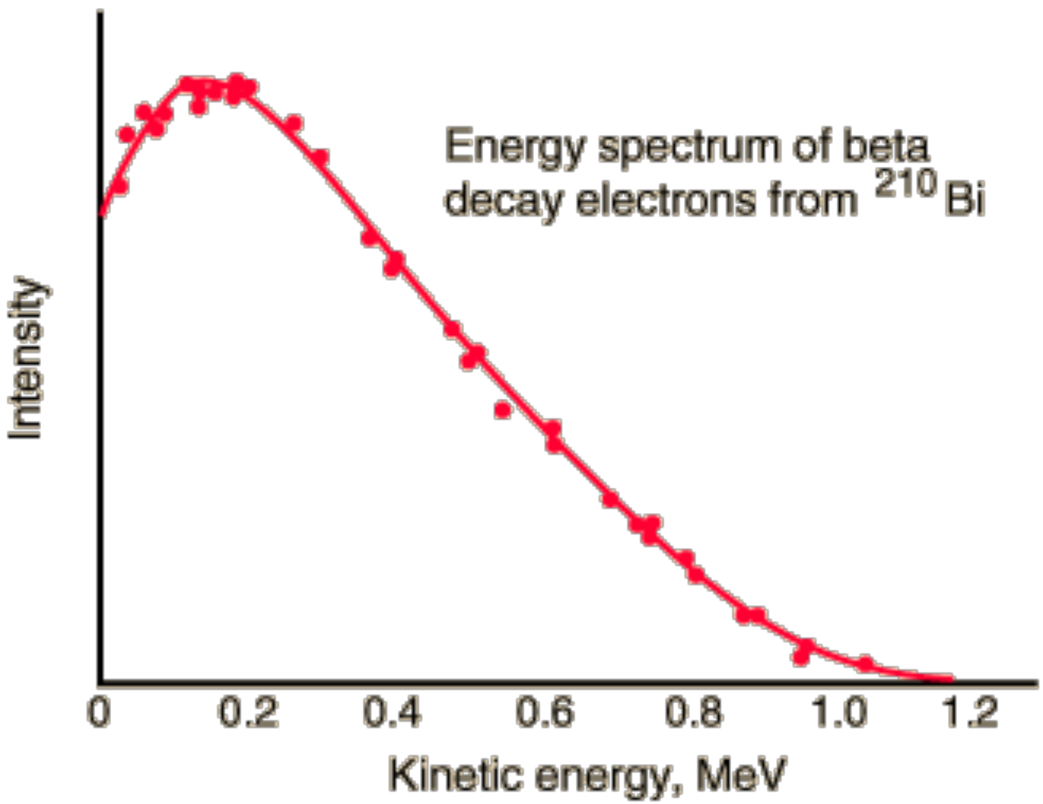
Depends on nuclear wave functions

$$\mathcal{W}_{i\to f}\left(p_{e^-}\right)dp_{e^-}=\frac{\left|M'_{fi}\right|^2}{2\pi^3\hbar^7c^3}F\left(Z_D,p_{e^-}\right)p_{e^-}^2\left(Q-E_{e^-}\right)^2\sqrt{1-\frac{m_{\bar{\nu}}^2c^4}{\left(Q-E_{e^-}\right)^2}}dp_{e^-}$$

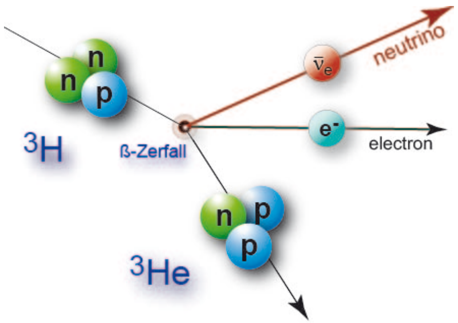
Fermi function: $F\left(Z,p_{e^-}\right)=\dfrac{2\pi\eta}{1-e^{-2\pi\eta}}$, $\eta\equiv\pm\dfrac{Ze^2}{\hbar v_{e^-}}$ positive (negative) sign used for β^- (β^+) decay



