Beta decay: axial vector coupling constant

The operators that are scalars, pseudoscalars and tensors produce leptons of both helicities under a parity transformation. Only vector operators *V* and axial vector operators *A* can accommodate the observed result. Furthermore, since *V* and A are of different parity, they must appear in a linear combination. This leads to the *V-A* theory of beta decay. In principle, both *V* and *A* parts should be characterized by different coupling constants, G_V and G_A, respectively. In this theory, the weak interaction acts only on left-handed particles (and right-handed antiparticles). Since the mirror reflection of a left-handed particle is right-handed, this explains the maximal violation of parity.

The vector current is known to be a conserved quantity (CVC hypothesis)

How to relate G_V and G_A?

pion decay constant → f_π ; pion-nucleon coupling constant → $g_{\pi n}$

$$g_A \equiv \frac{G_A}{G_V} = \frac{f_\pi g_{\pi n}}{M_N c^2}$$

Goldberger-Treiman relation

Experimentally, g_A=1.267(4). This value is very close, up to 3%, to the Goldberger-Treiman estimate. This relation can be obtained by assuming the so-called partially conserved axial-vector current (PCAC) hypothesis.

Now we are ready to estimate the nuclear operator!

Beta decay: nuclear matrix elements

$$V_{\text{int}} \approx g\delta\left(\vec{r}_n - \vec{r}_p\right)\delta\left(\vec{r}_n - \vec{r}_{e^-}\right)\delta\left(\vec{r}_n - \vec{r}_{\bar{v}}\right)\hat{O}(n \to p)$$ zero-range

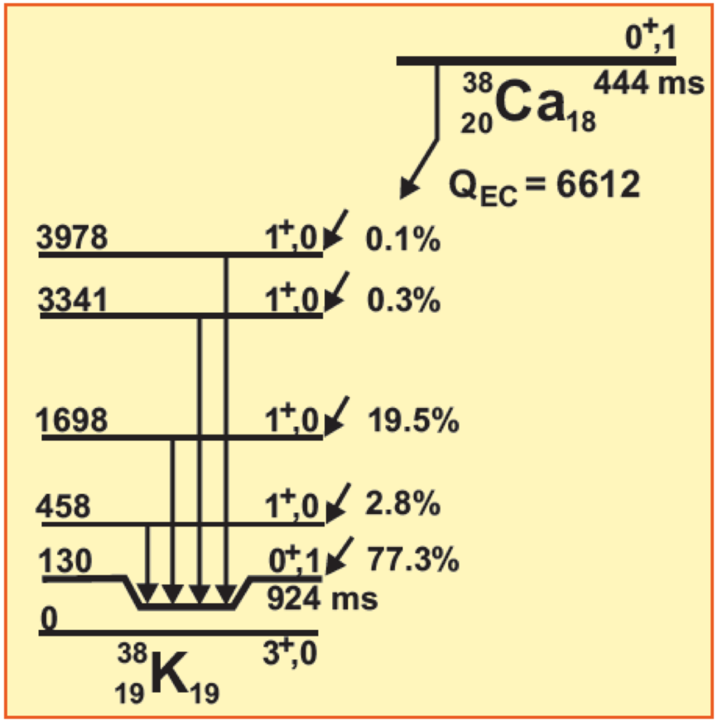
The nuclear operator transforming a neutron into a proton must be one body in nature. Hence it must involve the isospin raising or lowering operators.
In the non-relativistic limit, the vector part may be represented by the unity operator times $\tau_\pm$ and the axial-vector part by a product of $\tau_\pm$ and $\boldsymbol{\sigma}$. (A proper derivation requires manipulation with Dirac 4-component functions and γ matrices!)

$$V_{\text{int}} \to G_V \sum_{j=1}^{A} \left[\tau_\pm(j) + g_A \vec{\sigma}(j) \cdot \vec{\tau}(j)\right]$$

