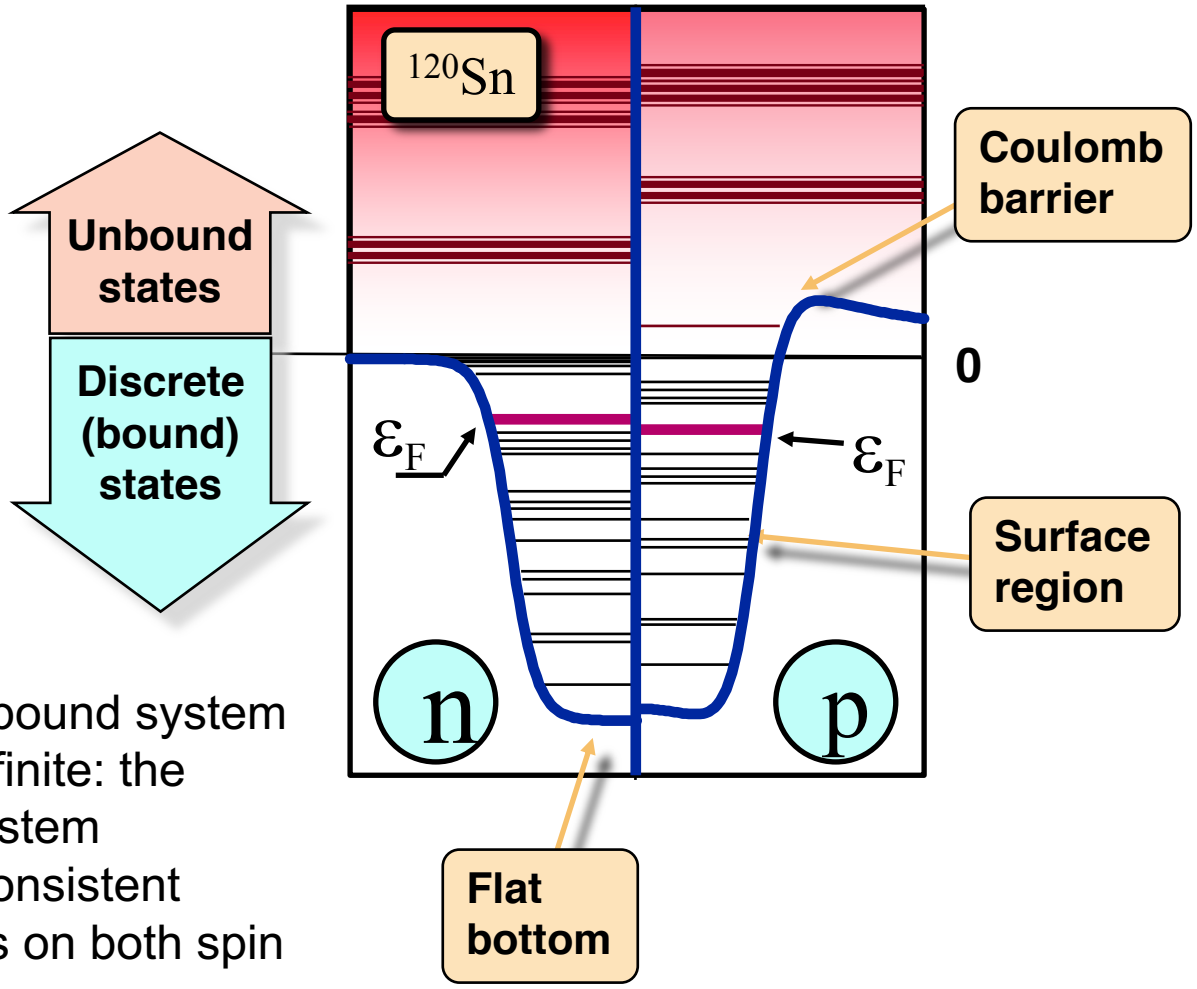
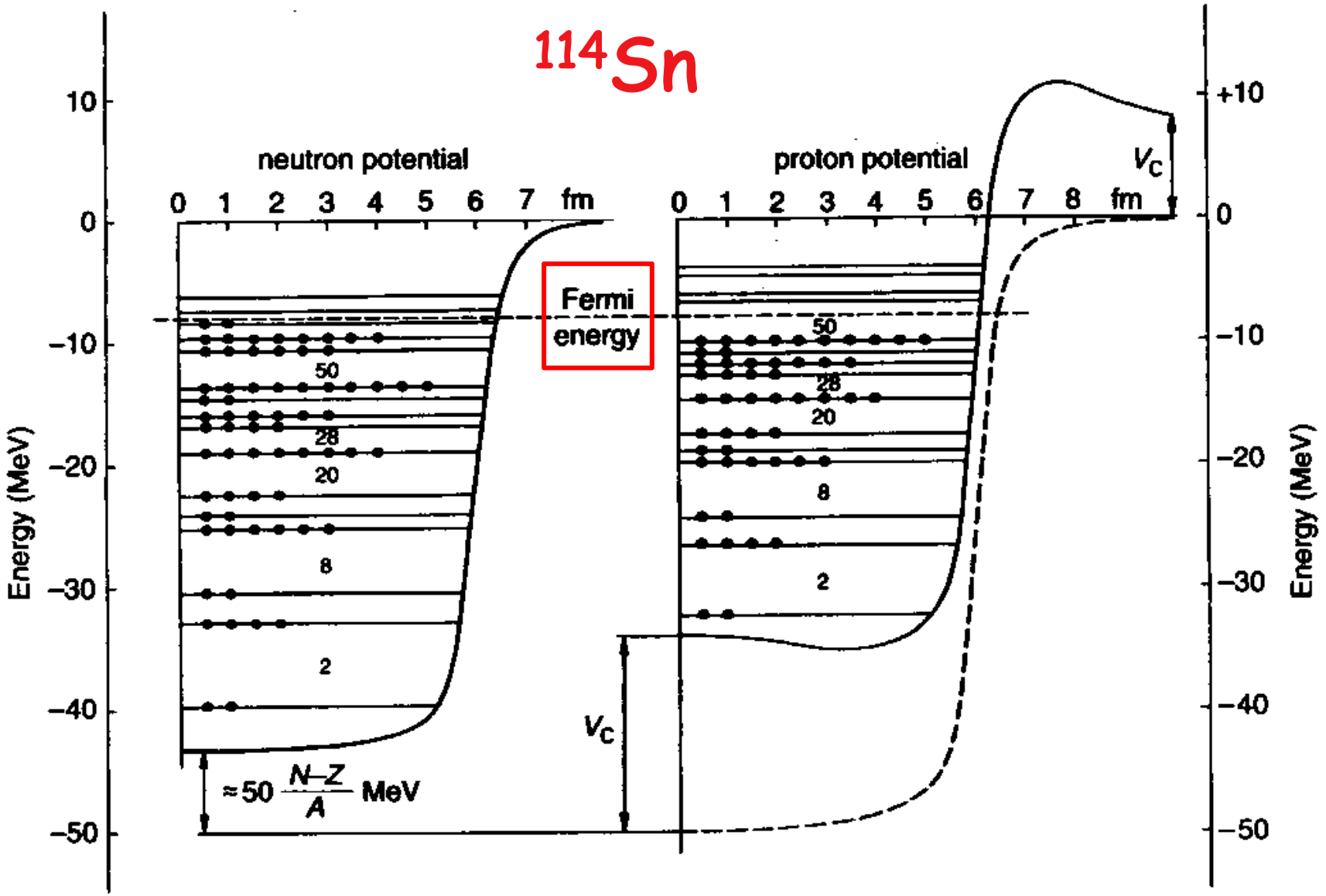
Average one-body Hamiltonian
(characteristic features)