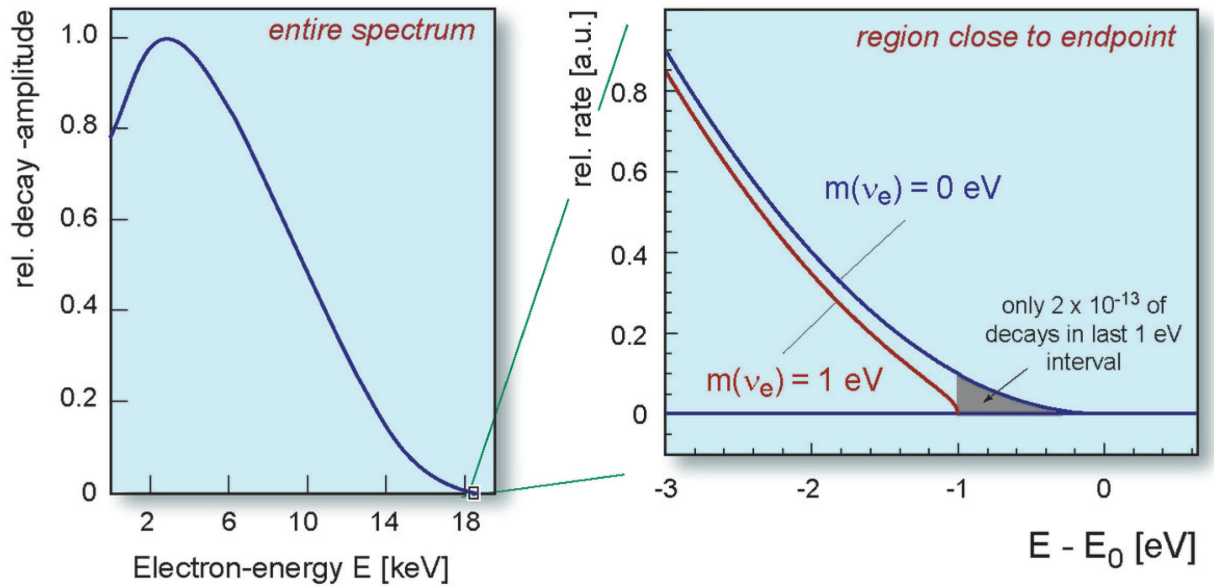
KArlsruhe **TRI**tium **N**eutrino (KATRIN) neutrino experiment

http://www.katrin.kit.edu

$T_{1/2}$=12.32 y



Deviations around the endpoint due to nonzero neutrino mass…



https://www.youtube.com/watch?v=iqkpjEI-UMo

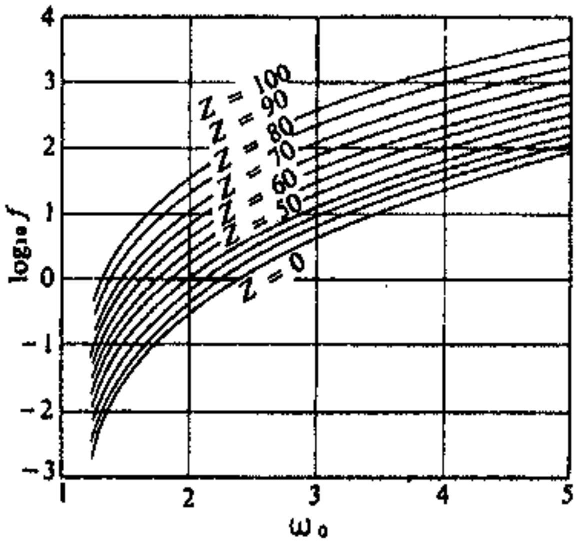
If we assume that the matrix element does not depend on E_{e^-}, and after taking out the strength g of the weak interaction, one obtains:

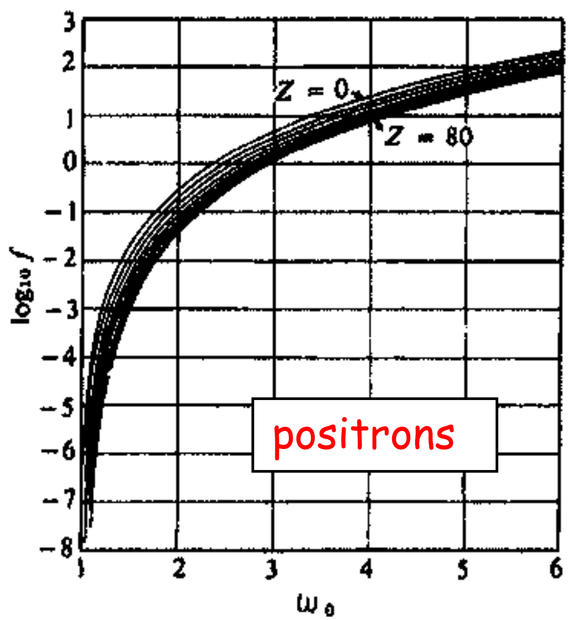
$$fT = 0.693 \frac{2\pi^3 \hbar^7}{g^2 m_e^5 c^4 \left| \hat{M}'_{fi} \right|^2}, \quad \hat{M}'_{fi} \equiv g M'_{fi}$$

