Symmetries of the nuclear Hamiltonian
(exact or almost exact)

1. **Translational invariance**
2. **Galilean invariance (or Lorentz invariance)**
3. **Rotational invariance**

} Continuous transformations (appear to be universally valid)

4. Time reversal
5. Parity (space reflection)
6. Charge independence and isobaric symmetry
7. Baryon and lepton number symmetry
8. Permutation between the two nucleons (imposed by the exclusion principle)

Dynamical symmetries
apply in certain cases, provide useful coupling schemes

1. Chiral symmetry (broken by a quark condensate; valid for massless quarks)
2. SU(4) symmetry (Wigner supermultiplet)
3. SU(2) symmetry (seniority)
4. SU(3) symmetry (Elliott model)

Symmetries in quantum mechanics

(see "Symmetry in Physics", J.P. Elliott and P.G. Dawber, The Macmillan Press, London)

- Wave equation for the Hamiltonian operator:

$$\hat{H}\Psi_k = E_k \Psi_k$$

- Group of transformations $\mathcal{G}$ whose elements G commute with H:

$$G\hat{H}G^{-1} = \hat{H}$$

- We say that H *is* invariant under $\mathcal{G}$ or totally symmetric with respect to the elements of $\mathcal{G}$
- What are the properties of $G\Psi_k$?
- Representation of the group (represents group elements as matrices so that the group operation can be represented by matrix multiplication)

$$\hat{G}\phi_p = \sum_{q=1}^{d} D^{\alpha}_{qp}(G)\phi_q$$

dimension of the representation

matrix representation of the group

basis

If all matrices D can be put into a block-diagonal form, the representation is irreducible

$$\hat{H}G\Psi_k = G\hat{H}G^{-1}G\Psi_k = G\hat{H}\Psi_k = E_k G\Psi_k$$

Hence $G\Psi_k$ is also an eigenfunction of H with eigenvalue E_k.

- If E_k is nondegenerate $G\Psi_k = \chi^\alpha(G)\Psi_k$

  one-dimensional irrep of $\mathcal{G}$

  We can thus label the wave function fully as Ψ_k^α

- If E_k is *n*-fold degenerate, there are *n* partner functions $\Psi_k^p \quad (p = 1 \ldots n)$

$$G\Psi_k^p = \sum_q^n D_{qp}^\alpha(G)\Psi_k^q$$

n-dimensional irrep of $\mathcal{G}$, except for accidental degeneracy

We can thus label the wave function fully as Ψ_k^α