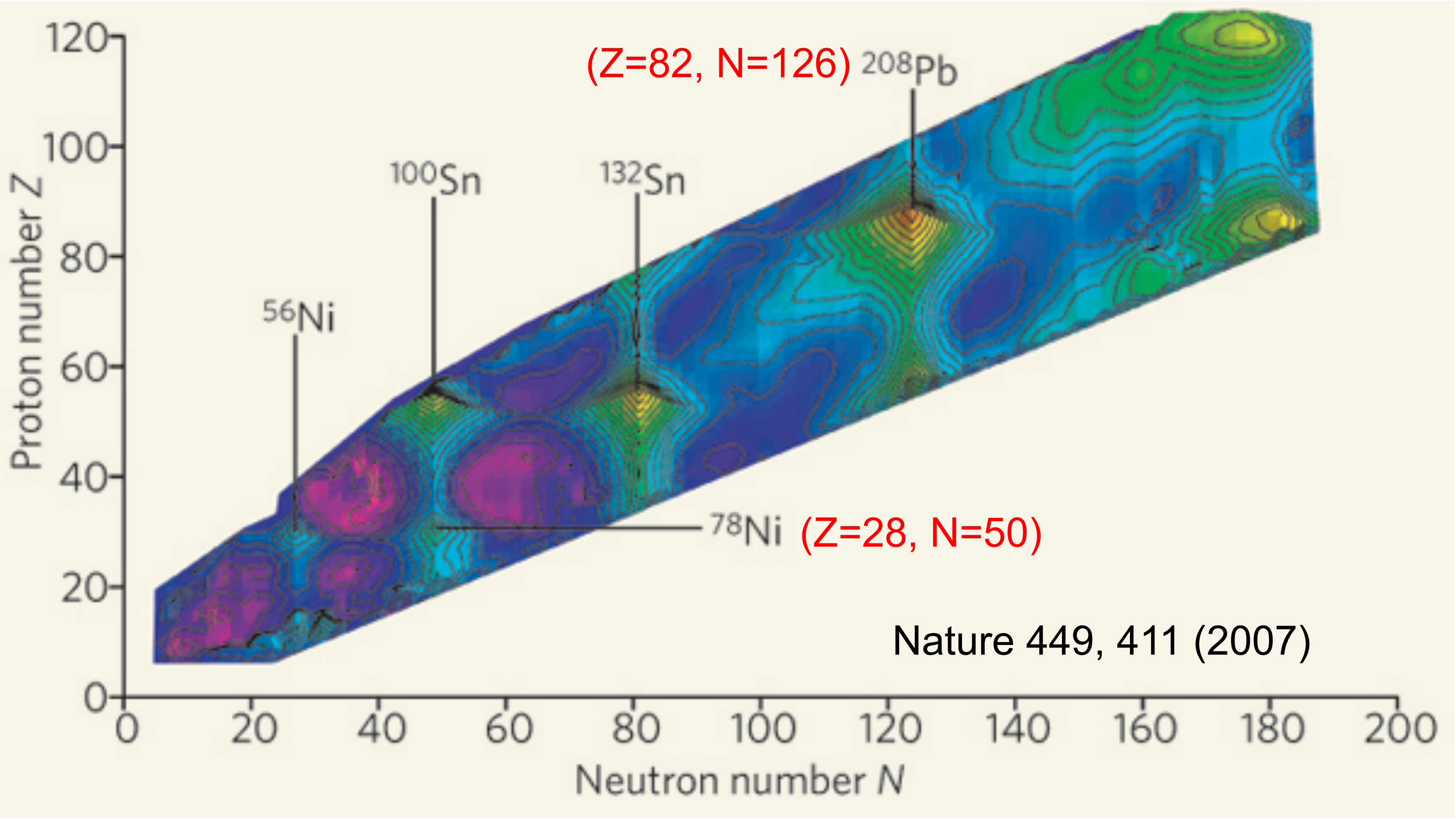
Nucleonic Shells

REMINDER: The semi-empirical mass formula, based **on the liquid drop model**, compared to the data