Fermi decay, carries zero angular momentum

Gamow-Teller decay, carries one unit of angular momentum

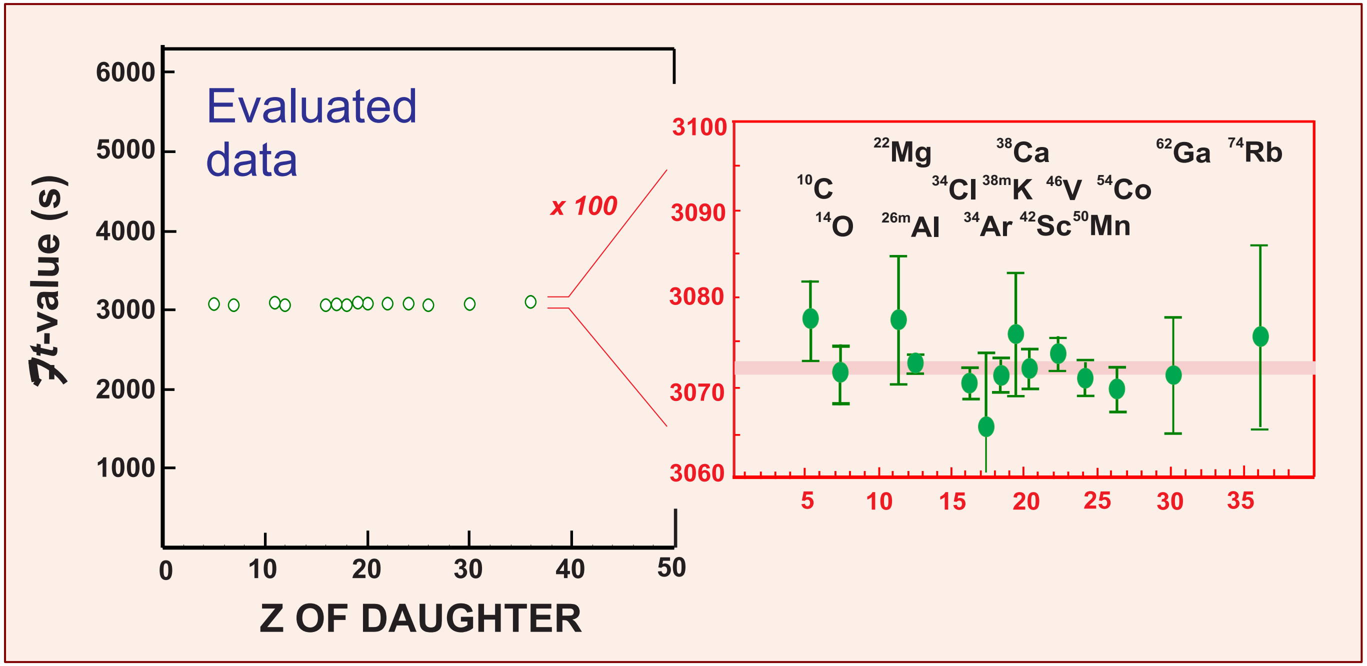
G_V determined from superallowed Fermi beta decays!



From the expression for fT, it is possible to determine the strength g of the beta-decay process, if one knows how to determine the reduced matrix element. For superallowed $0^+ \rightarrow 0^+$ Fermi transitions, the matrix element is $\sqrt{2}$ so the fT values should be identical.

Superallowed 0+ → 0+ beta decay between T=1 analog states

corrected for nuclear structure effects (isospin mixing) and radiative corrections

