Beta decay: energy relations

atomic mass $$Mc^2 = M'c^2 + Zm_ec^2 - B_{el}$$

nuclear mass

electron binding energy

P(arent) D(aughter)

a) β⁻ decay $${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}X_{N-1} + e^- + \bar{\nu}_e$$

Nuclear recoil is very small

$$Q_{\beta^-} = T_{e^-} + T_{\bar{\nu}_e} = M'_Pc^2 - M'_Dc^2 - m_ec^2$$

In the following, we assume that the neutrino mass is ~zero and that the very small differences in electron binding energy between the parent and daughter atoms can be neglected. This gives: $$Q_{\beta^-} = M_Pc^2 - M_Dc^2$$

Consequently, the β^- decay process is possible whenever $M_P > M_D$

b) β⁺ decay $${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}X_{N+1} + e^+ + \nu_e$$

$$Q_{\beta^+} = T_{e^+} + T_{\nu_e} = M'_Pc^2 - M'_Dc^2 - m_ec^2$$

$$= M_Pc^2 - \left(M_Dc^2 + 2m_ec^2\right)$$

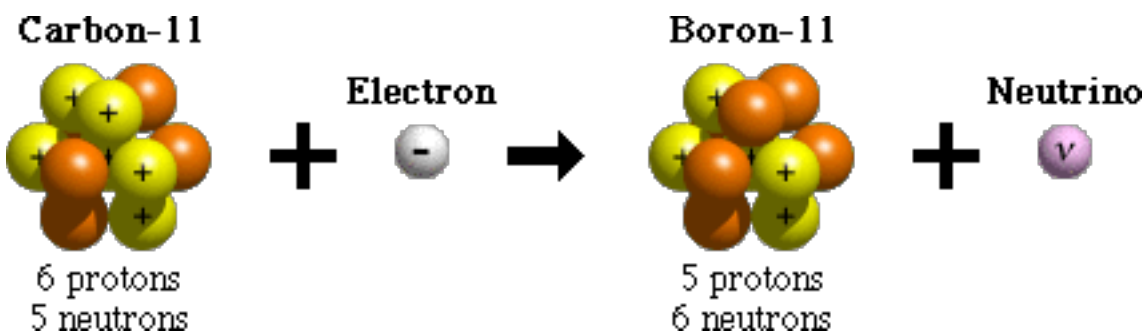
Consequently, the β^+ decay process has a *threshold* $2m_ec^2$

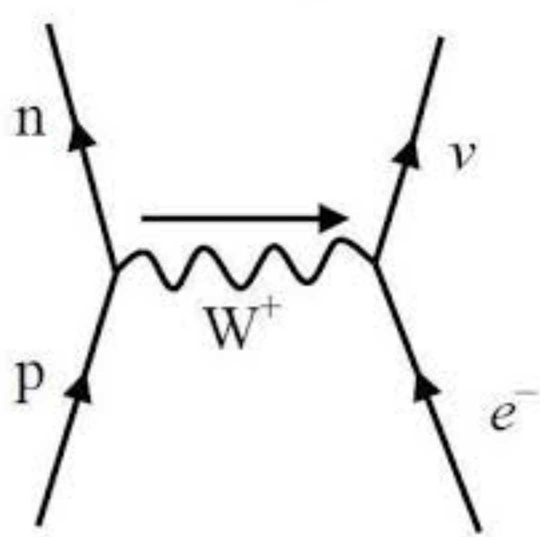
c) Electron capture (inverse beta decay)

