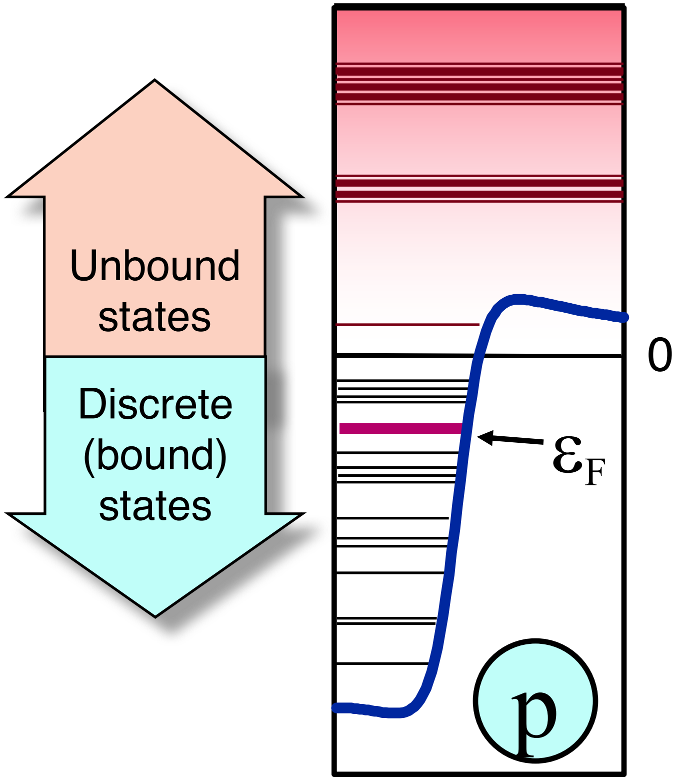
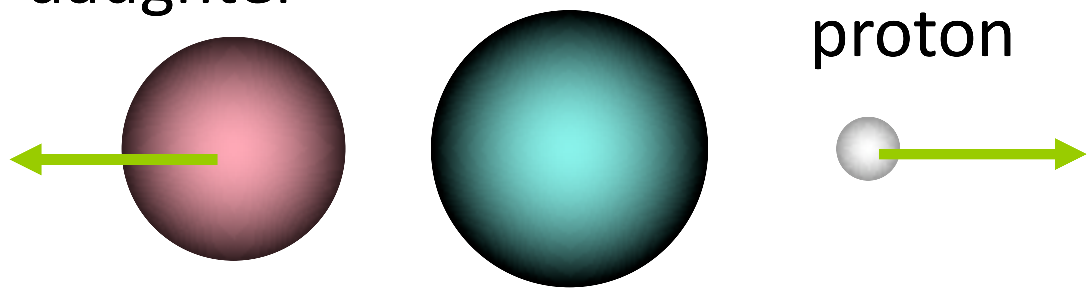