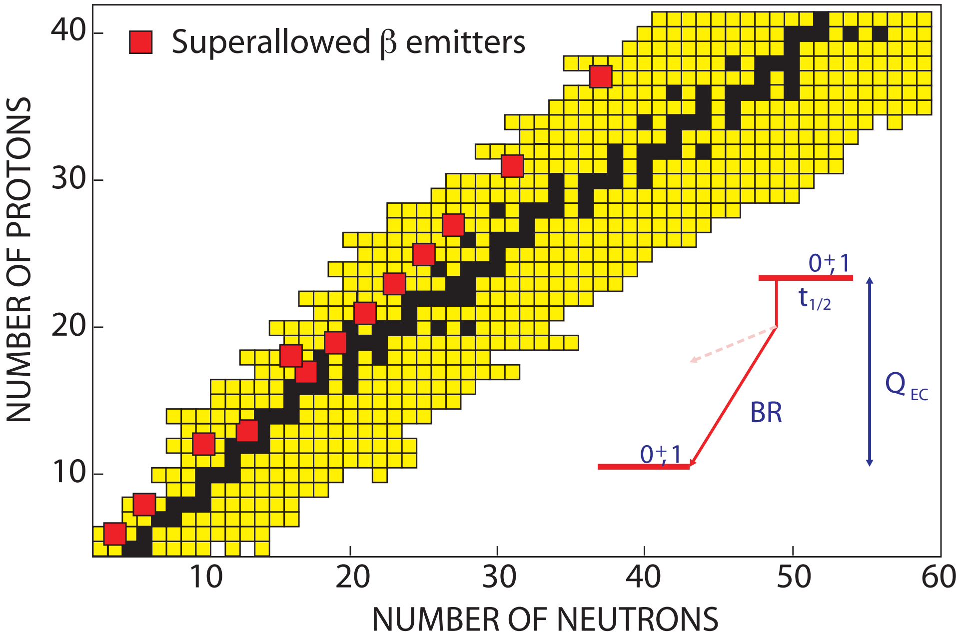


$$g = 0.88 \times 10^{-4} \text{ MeV fm}^3$$

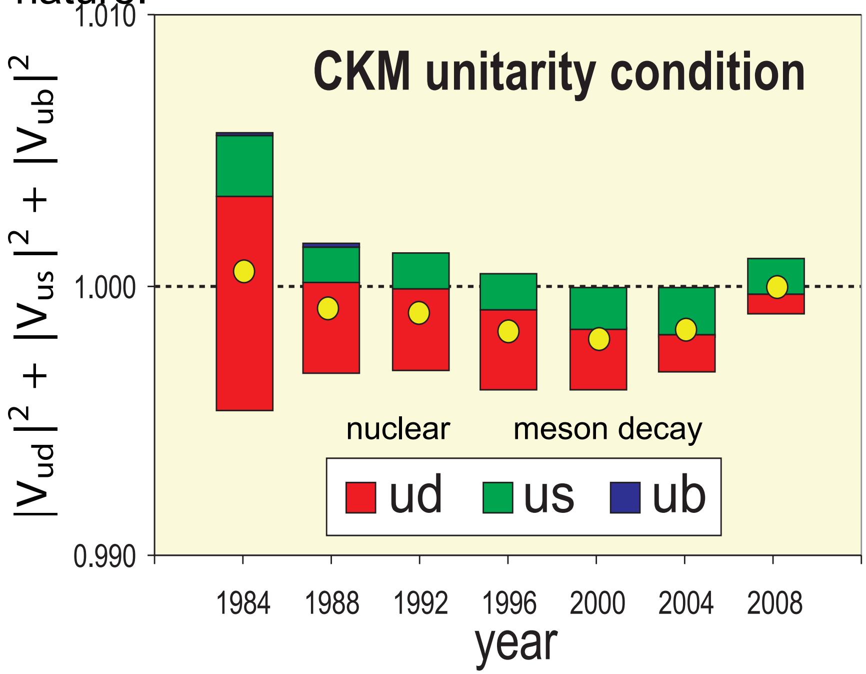
or, introducing the dimensionless constant G: $$G = g \frac{m_e^2 c}{\hbar} = 1.026 \times 10^{-5}$$

Superallowed Fermi $0^+ \rightarrow 0^+$ β-decay studies

Impressive experimental effort worldwide



Kobayashi and Maskawa (2008): … for "the discovery of the origin of broken symmetry, which predicts the existence of at least three families of quarks in nature."



Hardy and Towner survey (Feb. 2015)

http://journals.aps.org/prc/abstract/10.1103/PhysRevC.91.025501

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99992 \pm 0.00048$$

V_{ud}^2 nuclear decays, muon decay: 0.94900 ± 0.00042

V_{us}^2 kaon decays: 0.05090 ± 0.00022

V_{ub}^2 B decays: 0.00002 ± 0.00001

Beta decay: allowed transitions

Fermi transitions

$$\langle J_f M_f T_f T_{0f} | T_{\mp} | J_i M_i T_i T_{0i} \rangle = \sqrt{T_i(T_i+1) - T_{0i}(T_{0i} \mp 1)}\,\delta_{J_i J_f}\delta_{M_i M_f}\delta_{T_i T_f}\delta_{T_{0i}\mp 1 T_{0f}}$$

In reality, isospin is violated by the electromagnetic force, but the violation is weak.