$$f(Z_D, w_0) = \int_1^{w_0} F\left(Z_D, \sqrt{w^2 - 1}\right) \sqrt{w^2 - 1} (w_0 - w)^2 w dw$$ *f*-function

where $w = E_{e^-}/m_e c^2$ and w_0 is the reduced max. electron energy.

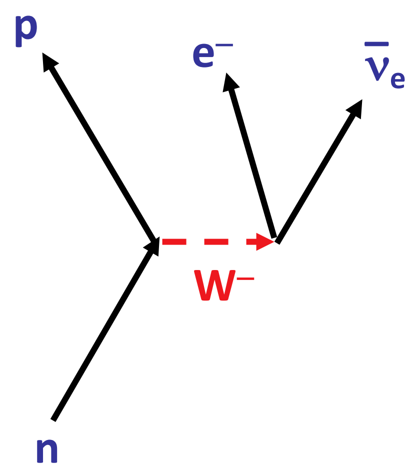
electrons

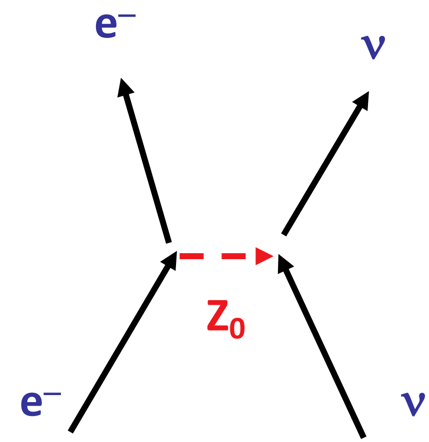




Beta decay: microscopic view

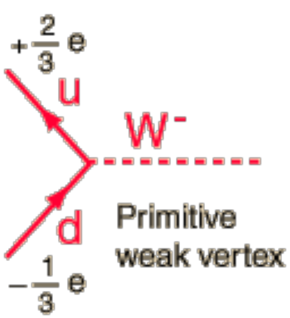
On a more fundamental level, beta decay of hadrons can be viewed as the transformation of one type of quark to another through exchange of charged weak currents (W bosons carry net charges; Z boson is neutral - it is the source of <u>neutral weak current</u>).

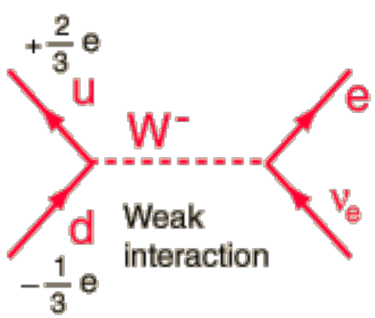


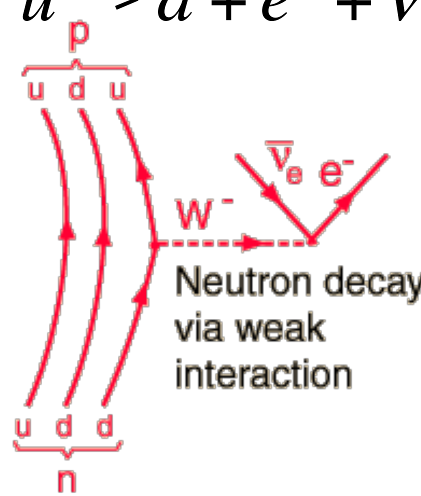


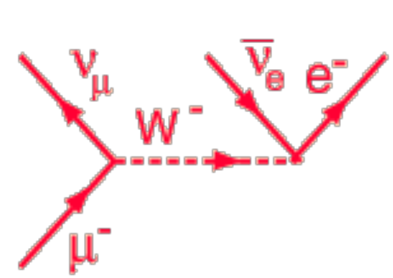
The flavor of quarks is conserved in strong interactions. However, weak interactions change flavor!