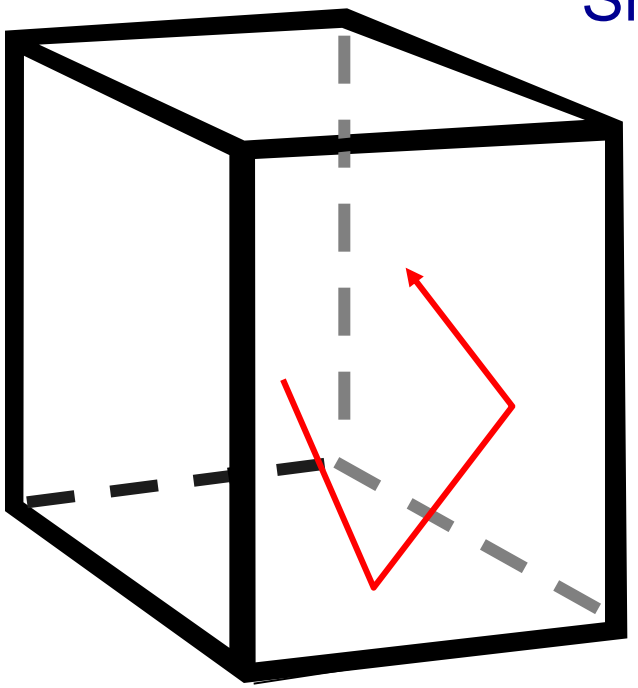


- The nucleus is a self-bound system
- The potential is not infinite: the nucleus is an open system
- The potential is self-consistent
- The potential depends on both spin and isospin

Nuclear shell model potential



Shell effects and classical periodic orbits



$$\hat{h}\varphi_\alpha = \epsilon_\alpha \varphi_\alpha$$

Shells

- Typical time scale: 0.1zs=babyseconds (10^{-22}s)
- Closed orbits and s.p. quantum numbers

$$g(\varepsilon) = \frac{dN}{d\varepsilon}$$

density of states (number of states per energy interval)

$$g(\varepsilon) = \tilde{g}(\varepsilon) + \sum_\gamma A_\gamma(\varepsilon)\cos\left[S_\gamma(\varepsilon)/\hbar - \alpha_\gamma\right]$$