$J_f = J_i$	$(\Delta J = 0)$
$T_f = T_i \neq 0$	$(\Delta T = 0,\ \text{but } T_i = 0 \to T_f = 0 \text{ forbidden})$
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

$$fT = \frac{const}{\langle F \rangle^2 + g_A^2 \langle GT \rangle^2}$$

Gamow-Teller transitions

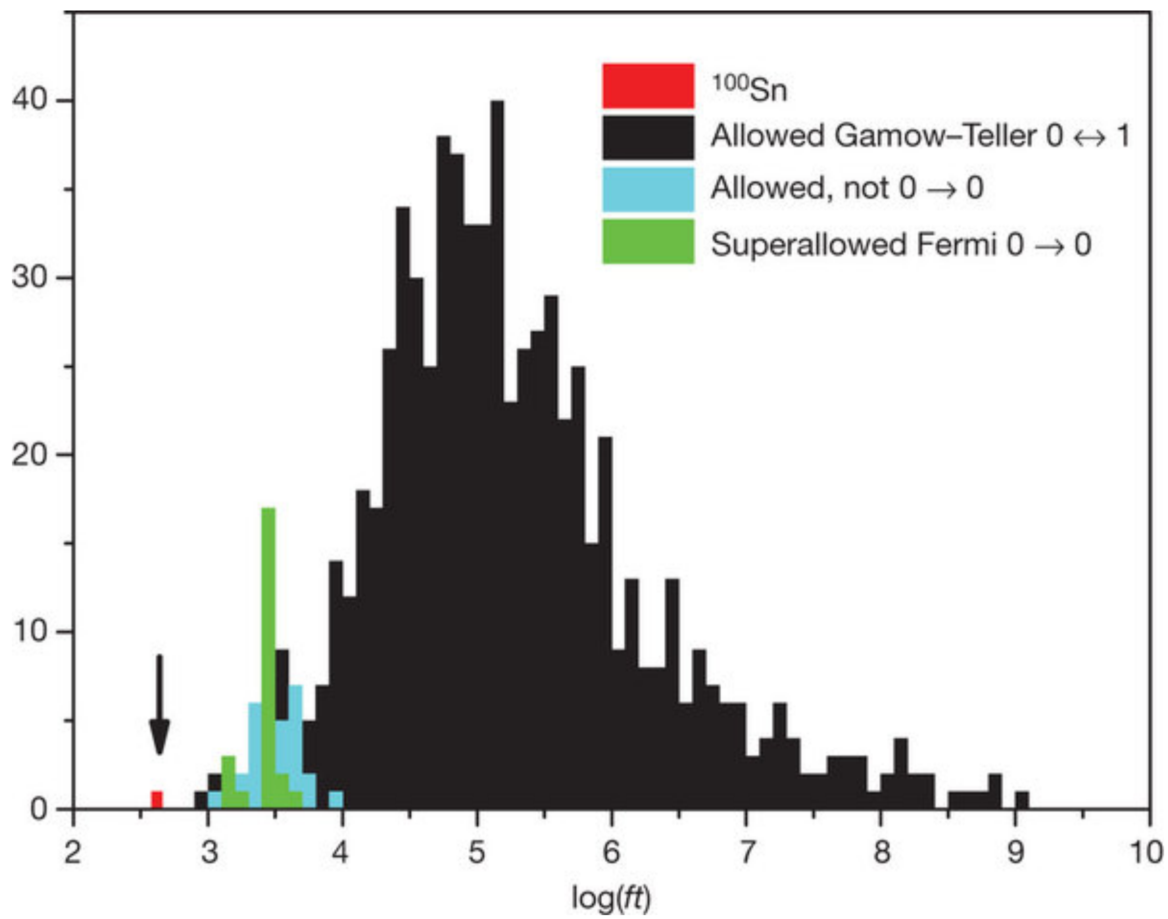
The matrix element strongly depends on the structure of the wave function!

$\Delta J = 0,1$	but $J_i = 0 \to J_f = 0$ forbidden
$\Delta T = 0,1$	but $T_i = 0 \to T_f = 0$ forbidden
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

The absolute values of GT matrix elements are generally smaller than those for Fermi transitions.

Superallowed Gamow-Teller decay of the doubly magic nucleus ^{100}Sn

Hinke et al., *Nature* **486**, 341 (2012)

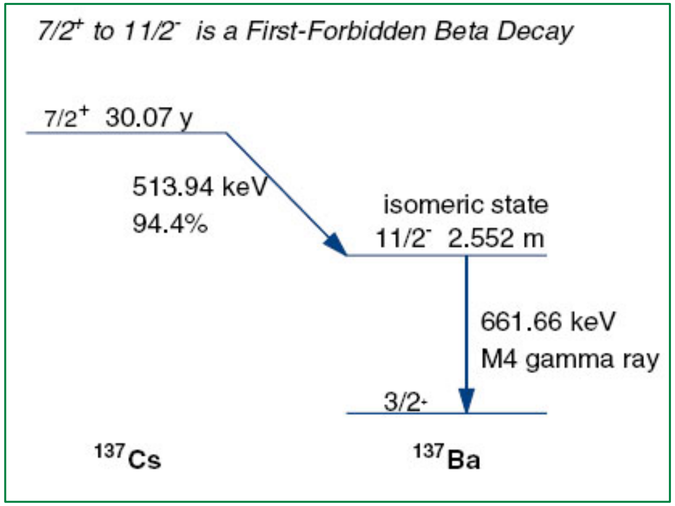


Number distribution of log(*ft*) values for allowed β-transitions (obeying the selection rules).