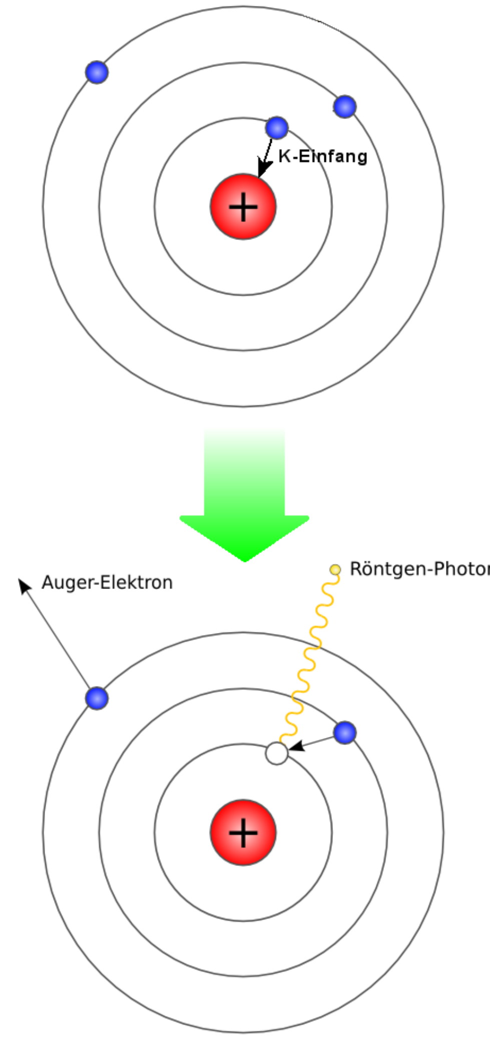
$${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}X_{N+1} + \nu_{e}$$

$$Q_{EC} = M_{P}c^{2} - \left(M_{D}c^{2} - B_{el}\right)$$

Atomic electron is captured by a proton. This process leaves the atom in an excited state: a vacancy has been created! The vacancy is quickly filled by producing the characteristic X-ray cascade

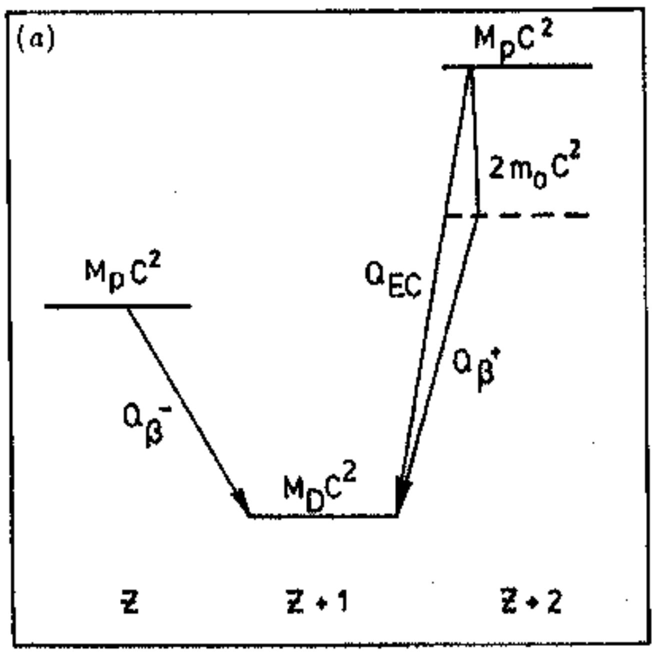


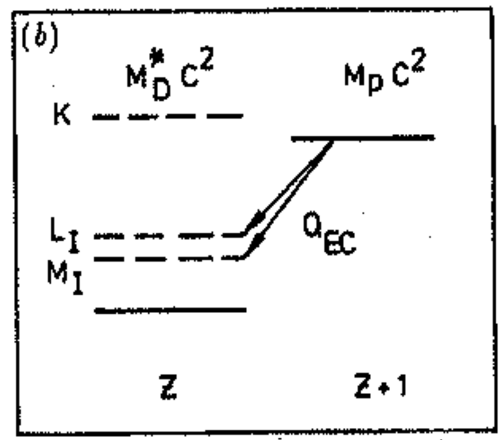




Decay	Type	Q (MeV)	T
$^{23}Ne \to {}^{23}Na + e^- + \bar{\nu}_e$	β^-	4.38	38 s
$^{99}Tc \to {}^{99}Ru + e^- + \bar{\nu}_e$	β^-	0.29	2.1×10^5 y
$^{25}Al \to {}^{25}Mg + e^+ + \nu_e$	β^+	3.26	7.2 s
$^{124}I \to {}^{124}Te + e^+ + \nu_e$	β^+	2.14	4.2 s
$^{15}O + e^- \to {}^{15}N + \nu_e$	EC	2.75	1.22 s
$^{41}Ca + e^- \to {}^{41}K + \nu_e$	EC	0.43	1.0×10^5 y