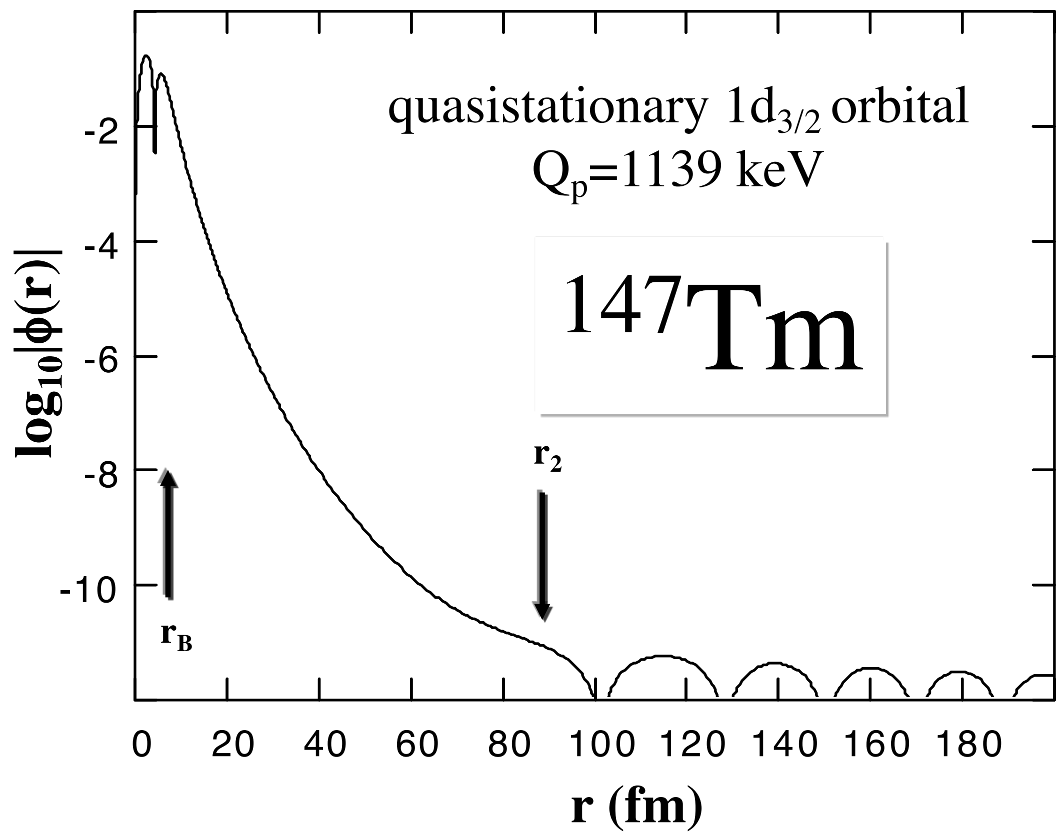
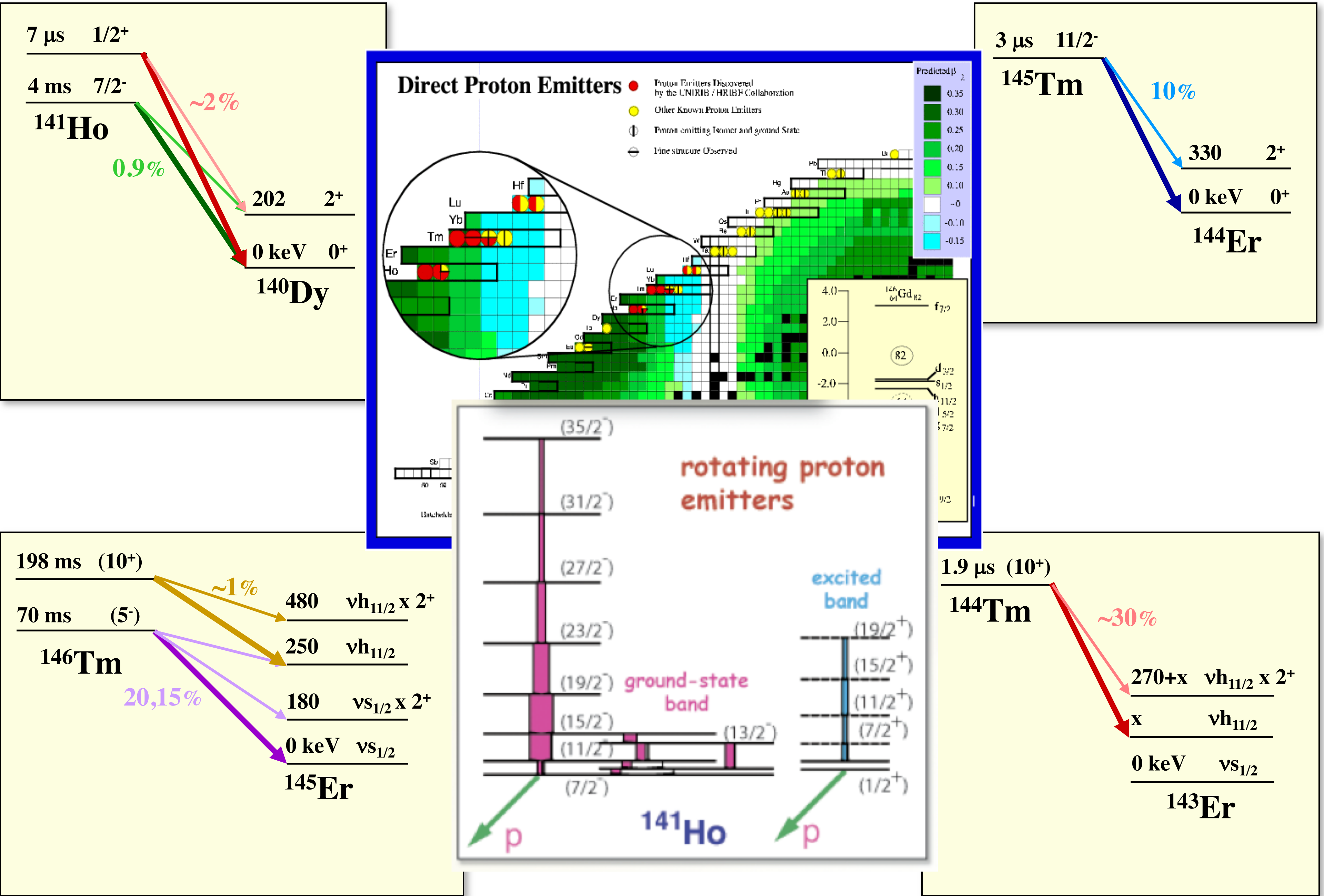
Proton emitters