$$d \rightarrow u + e^- + \bar{\nu}_e \qquad u \rightarrow d + e^+ + \nu_e$$



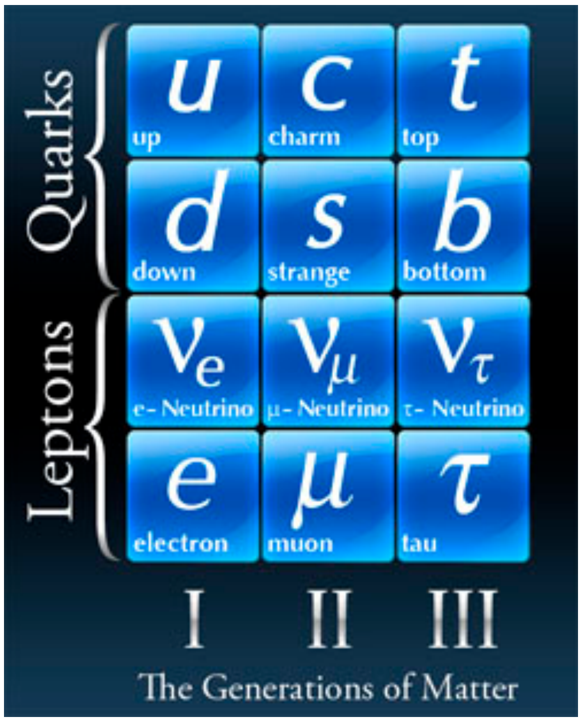






Three generations of matter

There are three "sets" of quark pairs and lepton pairs. Each "set" of these particles is called a generation, or family. The up/down quarks are first generation quarks, while the electron/electron neutrino leptons are first generation leptons.



up type (q=+2/3e)

down type (q=-1/3e)

neutral

charged (q=-e)

mass

particles of higher generations to decay to the first generation (everyday matter is made of particles from the first generation)

$$K^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$$
$$(s\bar{u})$$

http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.109.241802

The result of the statistical analysis is that the existence of further fermions can be excluded with a probability of 99.99999 percent (5.3 sigma). The most important data used for this analysis come from the recently discovered Higgs particle.

12 matter particles suffice in nature?

http://phys.org/news/2012-12-particles-suffice-nature.html

Beta decay: mass and weak eigenstates

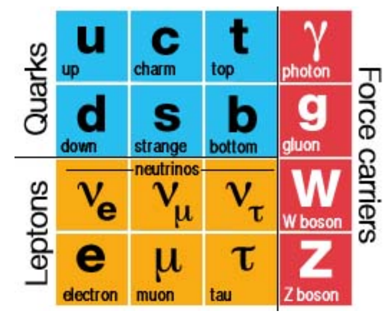
When a quark beta-decays, the new quark does not have a definite flavor. For instance (Cabibbo 1963):

$$u \rightarrow d' = d\cos\theta_c + s\sin\theta_c$$

Cabibbo angle θ_c=13.02°

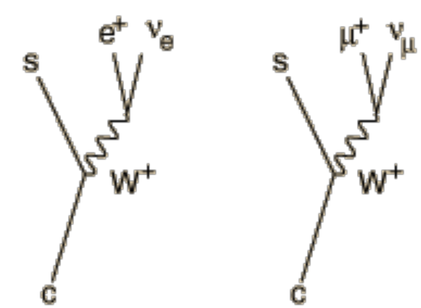
However, the observed weak transitions are between quarks of definite flavor. In general, the strong-interaction quark eigenstates:

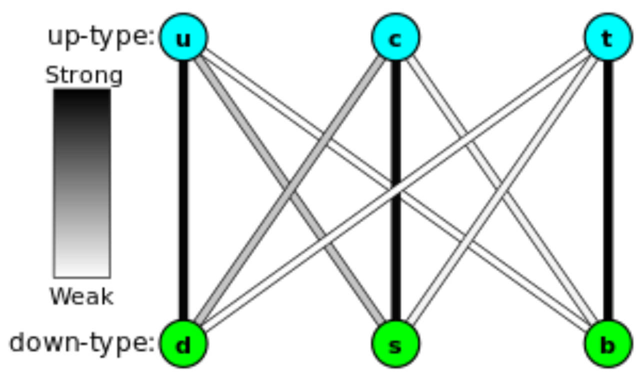
$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}$$



are different from weak interaction eigenstates:

$$\begin{pmatrix} u \\ d' \end{pmatrix}\begin{pmatrix} c \\ s' \end{pmatrix}\begin{pmatrix} t \\ b' \end{pmatrix}$$





A quark of charge +2/3 (u,c,t) is always transformed to a quark of charge -1/3 (d,s,b) and vice versa. This is because the transformation proceeds by the exchange of charged W bosons, which must change the charge by one unit.

