Symmetries of the nuclear Hamiltonian (exact or almost exact)

1. Translational invariance
2. Galilean invariance (or Lorentz invariance)
3. Rotational invariance
4. Time reversal
5. Parity (space reflection)
6. Charge independence and isobaric symmetry
7. Baryon and lepton number symmetry
8. Permutation between the two nucleons (imposed by the exclusion principle)

Continuous transformations (appear to be universally valid)

Dynamical symmetries apply in certain cases, provide useful coupling schemes

1. Chiral symmetry (broken by a quark condensate; valid for massless quarks)
2. SU(4) symmetry (Wigner supermultiplet)
3. SU(2) symmetry (seniority)
4. SU(3) symmetry (Elliott model)
Symmetries in quantum mechanics


• Wave equation for the Hamiltonian operator:

\[ \hat{H} \Psi_k = E_k \Psi_k \]

• Group of transformations \( \mathcal{G} \) whose elements \( G \) commute with \( H \):

\[ G \hat{H} G^{-1} = \hat{H} \]

• We say that \( H \) is invariant under \( \mathcal{G} \) or totally symmetric with respect to the elements of \( \mathcal{G} \)
• What are the properties of \( G \Psi_k \)?
• Representation of the group (represents group elements as matrices so that the group operation can be represented by matrix multiplication)

\[ \hat{G} \phi_p = \sum_{q=1}^{d} D_{qp}^{\alpha}(G) \phi_q \]

If all matrices D can be put into a block-diagonal form, the representation is irreducible
\[ \hat{H} G \Psi_k = G \hat{H} G^{-1} G \Psi_k = G \hat{H} \Psi_k = E_k G \Psi_k \]

Hence $ G \Psi_k $ is also an eigenfunction of $ H $ with eigenvalue $ E_k $. 

- If $ E_k $ is nondegenerate
  
  $ G \Psi_k = \chi^\alpha(G) \Psi_k $ 

  one-dimensional irrep of $ G $ 

  We can thus label the wave function fully as $ \Psi_k^\alpha $  

- If $ E_k $ is $ n $-fold degenerate, there are $ n $ partner functions $ \Psi_k^p $ ($ p = 1 \ldots n $)

  \[ G \Psi_k^p = \sum_{q}^{n} D_{qp}^\alpha(G) \Psi_k^q \]

  $ n $-dimensional irrep of $ G $, except for accidental degeneracy  

  We can thus label the wave function fully as $ \Psi_k^\alpha $