Magnetic Fields in the Quark Gluon Plasma

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Neglecting external fields, how do we quantify the magnetic fields in a static QGP? What about when we consider an expanding QGP? Are those effects significant?

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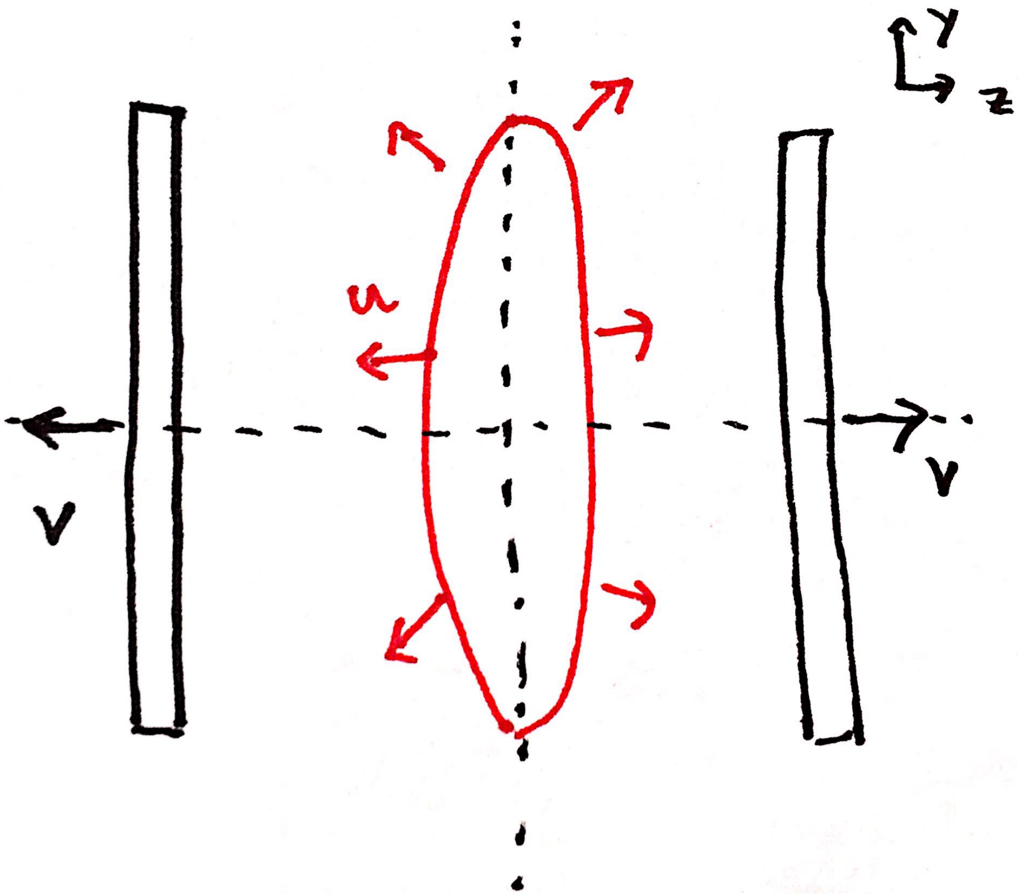
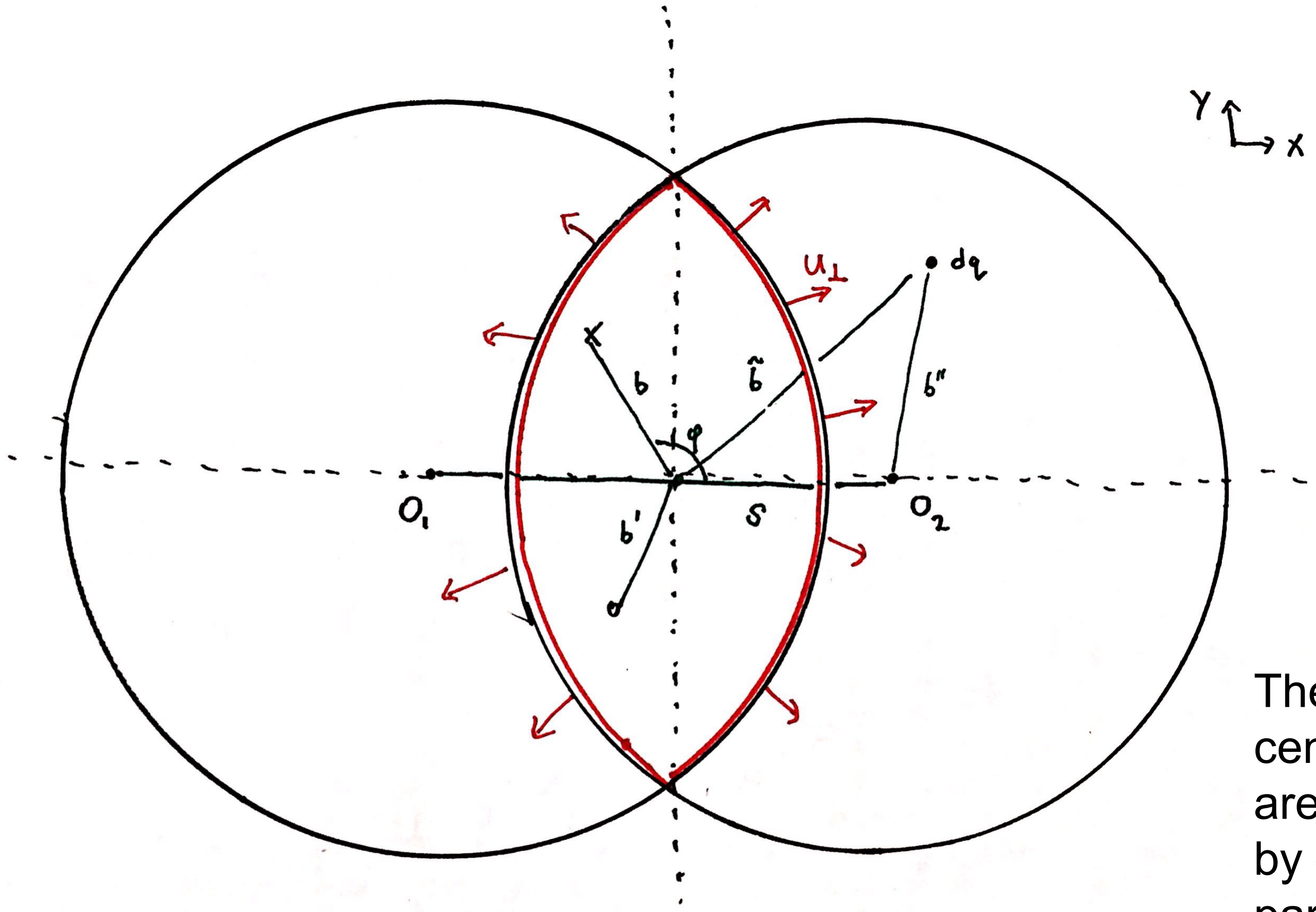
Side View Geometry of Relativistic Heavy Ion Collision (Post Collision)