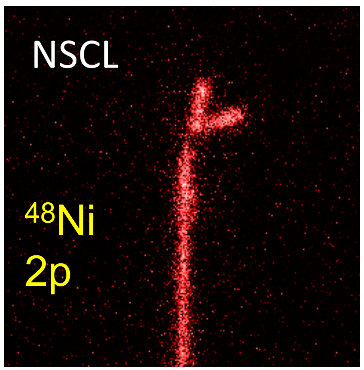
Proton emission

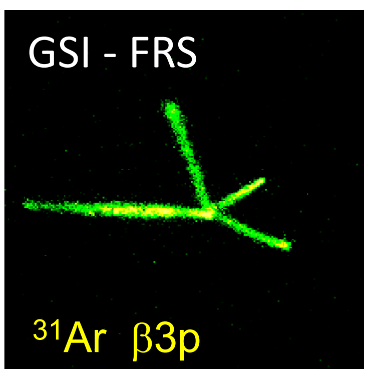


two-proton radioactivity

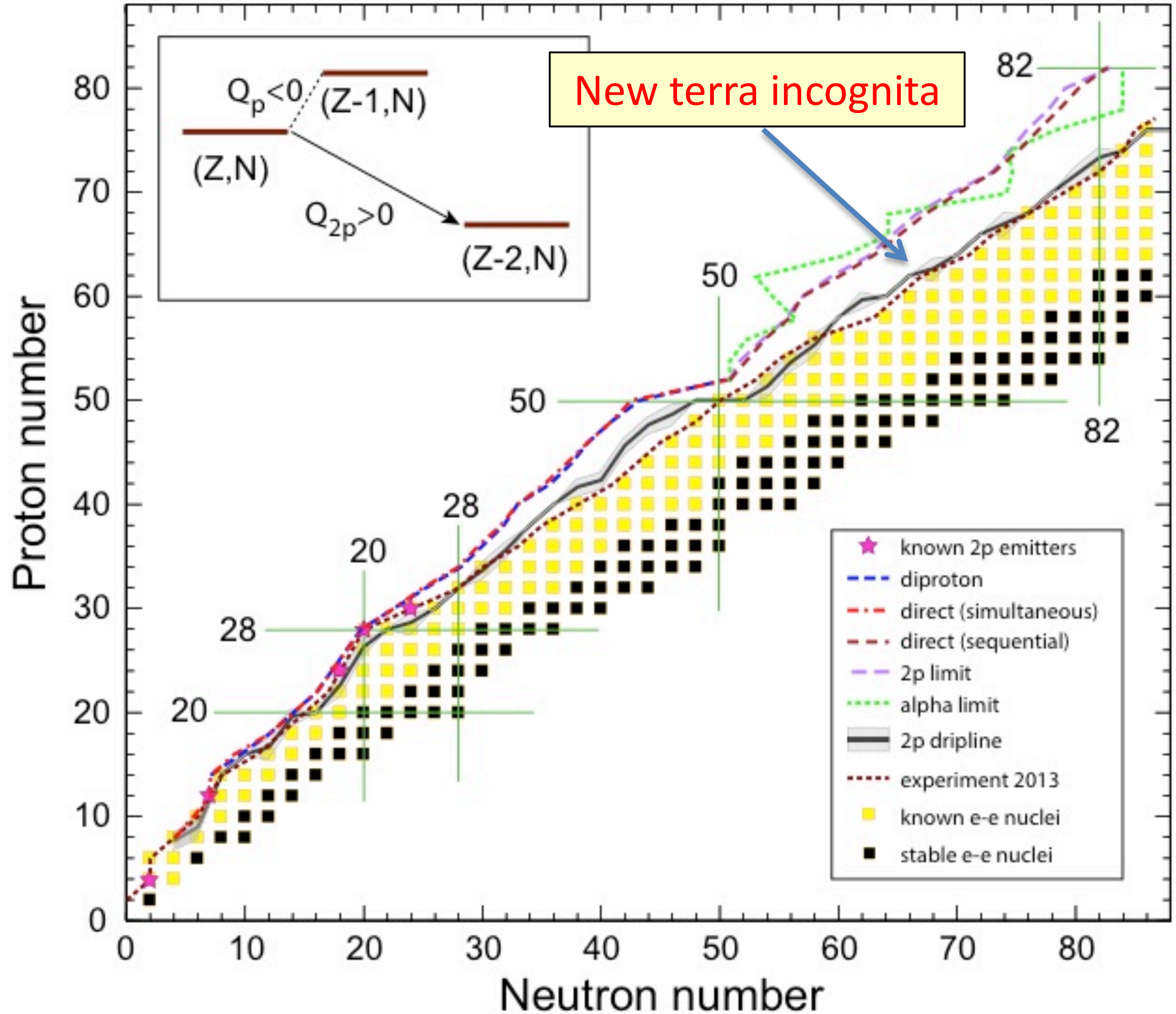
Nucleus	Experiment
^{19}Mg [7]	4.0(15) ps
^{45}Fe [10]	3.7(4) ms
^{48}Ni [8]	$3.0^{+2.2}_{-1.2}$ ms
^{54}Zn [9]	$1.98^{+0.73}_{-0.41}$ ms