examples…





mass relationship in electron capture between the parent and daughter atom

energy relations in various beta decay processes

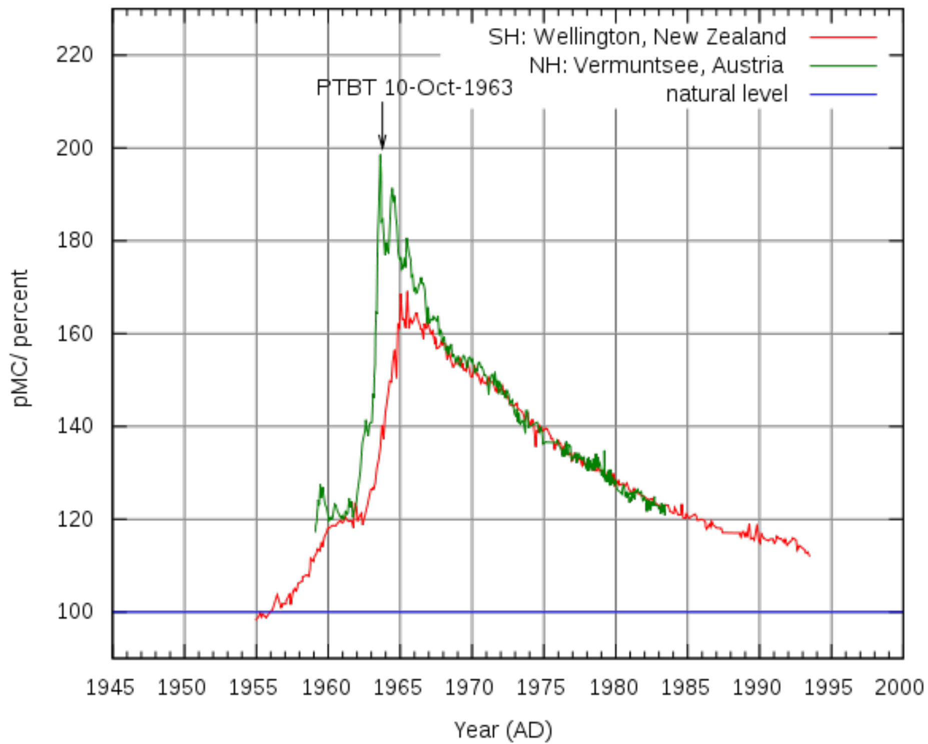
β^+ decay can occur when the mass of parent atom exceeds that of daughter atom by at least twice the mass of the electron

