The nuclear operator transforming a neutron into a proton must be one body in nature. Hence it must involve the isospin raising or lowering operators.

In the non-relativistic limit, the vector part may be represented by the unity operator times $\tau_{\pm}$ and the axial-vector part by a product of $\tau_{\pm}$ and $\sigma$. (A proper derivation requires manipulation with Dirac 4-component functions and $\gamma$ matrices!)

$$V_{\text{int}} \approx g\delta(\vec{r}_n - \vec{r}_p)\delta(\vec{r}_n - \vec{r}_e)\delta(\vec{r}_n - \vec{r}_\nu)\hat{O}(n \rightarrow p) \quad \text{zero-range}$$

$$V_{\text{int}} \rightarrow G_V \sum_{j=1}^{A} \left[ \tau_{\pm}(j) + g_A \vec{\sigma}(j) \cdot \vec{\tau}(j) \right]$$

Fermi decay, carries zero angular momentum

Gamow-Teller decay, carries one unit of angular momentum

$G_V$ determined from superallowed Fermi beta decays!
From the expression for $fT$, it is possible to determine the strength $g$ of the beta-decay process, if one knows how to determine the reduced matrix element. For superallowed $0^+ \rightarrow 0^+$ Fermi transitions, the matrix element is $\sqrt{2}$ so the $fT$ values should be identical.

Superallowed $0^+ \rightarrow 0^+$ beta decay between $T=1$ analog states

![Graph](image)

corrected for nuclear structure effects (isospin mixing) and radiative corrections

\[ g = 0.88 \times 10^{-4} \text{ MeV fm}^3 \]

or, introducing the dimensionless constant $G$:

\[ G = g \frac{m_e^2 c}{\hbar} = 1.026 \times 10^{-5} \]
Superallowed Fermi $0^+ \rightarrow 0^+ \beta$-decay studies

Impressive experimental effort worldwide

Kobayashi and Maskawa (2008): ... for "the discovery of the origin of broken symmetry, which predicts the existence of at least three families of quarks in nature."

Hardy and Towner survey (Feb. 2015)
http://journals.aps.org/prc/abstract/10.1103/PhysRevC.91.025501

\[ V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99992 \pm 0.00048 \]

\[ V_{ud}^2 \text{ nuclear decays} = 0.94900 \pm 0.00042 \]
\[ V_{us}^2 \text{ kaon decays} = 0.05090 \pm 0.00022 \]
\[ V_{ub}^2 \text{ B decays} = 0.00002 \pm 0.00001 \]
Beta decay: allowed transitions

Fermi transitions

\[ \langle J_f M_f T_f T_{0f} | T_\mp | J_i M_i T_i T_{0i} \rangle = \sqrt{T_i(T_i + 1) - T_{0i}(T_{0i} \mp 1)} \delta_{J_i J_f} \delta_{M_i M_f} \delta_{T_i T_f} \delta_{T_{0i} \mp T_{0f}} \]

In reality, isospin is violated by the electromagnetic force, but the violation is weak.

\[ J_f = J_i \quad (\Delta J = 0) \]
\[ T_f = T_i \neq 0 \quad (\Delta T = 0, \text{ but } T_i = 0 \rightarrow T_f = 0 \text{ forbidden}) \]
\[ T_{0f} = T_{0i} \mp 1 \quad (\Delta T_0 = 1) \]
\[ \Delta \pi = 0 \quad \text{no parity change} \]

Gamow-Teller transitions

The matrix element strongly depends on the structure of the wave function!

\[ fT = \frac{const}{\langle F \rangle^2 + g_A^2 \langle GT \rangle^2} \]

The absolute values of GT matrix elements are generally smaller than those for Fermi transitions.

\[ \Delta J = 0,1 \quad \text{but } J_i = 0 \rightarrow J_f = 0 \text{ forbidden} \]
\[ \Delta T = 0,1 \quad \text{but } T_i = 0 \rightarrow T_f = 0 \text{ forbidden} \]
\[ T_{0f} = T_{0i} \mp 1 \quad (\Delta T_0 = 1) \]
\[ \Delta \pi = 0 \quad \text{no parity change} \]
Superallowed Gamow-Teller decay of the doubly magic nucleus $^{100}$Sn


Number distribution of log($ft$) values for allowed $\beta$-transitions (obeying the selection rules).
Forbidden transitions

Forbidden transitions involve parity change and a spin change of more than one unit. They come from the higher-order terms in the expansion of electron and neutrino plane waves into spherical harmonics. Forbidden decays are classified into different groups by the $L$-value of the spherical harmonics involved.

<table>
<thead>
<tr>
<th>Decay type</th>
<th>$\Delta J$</th>
<th>$\Delta T$</th>
<th>$\Delta \pi$</th>
<th>$\log_{10} t_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superallowed</td>
<td>$0^+ \rightarrow 0^+$</td>
<td>0</td>
<td>no</td>
<td>3.1–3.6</td>
</tr>
<tr>
<td>Allowed</td>
<td>0, 1</td>
<td>0, 1</td>
<td>no</td>
<td>2.9–10</td>
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<tr>
<td>First forbidden</td>
<td>0, 1, 2</td>
<td>0, 1</td>
<td>yes</td>
<td>5–19</td>
</tr>
<tr>
<td>Second forbidden</td>
<td>1, 2, 3</td>
<td>0, 1</td>
<td>no</td>
<td>10–18</td>
</tr>
<tr>
<td>Third forbidden</td>
<td>2, 3, 4</td>
<td>0, 1</td>
<td>yes</td>
<td>17–22</td>
</tr>
<tr>
<td>Fourth forbidden</td>
<td>3, 4, 5</td>
<td>0, 1</td>
<td>no</td>
<td>22–24</td>
</tr>
</tbody>
</table>

Systematic Uncertainties in the Analysis of the Reactor Neutrino Anomaly
A. Hayes et al., PRL 112, 202501 (2014)
"...the corrections are nuclear-operator dependent and that an undetermined combination of matrix elements contributes to non-unique forbidden transitions."

Also: r-process simulations....
Beta decay: electron capture

Electron capture leads to a vacancy being created in one of the strongest bound atomic states, and secondary processes are observed such as the emission of X-rays and Auger electrons. Auger electrons are electrons emitted from one of the outer electron shells and take away some of the remaining energy.

Capture is most likely for a 1s-state electron. The K-electron wave function at the origin is maximal and is given by

$$\psi_e(0) = \frac{1}{\sqrt{\pi}} \left( \frac{Zm_e e^2}{\hbar^2} \right)^{3/2}$$

The electron capture probability is thus given by:

$$W_{EC} = E_v^2 \frac{M_i}{\pi^2 \hbar^4 c^3} \left( \frac{Zm_e e^2}{\hbar^2} \right)^3$$

Example

$$^7_4\text{Be} + e^- \rightarrow ^7_3\text{Li} + \nu_e$$