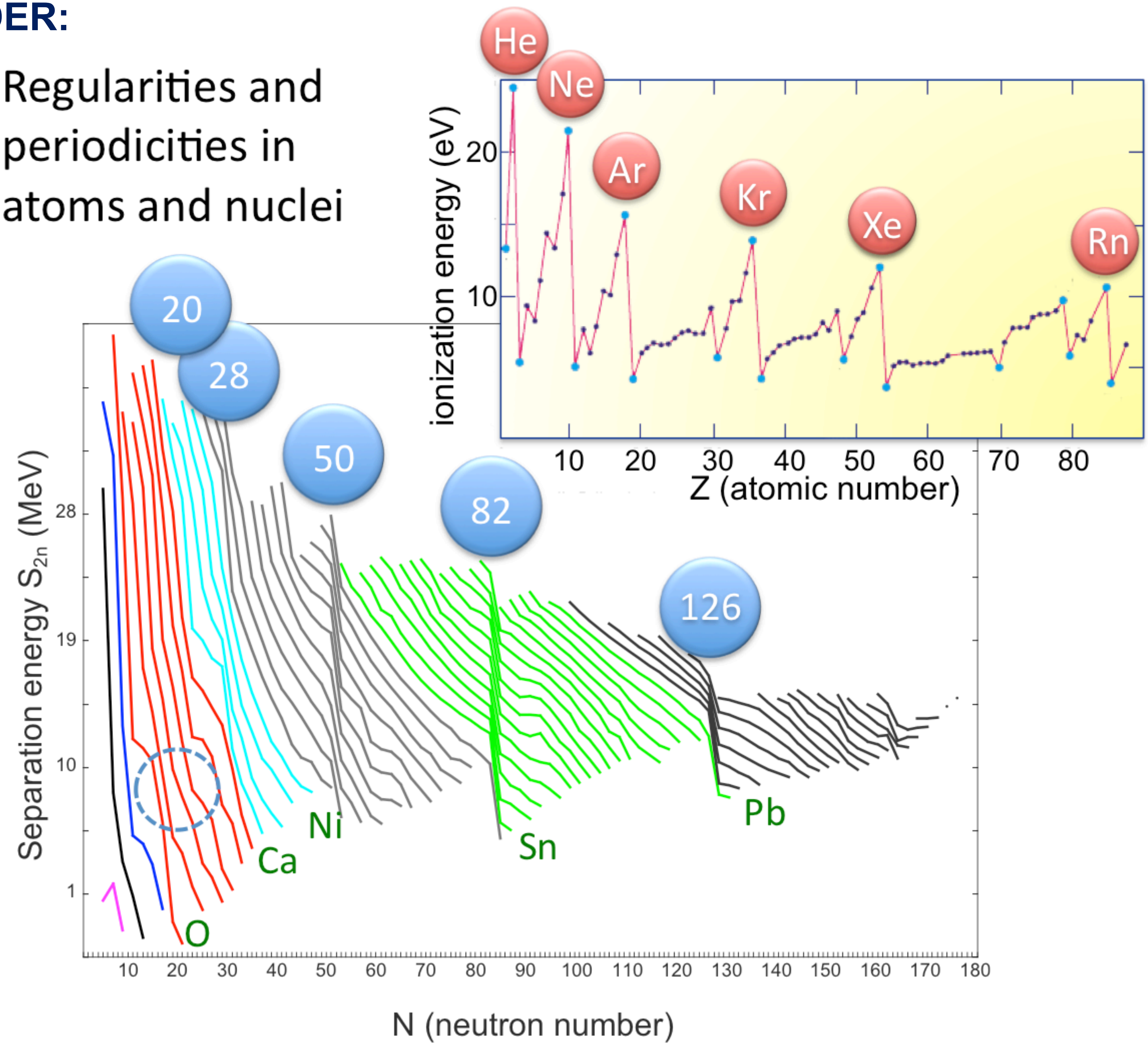
$$E_{\text{shell}} = E_{\text{total}} - E_{\text{LD}}$$