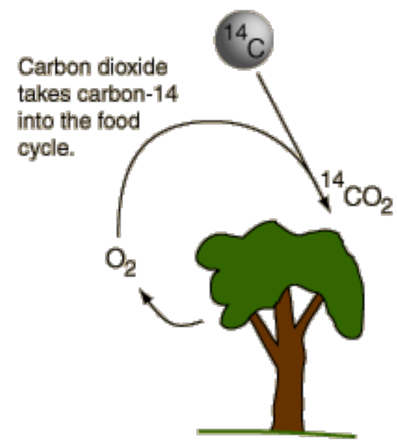
Radiocarbon dating

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + e^{-} + \bar{\nu}_e$$

half-life of 5730 years

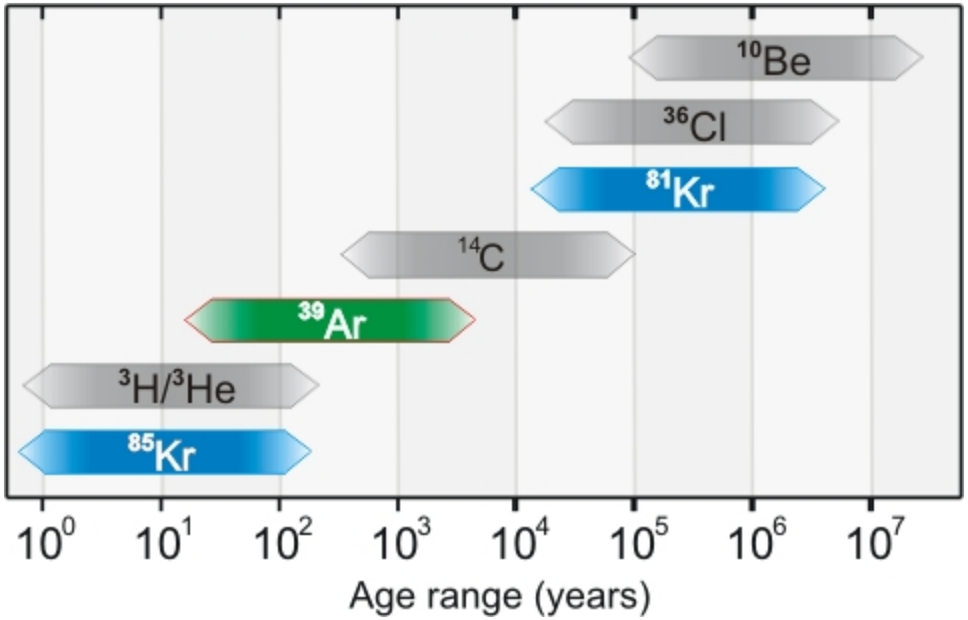
Radiocarbon dating is a radiometric dating method that uses ^{14}C to determine the age of carbonaceous materials up to about 60,000 years old. The technique was developed by Libby and his colleagues in 1949. In 1960, Libby was awarded the Nobel Prize in chemistry for this work. The level of ^{14}C in plants and animals when they die approximately equals the level of ^{14}C in the atmosphere at that time. However, it decreases thereafter from radioactive decay.





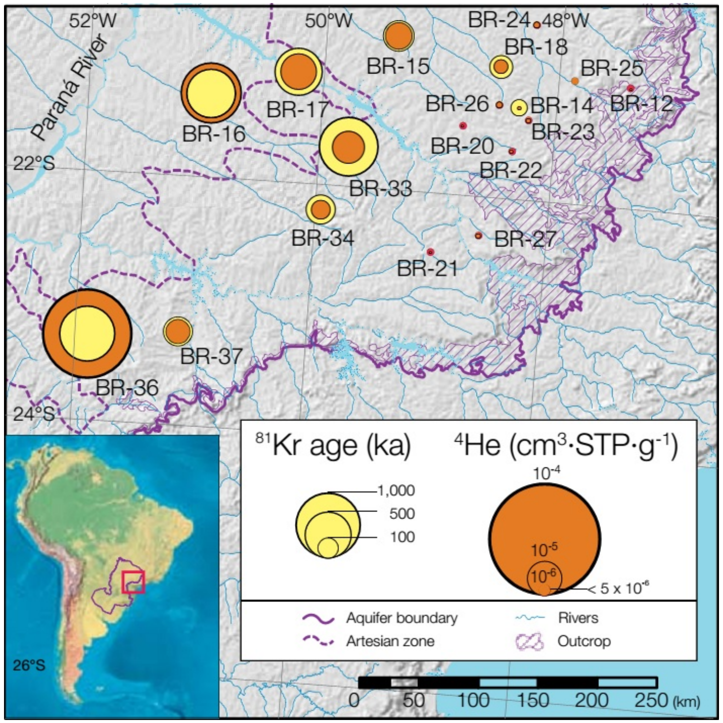
Atmospheric nuclear weapon tests almost doubled the concentration of ^{14}C in the Northern Hemisphere. The date that the Partial Test Ban Treaty (PTBT) went into effect is marked on the graph.