Trace formula, Gutzwiller, J. Math. Phys. 8 (1967) 1979

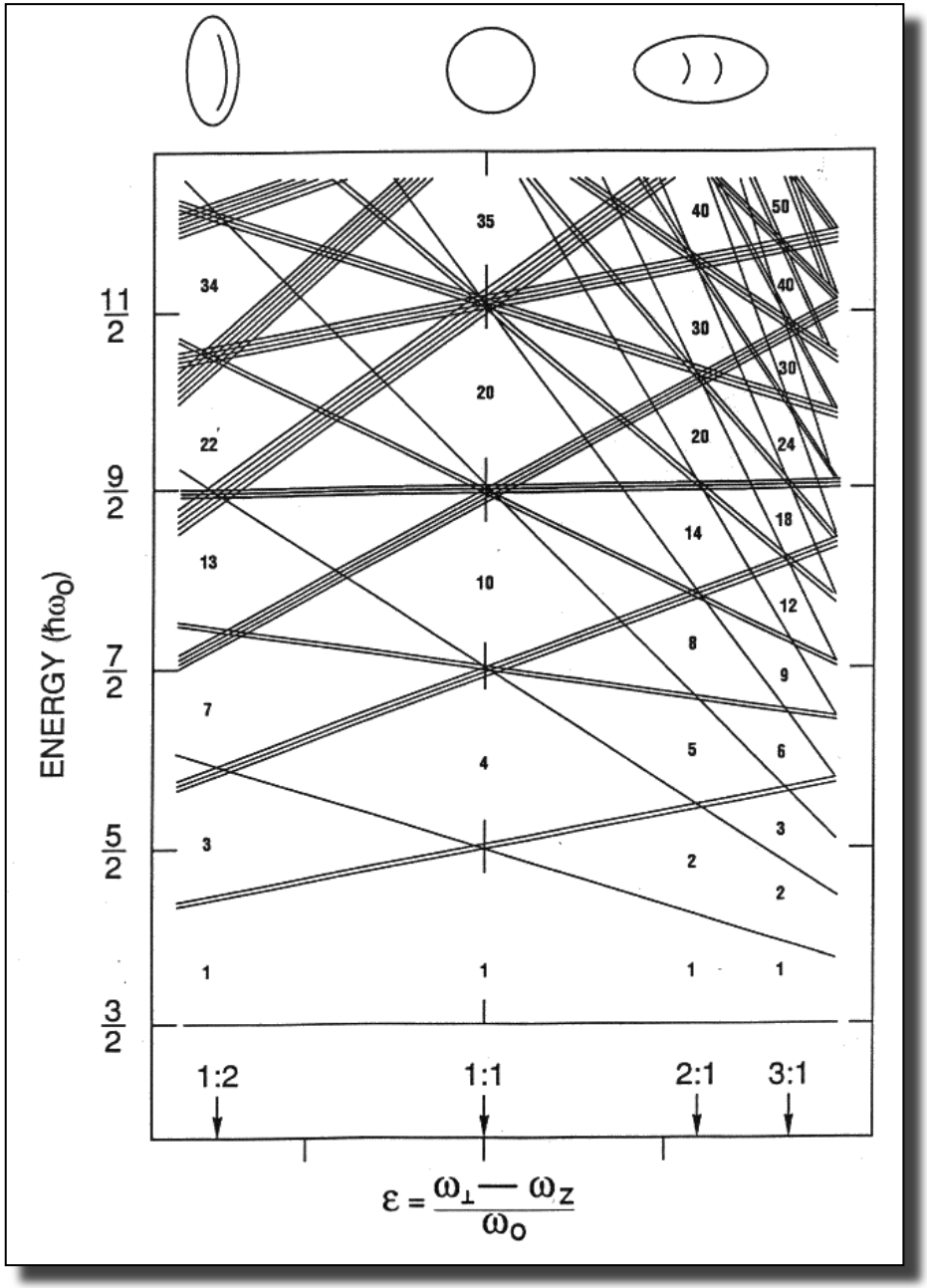
smooth part

oscillating part (shell effects)

$$S_\gamma(\varepsilon) = \oint_\gamma \vec{p}\, d\vec{q}$$

The action integral for the periodic (closed) orbit γ

Shell effects, degeneracies, and symmetries



Example" Rational Harmonic Oscillator: RHO

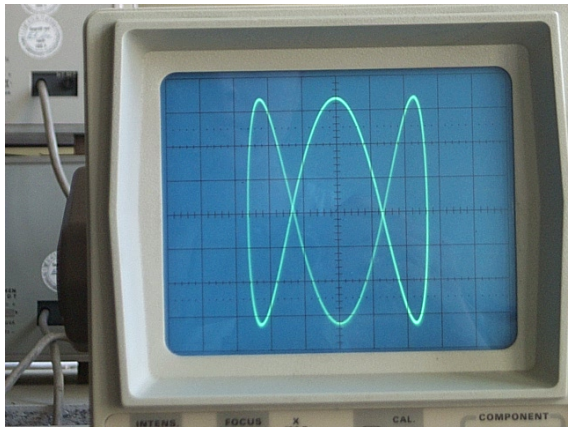
Phys. Rev. Lett. 68, 154 (1992)

$$\omega_i k_i = \tilde{\omega}$$

RHO explains the presence of superdeformed and cluster configurations in atomic nuclei

XC: Find the relation between magic numbers of spherical HO and superdeformed (2:1) and hyperdeformed (3:1) RHO

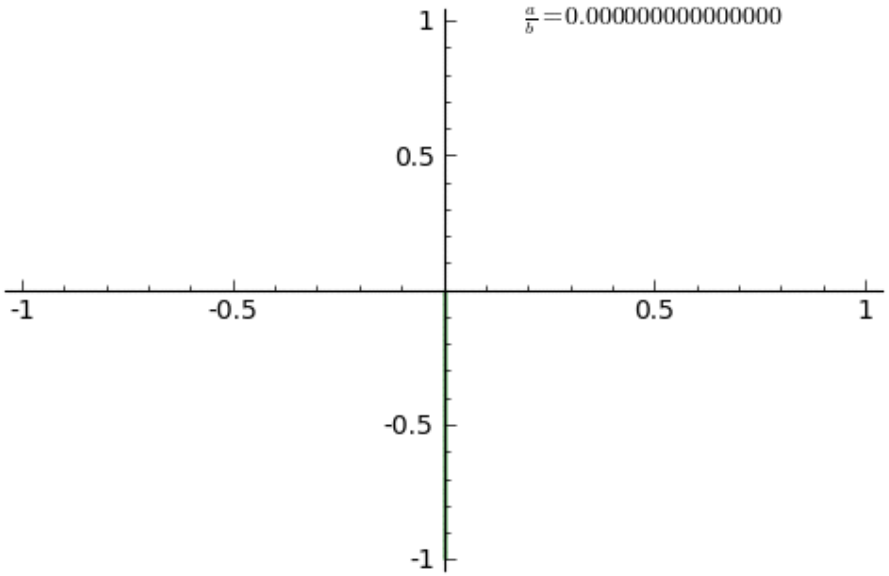
Lissajous curves (complex harmonic motion)