^{67}Kr 20(11) ms





Digital camera picture; optical time projection chamber

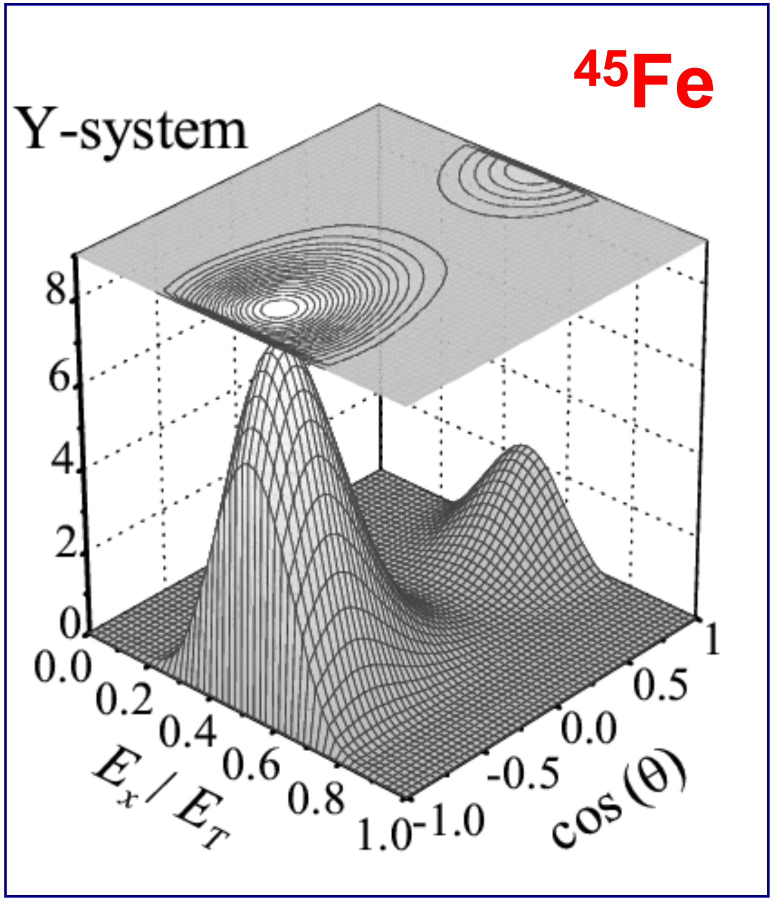


PRL 111, 139903 (2013); E: PRL 111, 139903 (2013)

