Since \( CP \) is violated, \( \Psi \) has to be violated as well!

Atomic/neutron electric dipole moment: The violation of \( CP \)-symmetry is responsible for the fact that the Universe is dominated by matter over anti-matter


• Closely spaced parity doublet gives rise to enhanced electric dipole moment
• Large intrinsic Schiff moment
  ○ \(^{199}\text{Hg}\) (Seattle, 1980’s – present)
  ○ \(^{225}\text{Ra}\) (ANL, KVI)
    Parker et al. 2015, \( d<5\times10^{-22} \) e cm
  ○ \(^{223}\text{Rn}\) at TRIUMF (E929)
  ○ FRIB
    • Widest search for octupole deformations
    • \(^{238}\text{U}\) beam, beam dump recovery: \(^{225}\text{Ra}\): \( 6\times10^9/s \)
    • \(^{232}\text{Th}\) beam: \(^{225}\text{Ra}\): \( 5\times10^{10}/s \), \(^{223}\text{Rn}\): \( 1\times10^9/s \)
    • \( 10^{12}/s \) with ISOL target FRIB upgrade

HW3: Using information from PDG.lbl.gov and nndc.bnl.gov determine whether the following decays/reactions are allowed by fundamental symmetries:

a. $\pi^0 \rightarrow \mu^+ e^-$
b. $p + \bar{p} \rightarrow \gamma$
c. Gamma decay of excited state of $^{16}$O at 6049 keV
d. Decay of meson $\eta \rightarrow \gamma \gamma$
e. Decay of meson $\eta \rightarrow \pi^0 + \pi^0$
Isospin Symmetry

Introduced 1932 by Heisenberg

• Protons and neutrons have almost identical mass: $\Delta m/m = 1.4 \times 10^{-3}$
• Low energy np scattering and pp scattering below $E = 5$ MeV, after correcting for Coulomb effects, is equal within a few percent in the $^1S$ scattering channel.
• Energy spectra of “mirror” nuclei, (N,Z) and (Z,N), are almost identical.

• up and down quarks are very similar in mass, and have the same strong interactions. Particles made of the same numbers of up and down quarks have similar masses and are grouped together.

\[
\varphi_n(\vec{r}, s) = \varphi(\vec{r}, s) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varphi_p(\vec{r}, s) = \varphi(\vec{r}, s) \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

wave functions

\[
\tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

Pauli isospin matrices

\[
\tau = (\tau_x, \tau_y, \tau_z), \quad \vec{t} = \frac{\tau}{2}
\]

SU(2) commutations

\[
[t_x, t_y] = it_z, \quad [\tau^2, t_i] = 0
\]
\[ \hat{T} = \sum_{i=1}^{A} \hat{t}_i \]  

\[
\left[ H, T_z \right] = 0 \Rightarrow T_z |\Psi\rangle = \frac{N - Z}{2} |\Psi\rangle
\]

Still Waiting For Electron Decay…

\[ e^- \rightarrow \nu_e + \gamma \]

The current bound from Borexino is \( \tau > 6.6 \times 10^{28} \) yr: Phys. Rev. Lett. 115, 231802 (2015)

“...The conservation of electric charge, suggested since the 19th century, is fundamental to the physics of the standard model as a direct consequence of Maxwell’s equations and the unbroken \( U(1) \) gauge symmetry of the electroweak theory. Despite the present undisputed validity of this law, experimental tests of charge conservation remain a way to search for physics beyond the standard model, and they deserve to be investigated with the highest possible sensitivity. An experimental search for the hypothetical charge nonconserving decay of the electron, which is the lightest known charged particle, into a neutrino and a photon is reported in this Letter. No presently viable theory predicts such a decay, and a large charge violation is excluded by the absence of macroscopic effects in matter.”
Using the NNDC website [http://www.nndc.bnl.gov](http://www.nndc.bnl.gov) find two examples of spectra of mirror nuclei. How good is isospin symmetry in those cases?