Magic numbers at Z or N= 2, 8, 20, 28, 50, 82,126

REMINDER:

Regularities and periodicities in atoms and nuclei



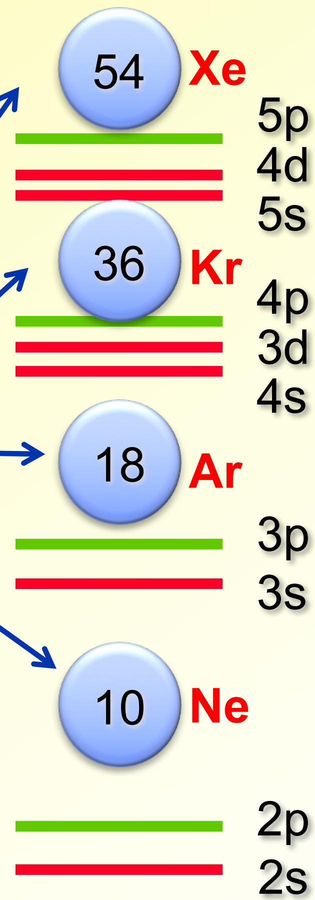
1912

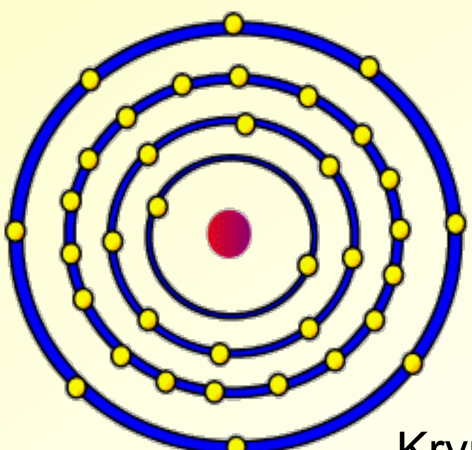
Nobel Prize 1922

electronic shells of the atom

Bohr's picture still serves as an elucidation of the physical and chemical properties of the elements.

noble gases (closed shells)





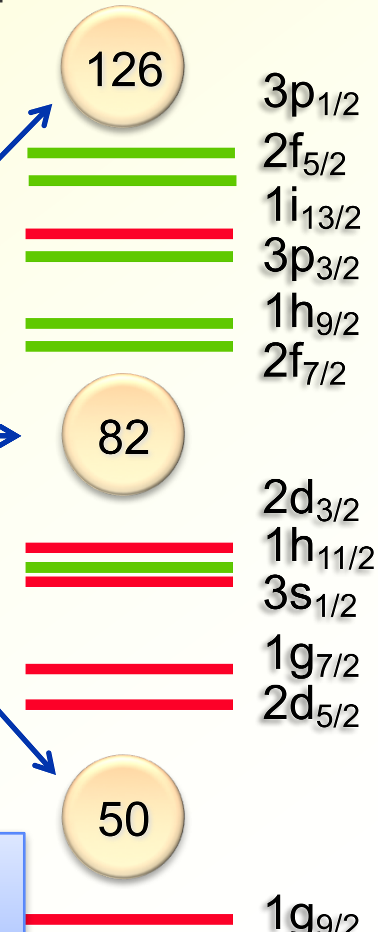
Krypton Atom

1949

Nobel Prize 1963

nucleonic shells of the nucleus

magic nuclei (closed shells)



We know now that this picture is ***very*** incomplete…

Spherical Harmonic Oscillator

$$\hat{h} = \hat{t} + \frac{m\omega_0^2 r^2}{2} \Rightarrow \quad \varepsilon_N = \left(N + \frac{3}{2}\right)\hbar\omega_0$$

For each shell, the allowed orbital angular momenta are:

$$\ell = N, N-2, \ldots, 1, \text{ or } 0, \quad j = \ell \pm \frac{1}{2}$$

Since each nucleon has an intrinsic spin s=1/2, the maximum number of nucleons in a HO shell is:

$$D_N = \sum_\ell 2(2\ell+1) = (N+1)(N+2) \underset{N>>1}{\approx} \left(N + \frac{3}{2}\right)^2$$