Radiokrypton dating



^{81}Kr half-life is $2.293 \cdot 10^5$y

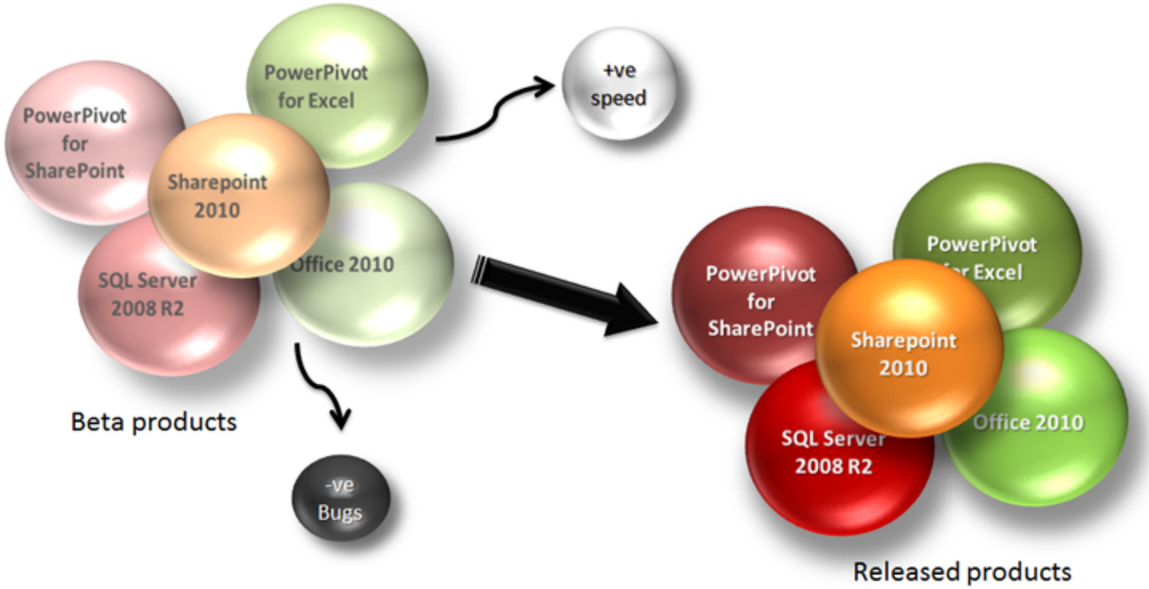
Guarani Aquifer, Brazil



https://www.phy.anl.gov/mep/atta/research/atta.html

Other applications ☺

http://blogs.technet.com/b/andrew/archive/2010/05/28/beta-decay.aspx



"I am pretty sure the term beta in software isn't related to atomic decay, but there are some similarities in that an atom that decays is unstable and decays after a period of time to something more stable e.g. Carbon14 to Nitrogen14. In the Microsoft world, the time to decay is usually 180 days (compared to a half life of 5,730 years for Carbon 14 to decay) and this results in fallout- the loss of bugs identified during the beat period, and some performance improvements and small enhancements leading to a very stable released product." (Andrew.Fryer)

Neutron beta decay

http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.111.222501

Astrophysicists rely on a precise value of the free neutron lifetime to calculate the rate of nucleosynthesis during the big bang, while particle physicists use it to constrain fundamental parameters of the standard model. Yet measured lifetimes have varied by about a percent (or 8 sec), depending on the experimental technique.

NIST: Phys. Rev. Lett. 111, 222501 (2013): T_n=(887.7±1.2[stat]±1.9[syst]) s

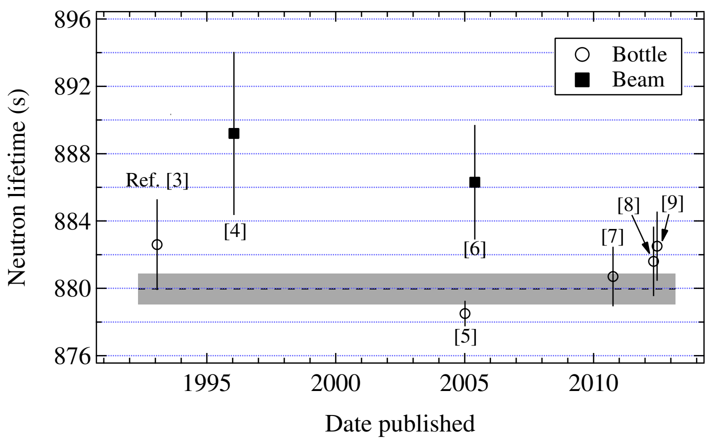


FIG. 1 (color online). The neutron lifetime measurements used in the 2013 PDG world average. The weighted mean and 1σ uncertainty (inflated by scale factor $\sqrt{\chi^2/\mathrm{d.o.f.}} = 1.53$, following PDG procedures) of the data set is represented by the dashed line and shaded band.

