Beta decay: nuclear matrix elements

$$V_{\text{int}} \approx g\delta\left(\vec{r}_n - \vec{r}_p\right)\delta\left(\vec{r}_n - \vec{r}_{e^-}\right)\delta\left(\vec{r}_n - \vec{r}_{\bar{\upsilon}}\right)\hat{O}(n \to p)$$ zero-range

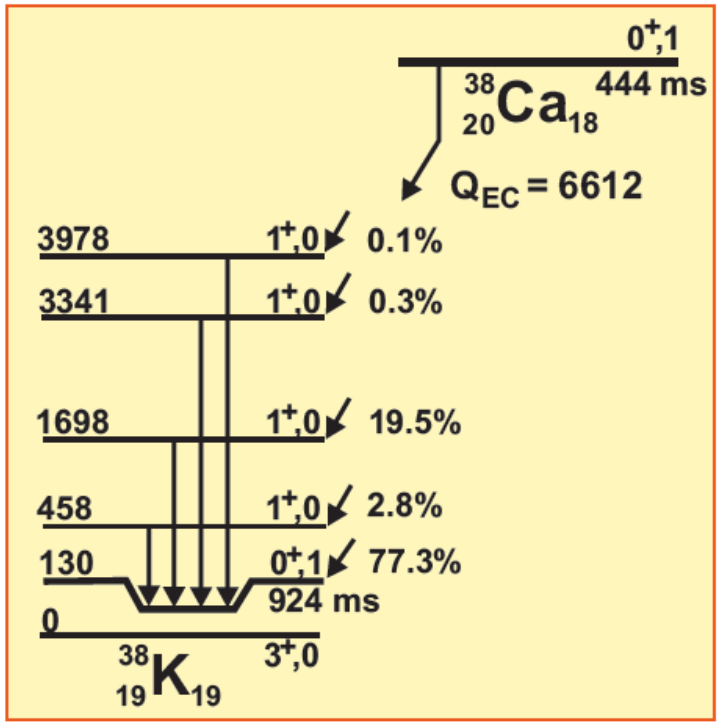
The nuclear operator transforming a neutron into a proton must be one body in nature. Hence it must involve the isospin raising or lowering operators.
In the non-relativistic limit, the vector part may be represented by the unity operator times $\tau_\pm$ and the axial-vector part by a product of $\tau_\pm$ and σ. (A proper derivation requires manipulation with Dirac 4-component functions and γ matrices!)

$$V_{\text{int}} \to G_V \sum_{j=1}^{A}\left[\tau_\pm(j) + g_A\vec{\sigma}(j)\cdot\vec{\tau}(j)\right]$$