The total number of states is:

$$\sum_{N'=0}^{N} (N'+1)(N'+2) = \frac{1}{3}(N+1)(N+2)(N+3)$$

$$\underset{N>>1}{\approx} \frac{1}{3}(N+2)^3$$

N		l	DEGEN.	TOTAL
5	———	1,3,5	42	112
4	———	0,2,4	30	70
3	———	1,3	20	40
2	———	0,2	12	20
1	———	1	6	8
0	———	0	2	2

Dimension of orbits:

$$\langle r^2 \rangle_{N\ell} = \frac{\hbar}{m\omega_0}\left(N + \frac{3}{2}\right)$$

$$\Rightarrow \quad \hbar\omega_0 \approx \frac{41}{A^{1/3}} (\text{MeV})$$

Spin-orbit potential

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Nuclear Configurations in the Spin-Orbit Coupling Model. I. Empirical Evidence

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(Received December 7, 1949)

An extreme one particle model of the nucleus is proposed. The model is based on the succession of energy levels of a single particle in a potential between that of a three-dimensional harmonic oscillator and a square well. (1) Strong spin orbit coupling leading to inverted doublets is assumed. (2) An even number of identical nucleons are assumed to couple to zero angular momentum, and, (3) an odd number to the angular momentum of the single odd particle. (4) A (negative) pairing energy, increasing with the *j* value of the orbit is assumed. With these four assumptions all but 2 of the 64 known spins of odd nuclei are satisfactorily explained, and all but 1 of the 46 known magnetic moments. The two spin discrepancies are probably due to failure of rule (3). The magnetic moments of the five known odd-odd nuclei are also in agreement with the model. The existence, and region in the periodic table, of nuclear isomerism is correctly predicted.

$$V_{\text{so}} \approx \kappa \vec{\ell} \cdot \vec{s}$$

$$\langle \vec{\ell} \cdot \vec{s} \rangle = \frac{\hbar^2}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]$$

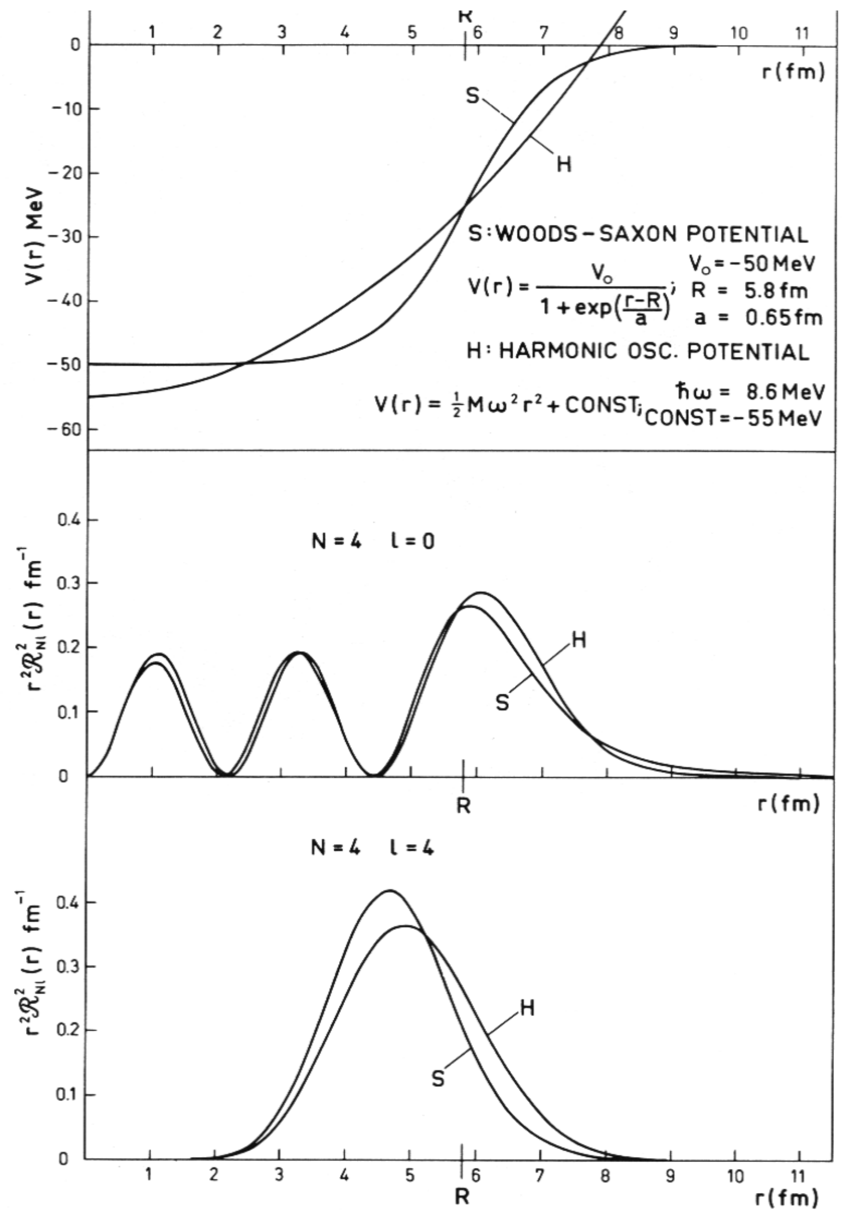
$$\langle \vec{\ell} \cdot \vec{s} \rangle = \frac{\hbar^2}{2} \begin{cases} \ell \text{ for } j = \ell + \frac{1}{2} \\ -(\ell+1) \text{ for } j = \ell - \frac{1}{2} \end{cases}$$