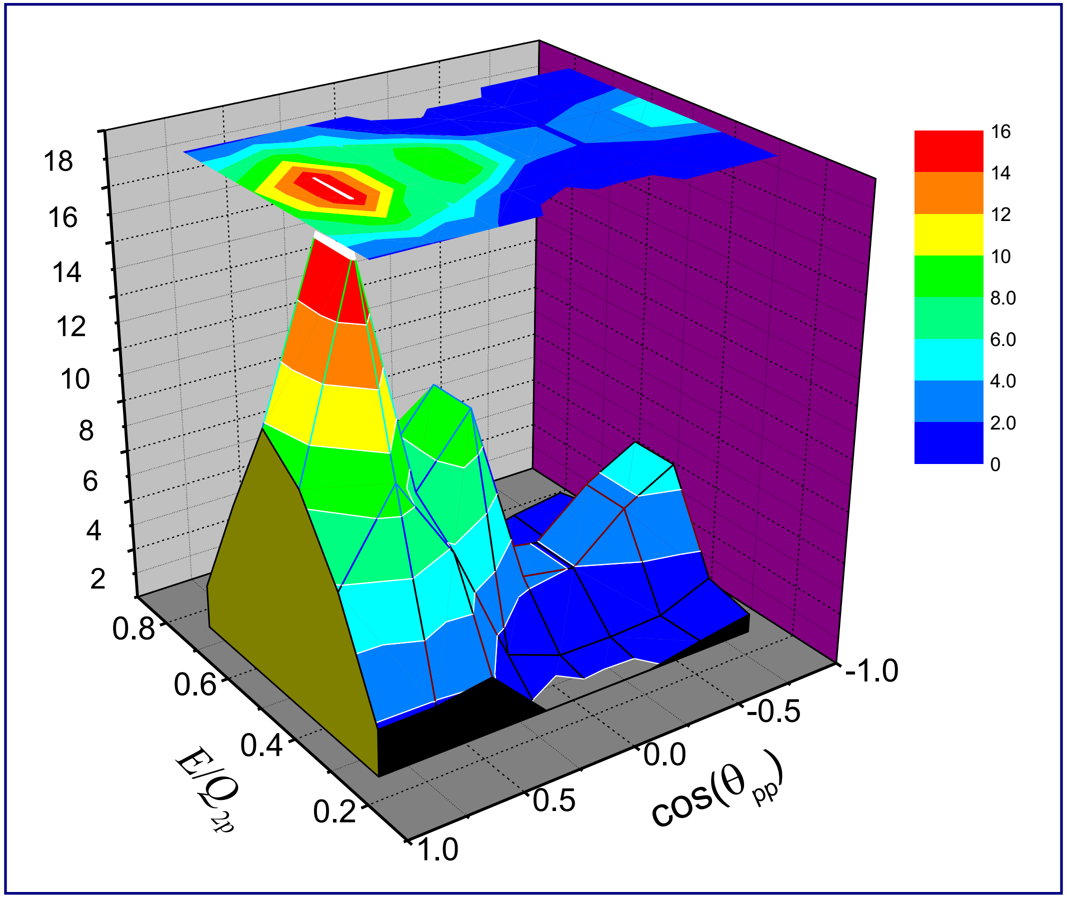
the feasibility of experimental observation of the 2p decay:

$$10^{-7}\ \text{s} < T_{2p} < 10^{-1}\ \text{s}$$

Energy - angle 2D correlation



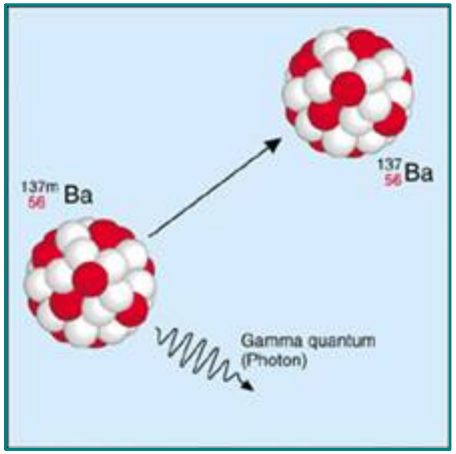
3-body model prediction

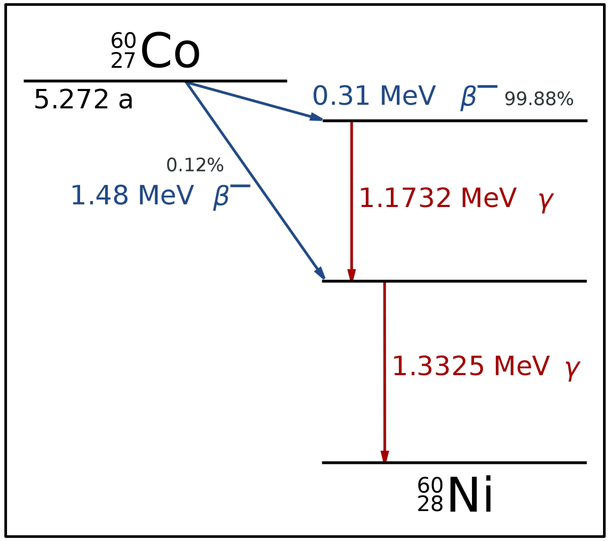


Experiment

Electromagnetic (gamma) decay

Coupling between nucleons and EM field

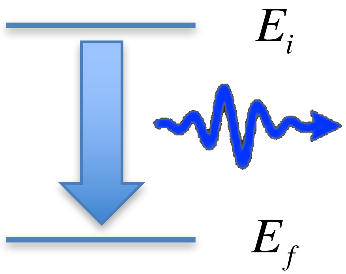


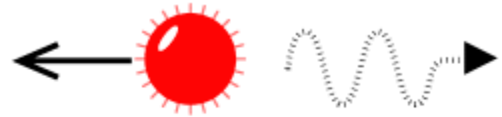


- Electromagnetic and weak interactions can be treated as perturbations
- Emission of a γ-ray is caused by the interaction of the nucleus with an external electromagnetic field
- Besides γ-decay, electromagnetic perturbation can also induce nuclear decay through *internal conversion* whereby one of the atomic electrons is ejected. This is particularly important for the heavy nuclei.
- The decay can also proceed by *creating an electron-positron pair* (internal pair creation)
- Since the nuclear wave function has a definite angular momentum, the external EM field has to be decomposed in spherical multipoles. The quantization and multipole expansion of EM field is straightforward by *tedious*.

Electromagnetic Decay

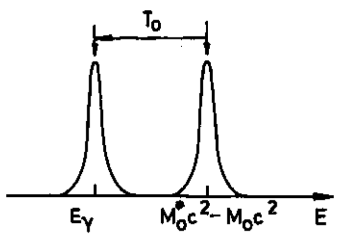
Kinematics of photon emission





$$E_i = E_f + E_\gamma + T_0$$

recoil term



$$T_0 \approx \frac{E_\gamma^2}{2M_0 c^2}$$

For A=100 and E_γ=1 MeV, the recoil energy is about 5 eV. But the natural linewidth of the radiation is even smaller.

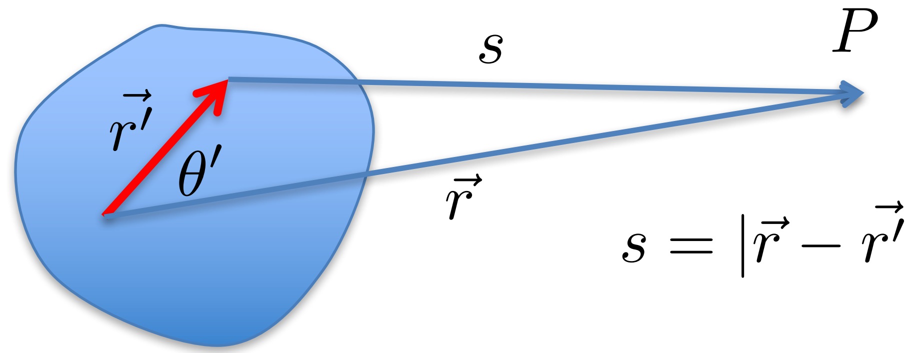
The emission of photons without recoil is possible if one implants the nucleus in a lattice. In such a case, the recoil is taken by the whole lattice and not by a single nucleus. If

$$\hbar\omega_{lattice} >> T_0$$

then, quantum-mechanically, the energy of the emitted gamma radiation takes away the total energy difference (Mössbauer effect – 1958 – or recoilless nuclear resonance fluorescence).