Fermi decay, carries zero angular momentum

Gamow-Teller decay, carries one unit of angular momentum

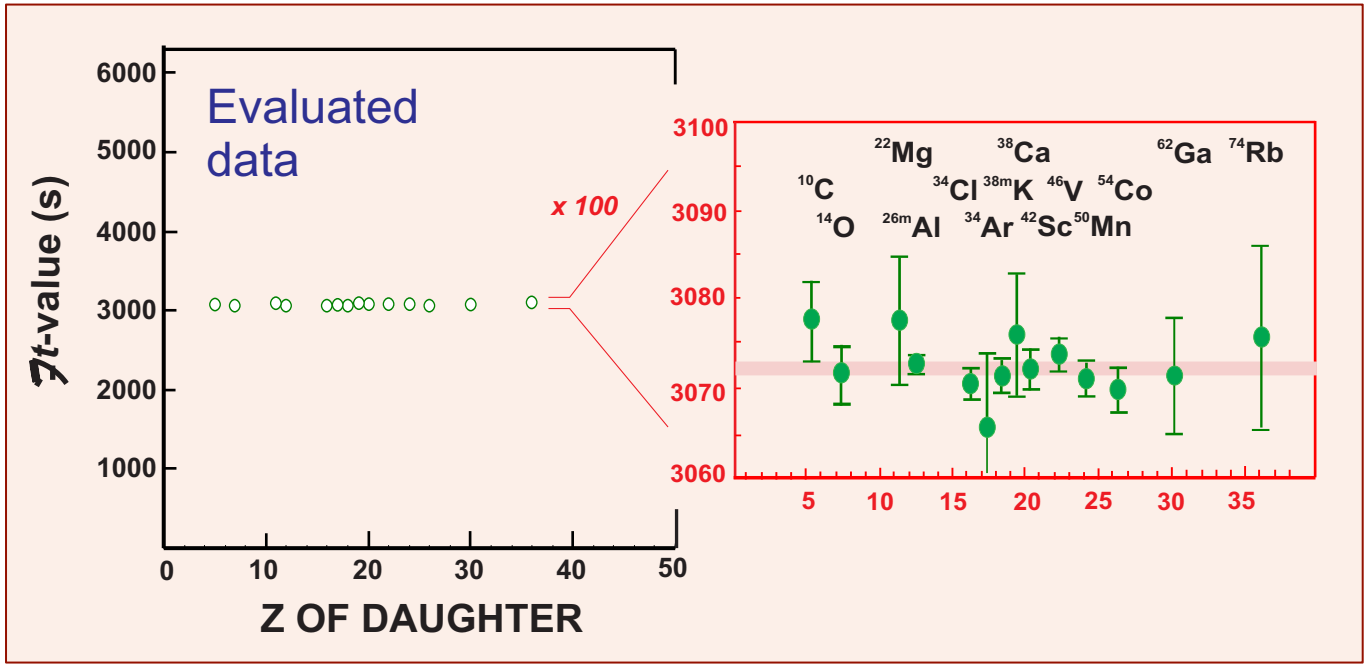
G_V determined from superallowed Fermi beta decays!



From the expression for *fT*, it is possible to determine the strength *g* of the beta-decay process, if one knows how to determine the reduced matrix element. For superallowed $0^+ \rightarrow 0^+$ Fermi transitions, the matrix element is √2 so the *fT* values should be identical.

Superallowed 0+ → 0+ beta decay between T=1 analog states

corrected for nuclear structure effects (isospin mixing) and radiative corrections



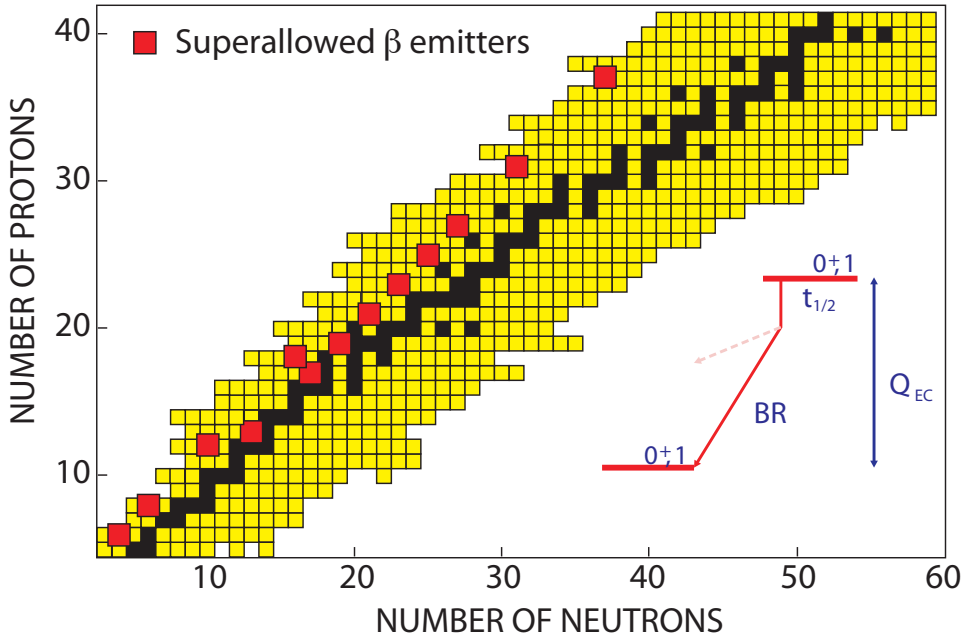
$$g = 0.88 \times 10^{-4} \text{ MeV fm}^3$$

or, introducing the dimensionless constant *G*:

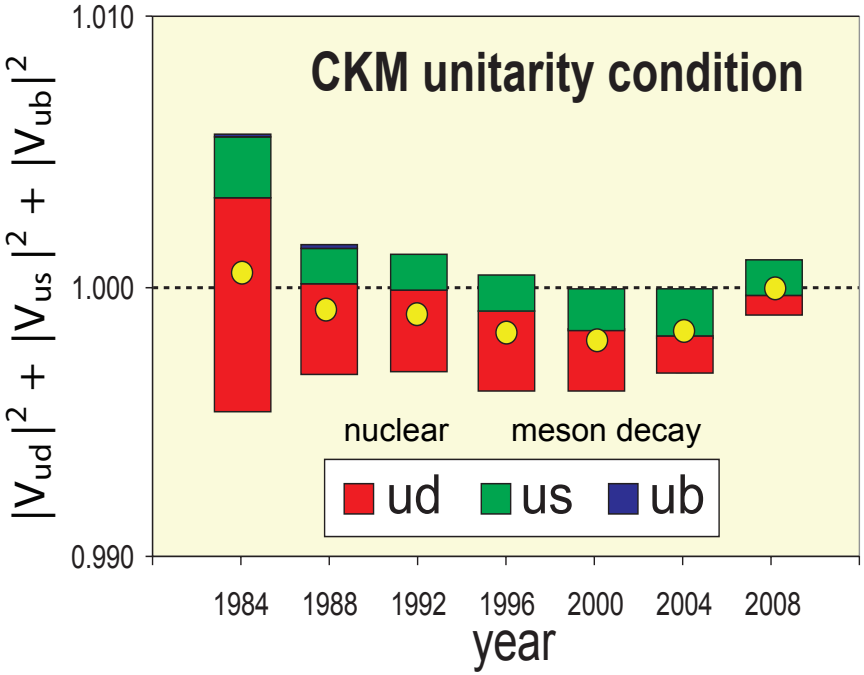
$$G = g\frac{m_e^2 c}{\hbar} = 1.026 \times 10^{-5}$$

Superallowed Fermi $0^+ \rightarrow 0^+$ β-decay studies

Impressive experimental effort worldwide



Kobayashi and Maskawa (2008): … for "the discovery of the origin of broken symmetry, which predicts the existence of at least three families of quarks in nature."



Hardy and Towner survey (Feb. 2015)

http://journals.aps.org/prc/abstract/10.1103/PhysRevC.91.025501

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99992 \pm 0.00048$$

V_{ud}^2 nuclear decays muon decay: 0.94900 ± 0.00042

V_{us}^2 kaon decays: 0.05090 ± 0.00022

V_{ub}^2 B decays: 0.00002 ± 0.00001

Beta decay: allowed transitions

Fermi transitions

$$\left\langle J_f M_f T_f T_{0f} \middle| T_\mp \middle| J_i M_i T_i T_{0i} \right\rangle = \sqrt{T_i(T_i+1) - T_{0i}(T_{0i} \mp 1)}\,\delta_{J_i J_f}\,\delta_{M_i M_f}\,\delta_{T_i T_f}\,\delta_{T_{0i} \mp 1 T_{0f}}$$

In reality, isospin is violated by the electromagnetic force, but the violation is weak.

$J_f = J_i$	$(\Delta J = 0)$
$T_f = T_i \neq 0$	$(\Delta T = 0,\ \text{but } T_i = 0 \rightarrow T_f = 0 \text{ forbidden})$
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