Forbidden transitions

Forbidden transitions involve parity change and a spin change of more than one unit. They come from the higher-order terms in the expansion of electron and neutrino plane waves into spherical harmonics. Forbidden decays are classified into different groups by the *L*-value of the spherical harmonics involved.

Decay type	ΔJ	ΔT	$\Delta\pi$	$\log_{10} ft_{1/2}$
Superallowed	$0^+ \rightarrow 0^+$	0	no	3.1–3.6
Allowed	0, 1	0, 1	no	2.9–10
First forbidden	0, 1, 2	0, 1	yes	5–19
Second forbidden	1, 2, 3	0, 1	no	10–18
Third forbidden	2, 3, 4	0, 1	yes	17–22
Fourth forbidden	3, 4, 5	0, 1	no	22–24



Systematic Uncertainties in the Analysis of the Reactor Neutrino Anomaly

A. Hayes et al., PRL 112, 202501 (2014)

"...the corrections are nuclear-operator dependent and that an undetermined combination of matrix elements contributes to non-unique forbidden transitions."

Also: r-process simulations….

Beta decay: electron capture

Electron capture leads to a vacancy being created in one of the strongest bound atomic states, and secondary processes are observed such as the emission of X-rays and Auger electrons. Auger electrons are electrons emitted from one of the outer electron shells and take away some of the remaining energy.

Capture is most likely for a 1s-state electron. The K-electron wave function at the origin is maximal and is given by

$$\psi_{e^-}(0) = \frac{1}{\sqrt{\pi}}\left(\frac{Zm_e e^2}{\hbar^2}\right)^{3/2}$$

The electron capture probability is thus given by:

$$\mathcal{W}_{EC} = E_\nu^2 \frac{\left|M'_{fi}\right|^2 g^2}{\pi^2 \hbar^4 c^3}\left(\frac{Zm_e e^2}{\hbar^2}\right)^3$$

Example $^{7}_{4}\mathrm{Be} + \mathrm{e}^- \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$ http://www.nndc.bnl.gov/chart/reCenter.jsp?z=4&n=3

Manipulating lifetimes in storage rings: http://www.sciencedirect.com/science/article/pii/S0146641013000744

Decay constants for β_c^+, EC decays and atomic losses for bare, H-like, He-like and neutral ^{140}Pr and ^{142}Pm ions together with the half-lives $T_{1/2}$ derived from time-resolved Schottky spectrometry.

Ion	λ_{β^+} $(s^{-1}) \cdot 10^{-3}$	λ_{EC} $(s^{-1}) \cdot 10^{-3}$	λ_{loss} $(s^{-1}) \cdot 10^{-3}$	$\lambda_{\beta^+} + \lambda_{EC}$ $(s^{-1}) \cdot 10^{-3}$	$T_{1/2}$ (s)	Ref.
$^{140}Pr^{59+}$	1.58(8)	–	0.4(1)	1.58(8)	439(22)	[155]
$^{140}Pr^{58+}$	1.61(10)	2.19(6)	0.4(1)	3.80(12)	182(6)	[155]
$^{140}Pr^{57+}$	1.54(11)	1.47(7)	0.4(1)	3.01(13)	230(10)	[155]
$^{140}Pr^{0+}$	1.74(5)	1.65(5)	–	3.39(7)	203(4)	[217,227]
$^{142}Pm^{61+}$	12.3(7)	–	0.5(1)	12.3(7)	56.4(32)	[157]
$^{142}Pm^{60+}$	12.6(3)	5.1(1)	0.5(1)	17.7(3)	39.2(7)	[157]
$^{142}Pm^{59+}$	13.9(6)	3.6(1)	0.5(1)	17.5(6)	39.6(14)	[157]
$^{142}Pm^{0+}$	13.2(5)	3.9(5)	–	17.1 (7)	40.5(17)	[217,227]