\[
\bar{T} = \sum_{i=1}^{A} \bar{t}_i
\]

\[
[H, T_z] = 0 \Rightarrow T_z \Psi = \frac{N-Z}{2} \Psi
\]

\[
[H, T_\pm] = 0
\]

\[
[H, T^2] = 0 \Rightarrow T^2 \Psi = T(T+1) \Psi
\]

**THE A = 30 ISOSPIN TRIPLET**

<table>
<thead>
<tr>
<th>E_x(MeV)</th>
<th>J^+</th>
<th>E_x(MeV)</th>
<th>J^+</th>
<th>E_x(MeV)</th>
<th>J^+</th>
</tr>
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<td>2^-</td>
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<td>4^-</td>
<td>4.18</td>
<td>2^-</td>
</tr>
<tr>
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<td></td>
<td>3.93</td>
<td>1(-2)</td>
<td>3.83</td>
<td>1(-2)</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.24</td>
<td>2^-</td>
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<td>1^-</td>
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<tr>
<td>1.97</td>
<td>3^-</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1.45</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**charge independence**

\[
[H, T^2] = 0 \Rightarrow T^2 \Psi = T(T+1) \Psi
\]

**T is conserved!**

**T_z component conserved!**

(charge conservation)

\[
T \text{ component conserved!}
\]
Group of permutations

Exchange operators

\[ \hat{P}_{ij} \] - exchanges particles \( i \) and \( j \)

\[ \hat{P}_{ij} \Psi (x_1 \cdots x_i \cdots x_j \cdots x_A) = \Psi (x_1 \cdots x_j \cdots x_i \cdots x_A) \]

\( \hat{P}_{ij} \) is hermitian and unitary:

\[ \hat{P}_{ij}^+ = \hat{P}_{ij}, \quad \hat{P}_{ij}^2 = 1 \]

The eigenvalues of \( \hat{P}_{ij} \) are \( \pm 1 \)

(identical particles cannot be distinguished)

For identical particles, measurements performed on quantum states \( \Psi \) and \( \hat{P}_{ij} \Psi \) have to yield identical results

A principle, supported by experiment
This principle implies that all many-body wave functions are eigenstates of \( \hat{P}_{ij} \)

\[
\hat{P}_{ij} \Psi = p_{ij} \Psi, \quad p_{ij} = \pm 1
\]

is a basis of one-dimensional representation of the permutation group.

There are only two one-dimensional representations of the permutation group:

\[
p_{ij} = +1 \quad \text{for all } i,j - \text{fully symmetric representation}
\]

\[
p_{ij} = -1 \quad \text{for all } i,j - \text{fully antisymmetric representation}
\]

Consequently, systems of identical particles form two separate classes:

\[
\hat{P}_{ij} \Psi = \Psi \quad \text{bosons (integer spins)}
\]

\[
\hat{P}_{ij} \Psi = -\Psi \quad \text{fermions (half-integer spins)}
\]

For spin-statistics theorem, see W. Pauli, Phys. Rev. 58, 716-722(1940)

\[
\left[ \hat{P}_{ij}, \hat{H} \right] = 0
\]
The concept of isospin symmetry can be broadened to an even larger symmetry group, now called flavor symmetry. Once the kaons and their property of strangeness became better understood, it started to become clear that these, too, seemed to be a part of an enlarged symmetry that contained isospin as a subgroup. The larger symmetry was named the *Eightfold Way* by Gell-Mann, and was recognized to correspond to the adjoint representation of SU(3). While isospin symmetry is broken slightly, SU(3) symmetry is badly broken, due to the much higher mass of the strange quark compared to the up and down.

**XC:** Using spin and isospin algebra, and Pauli principle, find two-nucleon wave functions. Assume that the spatial part of the wave functions corresponds to an s-wave (i.e., is symmetric).