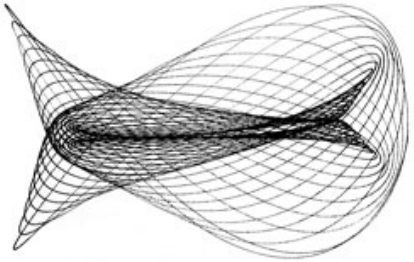
$$x = A\sin(at + \delta)$$

$$y = B\sin(bt)$$

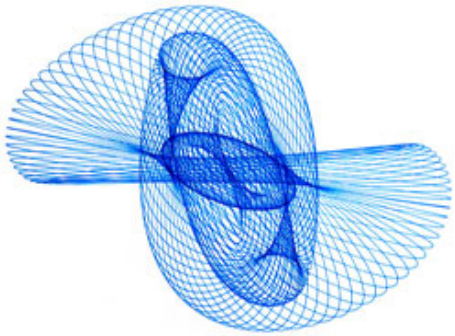
The appearance of the figure is highly sensitive to the ratio *a/b*. For a/b=1, the figure is an ellipse, with special cases including circles (A = B, $\delta=\pi/2$) and lines ($\delta=0$). Another simple Lissajous figure is the parabola (a/b=2, $\delta=\pi/4$). Other ratios produce more complicated curves, which are *closed only if a/b is rational*.



2D



3D



http://en.wikipedia.org/wiki/Lissajous_curve

Pronounced shell structure
(quantum numbers)