Beta decay: CKM Matrix

Kobayashi and Maskawa generalized the Cabbibo matrix into the Cabibbo–Kobayashi–Maskawa matrix (or CKM matrix) to keep track of the weak decays of three generations of quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix

weak eigenstates mass eigenstates

The choice of usage of down-type quarks in the definition is purely arbitrary and does not represent some sort of deep physical asymmetry between up-type and down-type quarks.

The current determination:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{bmatrix}.$$

For nuclear beta-decay, we are mainly concerned with the transition between u- and d-quarks. As a result, only the product

$$G_V = G_F \cos\theta_c$$

enters into the process.

HW: Consider the isobaric mass chain A=141. Using NNDC, determine what types of ground-state beta decays are possible for different elements within this chain (β^-, β^+, EC, etc.) as well as the nature (allowed, first-forbidden, etc.)

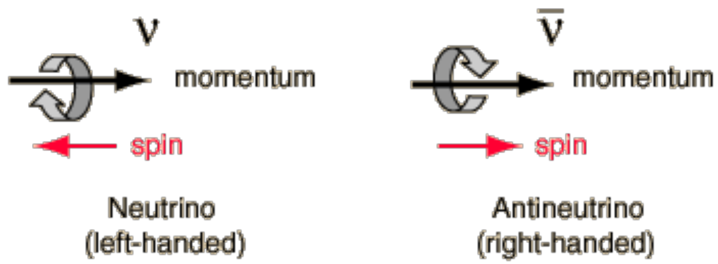
Beta decay: helicity

What are the consequences of parity violation in beta decay?

$$h = \frac{\vec{\sigma} \cdot \vec{p}}{p}$$ helicity (helicity flips under parity)

The eigenvalue of *h* is *v/c*. For a massless particle, the eigenvalues of *h* can be only +1 or -1. In general, the particle with

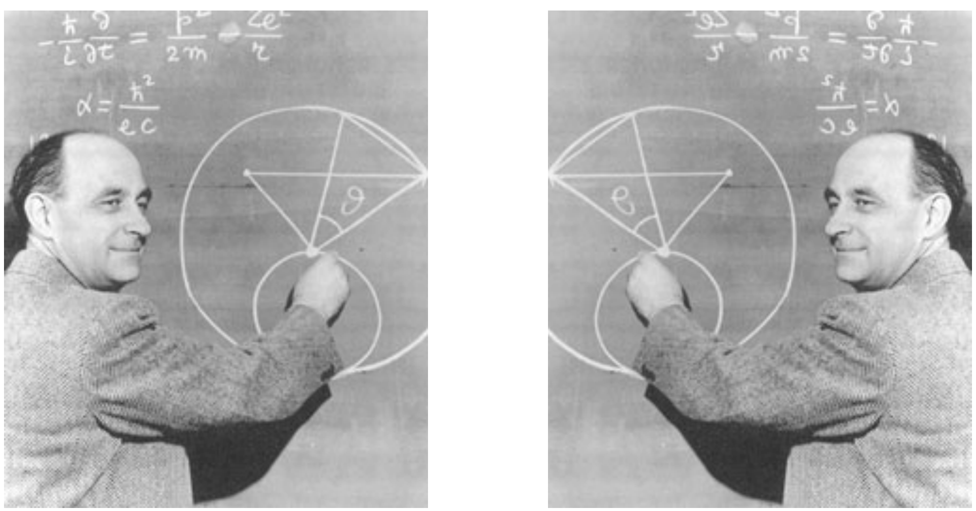
- *h>0* is called "right-handed"
- *h*<0 is called "left-handed"



Experimentally, $h(\nu_e) \approx -1, \quad h(\bar{\nu}_e) \approx +1$

http://journals.aps.org/pr/abstract/10.1103/PhysRev.109.1015

http://hyperphysics.phy-astr.gsu.edu



left-handed or right-handed?

The global characterization in terms of handedness is not meaningful for other particles, like electrons. An electron could have spin to the right and be traveling right and therefore be classified as right-handed. But from the reference frame of someone traveling *faster* than the electron, its velocity would be to the left, while its spin would be unchanged. This would mean that the electron is a left-handed particle with respect to that reference frame.