Multipole expansion of electrostatic potential

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{|\vec{r} - \vec{r'}|} \rho(\vec{r'})\, d^3 r'$$



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta') \rho(\vec{r'}) d^3 r'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \left(\frac{Q}{r} + \frac{\vec{p} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \cdots \right)$$

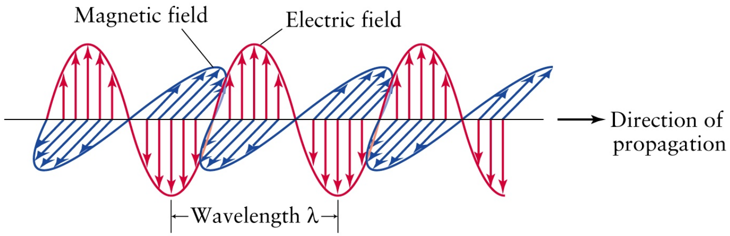
$$\vec{p} = \int \vec{r'} \rho(\vec{r'}) d^3 r' = \sum_{k=1}^{N} q_k \vec{r'_k}$$ dipole

$$Q_{ij} = \int (3 x'_i x'_j - r'^2 \delta_{ij}) \rho(\vec{r'}) d^3 r'$$ quadrupole

$$V = -\frac{1}{c} j_\mu A^\mu$$

This contains both electric and magnetic interactions

nuclear current — external EM field



$$\vec{A}(\vec{r},t) = \frac{1}{N}\sum_{\vec{k},\eta}\left\{ b_{\vec{k}\eta}\vec{\varepsilon}_\eta e^{i(\vec{k}\vec{r}-\omega t)} + b^{+}_{\vec{k}\eta}\vec{\varepsilon}_\eta e^{-i(\vec{k}\vec{r}+\omega t)} \right\}$$

two polarization states

photon creation operator

$$\vec{A}(\vec{r},t) = \sum_{\lambda\mu} \vec{A}_{\lambda\mu}(\vec{r},t)$$

← multipole expansion λ=1,2,3…

But what about λ=0?

The typical gamma-rays in nuclear transitions have energies less than 10 MeV, corresponding to wave numbers of the order k~1/20 fm^{-1} or less. The multipole operators give contributions only within the nuclear volume. That is, in most cases kr<<1 and the above series may be approximated by the first term in the expansion alone ("The long-wavelength limit"). We are now ready to calculate the contribution of each multipole order to the transition probability from an initial nuclear state to a final state.

$$\mathcal{W}(\lambda; J_i\zeta \to J_f\xi) = \frac{8\pi(\lambda+1)k^{2\lambda+1}}{\lambda\left[(2\lambda+1)!!\right]^2\hbar} B(\lambda; J_i\zeta \to J_f\xi)$$

transition probability — reduced transition probability

Electromagnetic Rates

$$B(\lambda; J_i\zeta \rightarrow J_f\xi) = \sum_{\mu M_f} \left|\langle J_f M_f \zeta | O_{\lambda\mu} | J_i M_i \xi\rangle\right|^2 = \frac{1}{2J_i+1}\left|\langle J_f \zeta \| O_{\lambda\mu} \| J_i \xi\rangle\right|^2$$

$$O_{\lambda\mu}(E\lambda) = \sum_{i=1}^{A} e(i) r_i^{\lambda} Y_{\lambda\mu}(\Omega_i) \qquad \vec{j}_i = \vec{s}_i + \vec{l}_i$$

$$O_{\lambda\mu}(M\lambda) = \sum_{i=1}^{A}\left[g_s(i)\vec{s}_i + g_l(i)\frac{2\vec{l}_i}{\lambda+1}\right]\vec{\nabla}_i\left[r_i^{\lambda} Y_{\lambda\mu}(\Omega_i)\right]$$

gyromagnetic factors

Selection Rules

$$\frac{\mathcal{W}(\lambda+1)}{\mathcal{W}(\lambda)} \sim (kR)^2$$

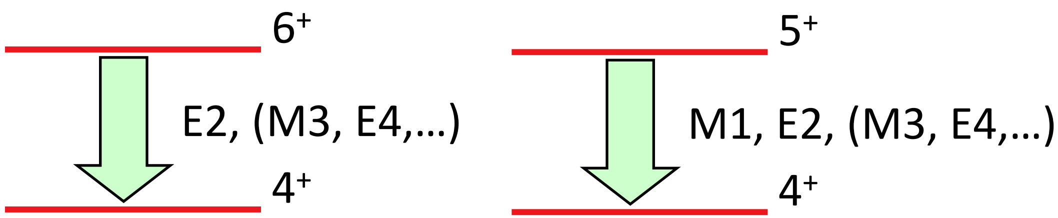
large reduction in probability
with increasing multipolarity order!