$$fT = \frac{const}{\langle F \rangle^2 + g_A^2 \langle GT \rangle^2}$$

Gamow-Teller transitions

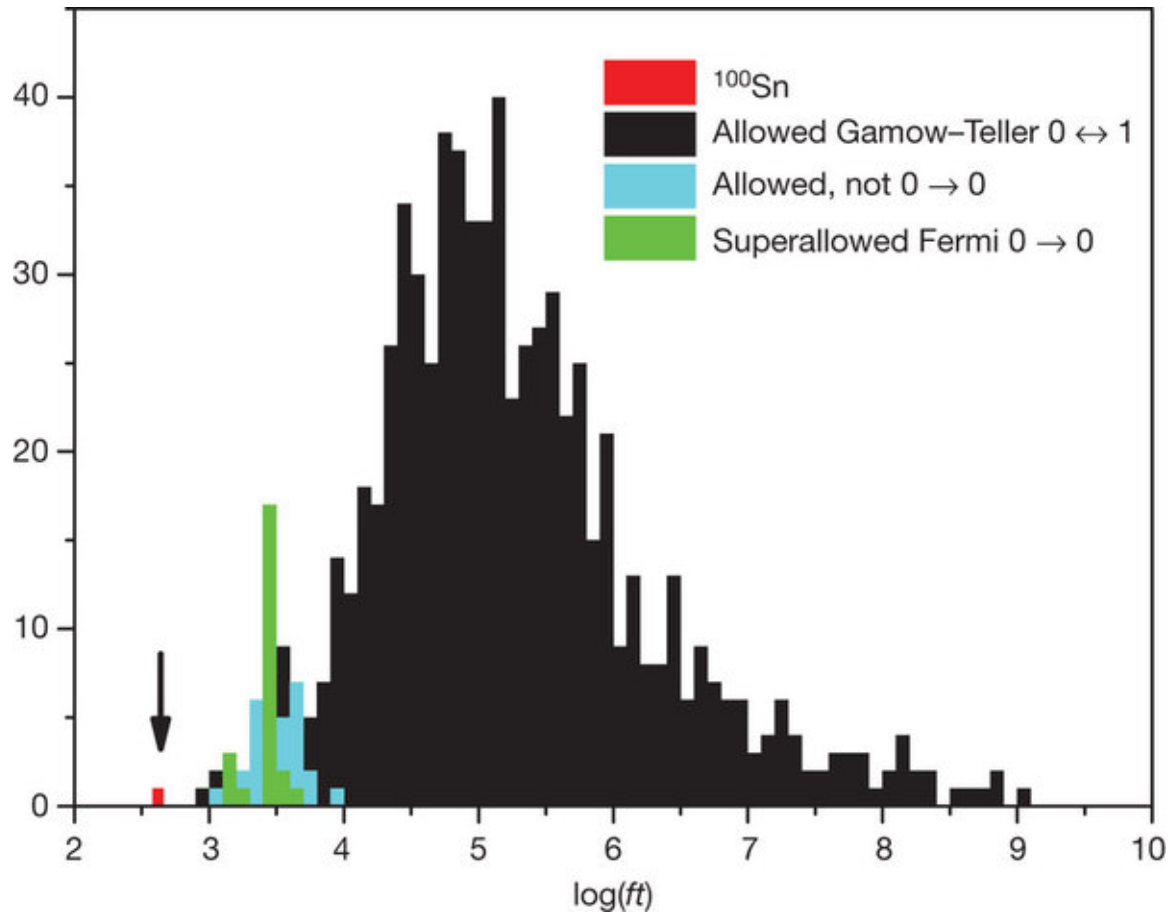
The matrix element strongly depends on the structure of the wave function!

$\Delta J = 0,1$	but $J_i = 0 \rightarrow J_f = 0$ forbidden
$\Delta T = 0,1$	but $T_i = 0 \rightarrow T_f = 0$ forbidden
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

The absolute values of GT matrix elements are generally smaller than those for Fermi transitions.

Superallowed Gamow-Teller decay of the doubly magic nucleus ^{100}Sn

Hinke et al., *Nature* **486**, 341 (2012)

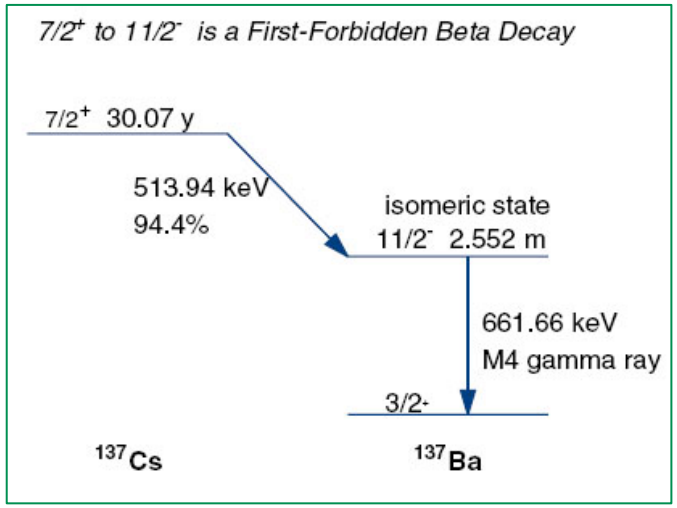


Number distribution of log(*ft*) values for allowed β-transitions (obeying the selection rules).