Shell structure absent

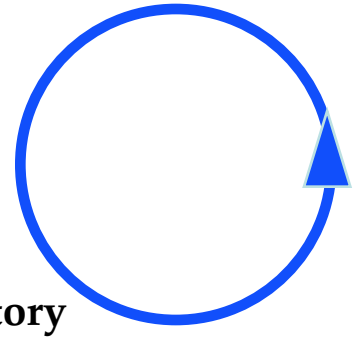
shell

gap

shell

gap

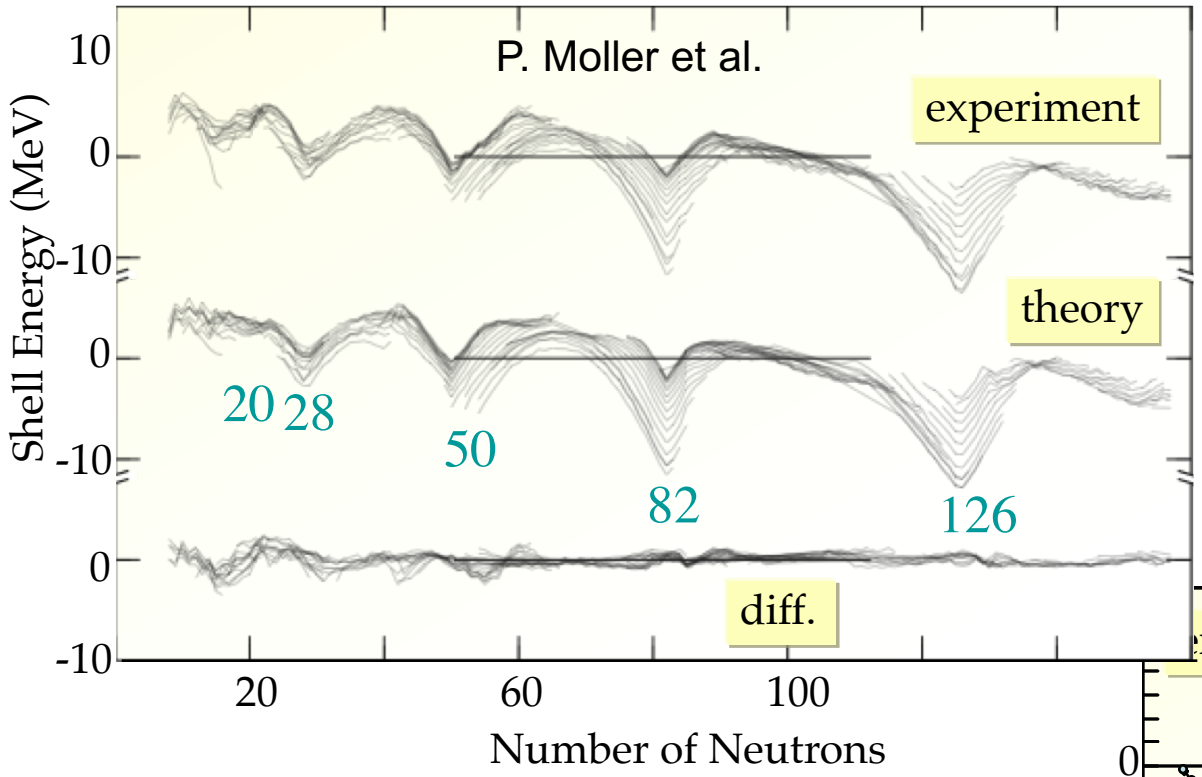
shell



closed trajectory
(regular motion)

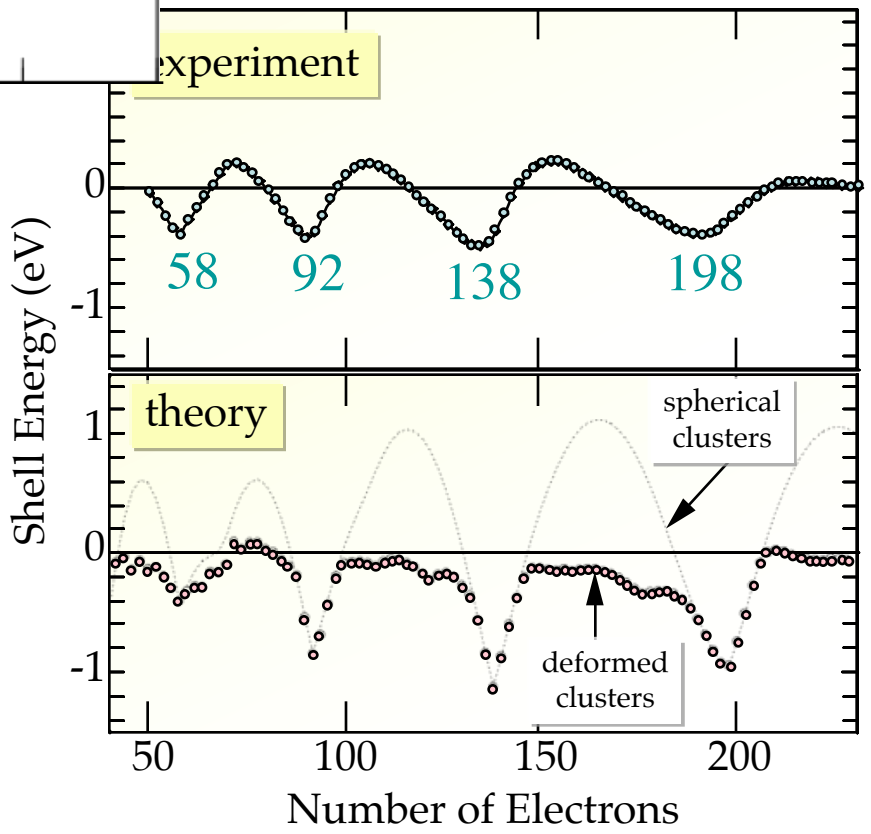
trajectory does not close

Shells in mesoscopic systems



Nuclei

S. Frauendorf et al.



Sodium Clusters

- Jahn-Teller Effect (1936)
- Symmetry breaking and deformed mean-field

HW: Extend the nuclear shell model scheme beyond Z=82, N=126. What should be the next neutron and proton magic numbers in superheavy/hyperheavy nuclei?