$$|J_f - J_i| \le \lambda \le J_f + J_i$$

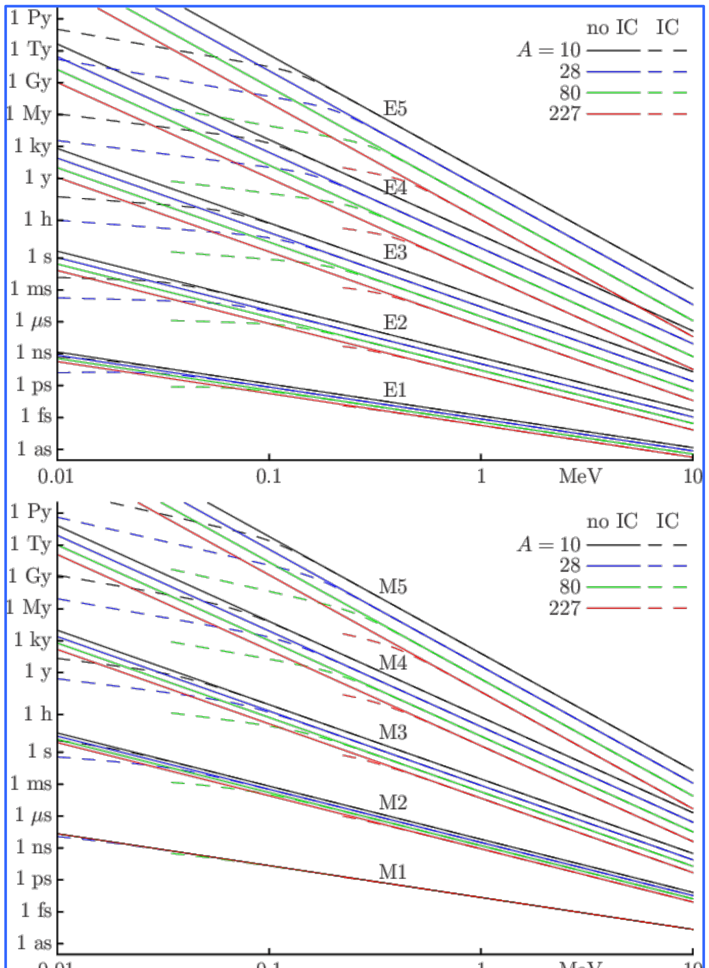
$$PO_{\lambda\mu}(E\lambda)P^{-1} = (-1)^{\lambda} O_{\lambda\mu}(E\lambda), \quad PO_{\lambda\mu}(M\lambda)P^{-1} = (-1)^{\lambda+1} O_{\lambda\mu}(M\lambda)$$

$$\pi_i \pi_f = (-1)^{\lambda} \qquad \pi_i \pi_f = (-1)^{\lambda+1}$$

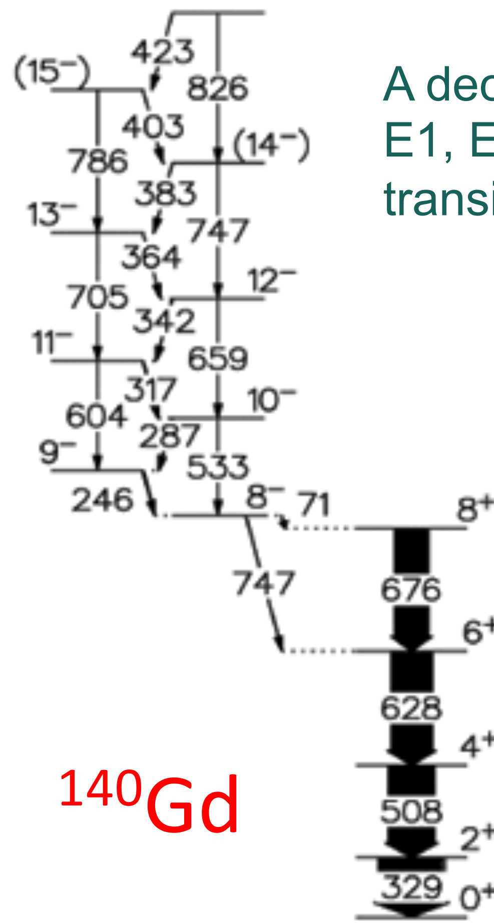
Magnetic transitions are weaker than electric transitions of the same multipolarity



mixed M1+E2 transition



A decay scheme: E1, E2 and M1 transitions



^{140}Gd