κ is negative!

The value of spin-orbit strength κ ***cannot*** be derived from a simple Thomas precession, as incorrectly stated in Jackson (next slide)

$$V_{\ell s} = \frac{g}{2m^2c^2} \frac{1}{r} \frac{dV}{dr} \vec{\ell} \cdot \vec{s}$$

Flat bottom



orbits with higher angular momentum shifted down!

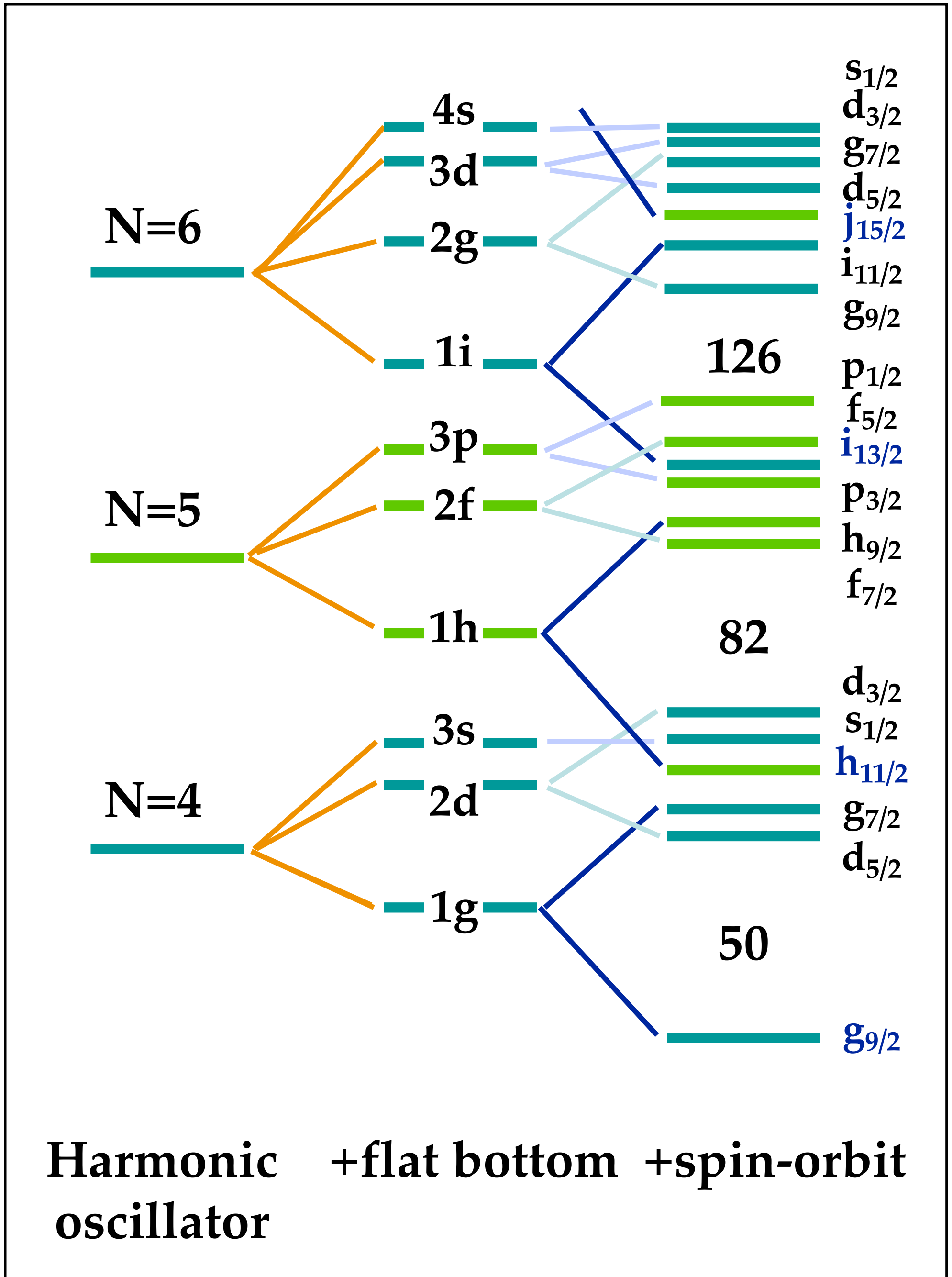
In atomic nuclei th ... celerations due to the specifically nuclear fo ... es are comparatively weak. In an approx ... nucleons as moving separately in a short ... attractive, potential well, $V_N(r)$. Then ea ... addition a spin-orbit interaction given by ... ctromagnetic contribution U' omitted:

$$U_N = -\mathbf{S}\cdot\boldsymbol{\omega}_T \tag{11.57}$$

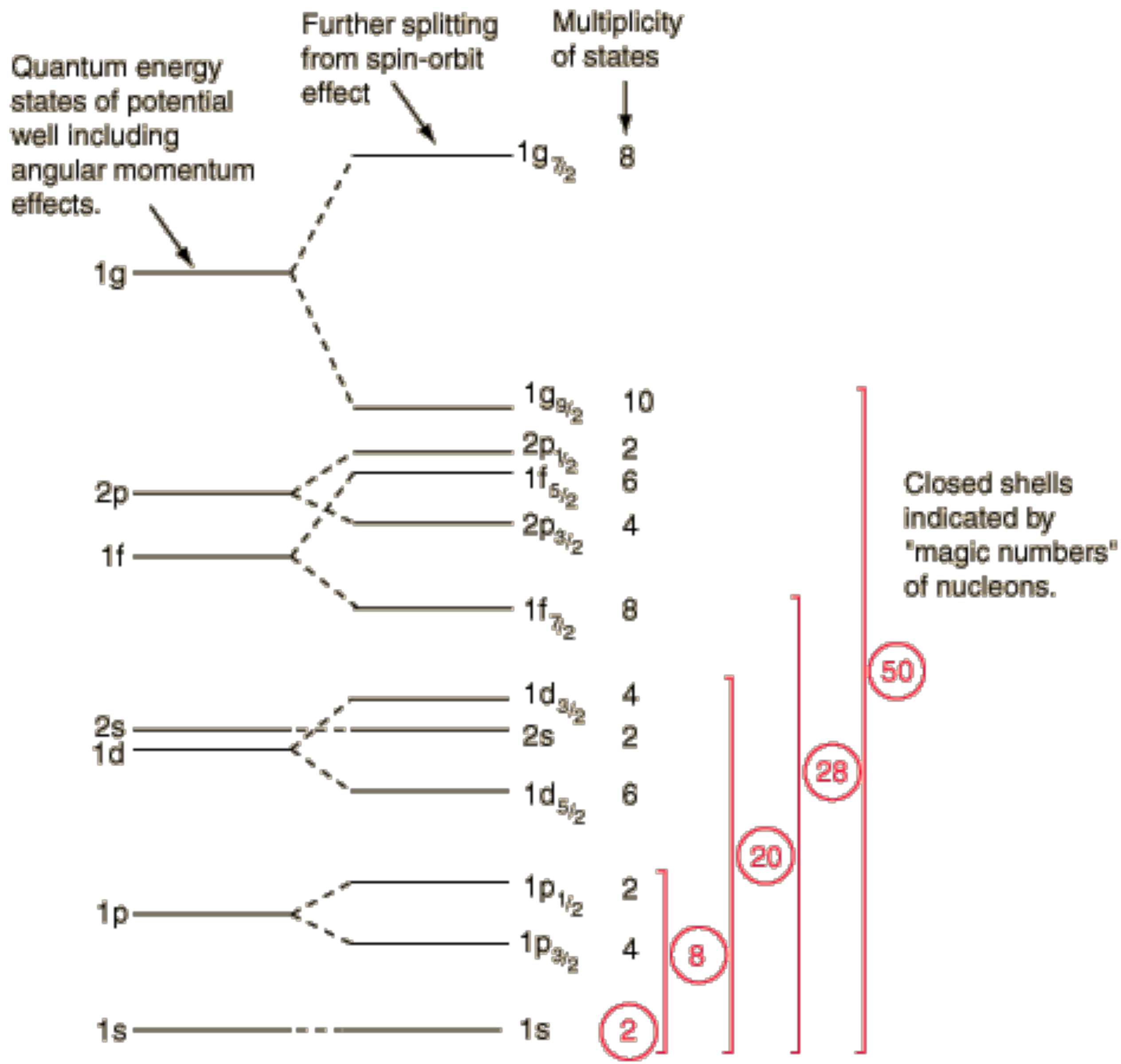
where the acceleration in $\boldsymbol{\omega}_T$ is determined by $V_N(r)$. The form of $\boldsymbol{\omega}_T$ is the same as (11.55) with V replaced by V_N. Thus the nuclear spin-orbit interaction is approximately

$$U_N \simeq -\frac{1}{2M^2c^2}\,\mathbf{S}\cdot\mathbf{L}\,\frac{1}{r}\frac{dV_N}{dr} \tag{11.58}$$

In comparing (11.58) with atomic formula (11.56) we note that both V and V_N are attractive (although V_N is much larger), so that the signs of the spin-orbit energies are opposite. This means that in nuclei the single particle levels form "inverted" doublets. With a reasonable form for V_N, (11.58) is in qualitative agreement with the observed spin-orbit splittings in nuclei.



N=6
N=5
N=4
4s
3d
2g
1i
3p
2f
1h
3s
2d
1g
s1/2
d3/2
g7/2
d5/2
j15/2
i11/2
g9/2
126
p1/2
f5/2
i13/2
p3/2
h9/2
f7/2
82
d3/2
s1/2
h11/2
g7/2
d5/2
50
g9/2
Harmonic oscillator
+flat bottom
+spin-orbit


Quantum energy states of potential well including angular momentum effects.
Further splitting from spin-orbit effect
Multiplicity of states
1g
2p
1f
2s
1d
1p
1s
$1g_{7/2}$ 8
$1g_{9/2}$ 10
$2p_{1/2}$ 2
$1f_{5/2}$ 6
$2p_{3/2}$ 4
$1f_{7/2}$ 8
$1d_{3/2}$ 4
2s 2
$1d_{5/2}$ 6
$1p_{1/2}$ 2
$1p_{3/2}$ 4
1s
2
8
20
28
50
Closed shells indicated by "magic numbers" of nucleons.