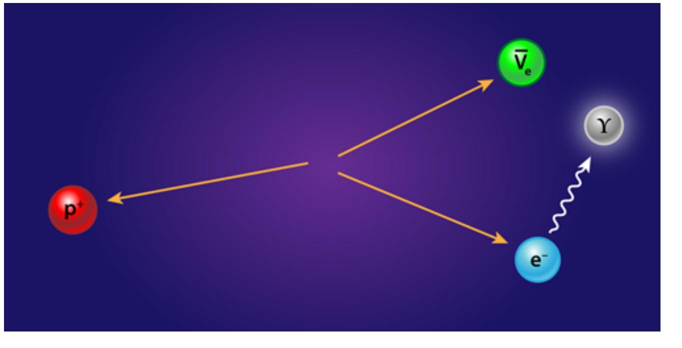
Neutron radiative beta decay

http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.116.242501

According to QED, the neutron decay also produces photons, most of which come from the deceleration of the emitted electron. Now the spectrum of these photons has been measured with the greatest precision to date in an experiment at the National Institute of Standards and Technology (NIST), Maryland. The measurement comes close to the level of precision needed to look for deviations from QED predictions, which would signal a break from standard model physics.

In 2006, the NIST team used this setup to measure the photon branching ratio (the fraction of radiation-producing decays) with an uncertainty of about 10% over a limited energy range. The new experiment reduces this uncertainty to less than 5% by measuring more neutrons, collecting more photons, and using new techniques to characterize the detectors. Based on 22 million electron-proton events, the researchers report an average branching ratio of 3.35×10^{-3} for product photons with energies between 14.1 and 782 keV. They are now working on reducing the experimental uncertainty to the 1% level needed to test predictions that go beyond QED.

http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.116.242501

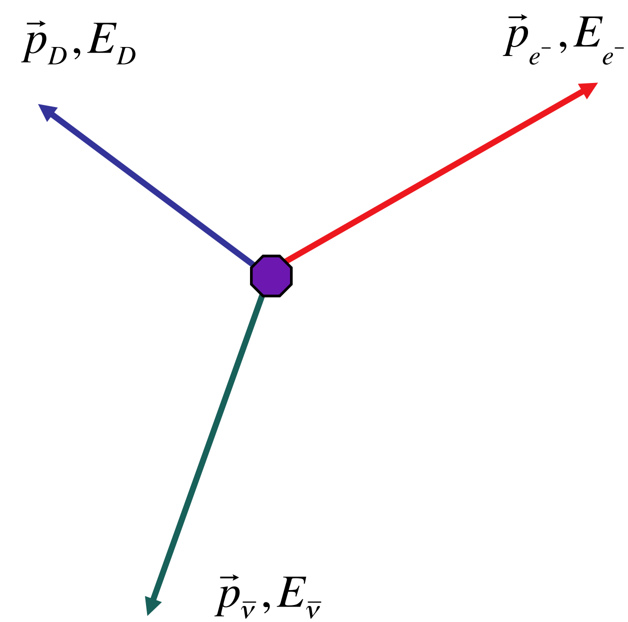


Beta decay: spectrum and lifetime

$$\mathcal{W}_{i\to f} = \frac{2\pi}{\hbar}\left|\left\langle \phi_f \left| V_{\text{int}} \right| \phi_i \right\rangle\right|^2 \frac{dn}{dE}\left(e^-, \bar{\nu}_e\right)$$

product wave function of the daughter nucleus, electron, and antineutrino

wave function of the parent nucleus



$$\boxed{\begin{aligned} &\vec{p}_D + \vec{p}_{e^-} + \vec{p}_{\bar{\nu}} = 0 \\ &T_D + T_{e^-} + T_{\bar{\nu}} = Q \end{aligned}}$$

The expression for the density of final states of en electron emitted with a given energy and momentum (integrated over all angles) is:

$$\frac{dn}{dE}\left(e^-, \bar{\nu}_e\right) = \frac{V^2}{4\pi\hbar^6 c^3} p_{e^-}^2 \left(E - E_{e^-}\right)^2 \sqrt{1 - \frac{m_{\bar{\nu}}^2 c^4}{\left(E - E_{e^-}\right)^2}}\, dp_{e^-}$$

$$V_{\text{int}} \approx g\delta\left(\vec{r}_n - \vec{r}_p\right)\delta\left(\vec{r}_n - \vec{r}_{e^-}\right)\delta\left(\vec{r}_n - \vec{r}_{\bar{\nu}}\right)\hat{O}(n \to p)$$ zero-range