Forbidden transitions

Forbidden transitions involve parity change and a spin change of more than one unit. They come from the higher-order terms in the expansion of electron and neutrino plane waves into spherical harmonics. Forbidden decays are classified into different groups by the *L*-value of the spherical harmonics involved.

Decay type	ΔJ	ΔT	$\Delta\pi$	$\log_{10} ft_{1/2}$
Superallowed	$0^+ \rightarrow 0^+$	0	no	3.1–3.6
Allowed	0, 1	0, 1	no	2.9–10
First forbidden	0, 1, 2	0, 1	yes	5–19
Second forbidden	1, 2, 3	0, 1	no	10–18
Third forbidden	2, 3, 4	0, 1	yes	17–22
Fourth forbidden	3, 4, 5	0, 1	no	22–24



Systematic Uncertainties in the Analysis of the Reactor Neutrino Anomaly

A. Hayes et al., PRL 112, 202501 (2014)

"...the corrections are nuclear-operator dependent and that an undetermined combination of matrix elements contributes to non-unique forbidden transitions."

Also: r-process simulations....

Beta decay: electron capture

Electron capture leads to a vacancy being created in one of the strongest bound atomic states, and secondary processes are observed such as the emission of X-rays and Auger electrons. Auger electrons are electrons emitted from one of the outer electron shells and take away some of the remaining energy.

Capture is most likely for a 1s-state electron. The K-electron wave function at the origin is maximal and is given by

$$\psi_{e^-}(0) = \frac{1}{\sqrt{\pi}}\left(\frac{Zm_e e^2}{\hbar^2}\right)^{3/2}$$

The electron capture probability is thus given by:

$$\mathcal{W}_{EC} = E_\nu^2 \frac{\left|M'_{fi}\right|^2 g^2}{\pi^2 \hbar^4 c^3}\left(\frac{Zm_e e^2}{\hbar^2}\right)^3$$

Example $\quad {}^7_4\mathrm{Be} + \mathrm{e}^- \rightarrow {}^7_3\mathrm{Li} + \nu_e$ http://www.nndc.bnl.gov/chart/reCenter.jsp?z=4&n=3

Manipulating lifetimes in storage rings: http://www.sciencedirect.com/science/article/pii/S0146641013000744

Decay constants for β_c^+, EC decays and atomic losses for bare, H-like, He-like and neutral ^{140}Pr and ^{142}Pm ions together with the half-lives $T_{1/2}$ derived from time-resolved Schottky spectrometry.

Ion	λ_{β^+} (s^{-1}) · 10^{-3}	λ_{EC} (s^{-1}) · 10^{-3}	λ_{loss} (s^{-1}) · 10^{-3}	$\lambda_{\beta^+} + \lambda_{EC}$ (s^{-1}) · 10^{-3}	$T_{1/2}$ (s)	Ref.
^{140}Pr^{59+}	1.58(8)	–	0.4(1)	1.58(8)	439(22)	[155]
^{140}Pr^{58+}	1.61(10)	2.19(6)	0.4(1)	3.80(12)	182(6)	[155]
^{140}Pr^{57+}	1.54(11)	1.47(7)	0.4(1)	3.01(13)	230(10)	[155]
^{140}Pr^{0+}	1.74(5)	1.65(5)	–	3.39(7)	203(4)	[217,227]
^{142}Pm^{61+}	12.3(7)	–	0.5(1)	12.3(7)	56.4(32)	[157]
^{142}Pm^{60+}	12.6(3)	5.1(1)	0.5(1)	17.7(3)	39.2(7)	[157]
^{142}Pm^{59+}	13.9(6)	3.6(1)	0.5(1)	17.5(6)	39.6(14)	[157]
^{142}Pm^{0+}	13.2(5)	3.9(5)	–	17.1 (7)	40.5(17)	[217,227]