v is the velocity of ion remnants, each moving away from center point ($\pm v$ in the z direction).

The plasma expands with velocity u.

Geometry of Relativistic Heavy Ion Collision in Transverse Plane



The nuclear centers (O_1, O_2) are connected by impact parameter s.

Intense Fields are Created. What are the Effects on the QGP?

- Neglecting magnetic fields generated before the plasma emergence (~0.2 fm/c), the EM fields are created by the valence quarks present in the ion remnants flying in the z-direction as shown in the first image.
  - These valence quarks are external to the plasma.
- Since QGP dynamics are determined mostly by strong force interactions, we can treat the EM interactions perturbatively.
  - This effectively decouples the EM dynamics from the plasma dynamics.
- The calculations presented here consider plasma expansion using the blast-wave model.

II. MAXWELL EQUATIONS IN EXPANDING PLASMA

An electromagnetic field in flowing conducting medium satisfies the equations

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) + \boldsymbol{j}, \tag{1a}$$

$$\nabla \cdot \boldsymbol{E} = \rho, \tag{1b}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1c}$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}, \tag{1d}$$

where $\boldsymbol{u}$ is the fluid velocity, σ is electrical conductivity and $j^\mu = (\rho, \boldsymbol{j})$ is the external current created by the valence charges .

For Stationary Plasma

Reducing Maxwell's equations by looking at potentials and using a gauge condition, we can reduce to two equations. The first of which describes the fields created by the external currents in the stationary plasma.

$$-\nabla^2 \boldsymbol{A}^{(0)} + \partial_t^2 \boldsymbol{A}^{(0)} + \sigma \partial_t \boldsymbol{A}^{(0)} = \boldsymbol{j}, \quad (1)$$

$$-\nabla^2 \boldsymbol{A}^{(1)} + \partial_t^2 \boldsymbol{A}^{(1)} + \sigma \partial_t \boldsymbol{A}^{(1)} = \sigma \boldsymbol{u} \times \boldsymbol{B}^{(0)}. \quad (2)$$

The solution using retarded Green's potentials is:

$$\boldsymbol{A}^{(0)}(\boldsymbol{r},t) = \int G(\boldsymbol{r},t|\boldsymbol{r}',t')\boldsymbol{j}(\boldsymbol{r}',t')d^3r'dt', \quad (3)$$

The magnetic field is:

$$\boldsymbol{B}_a^{(0)} = \frac{ev}{4\pi}\hat{\boldsymbol{\phi}}\left\{\frac{\sigma b/2}{\xi^2 + b^2/\gamma^2} + \frac{b}{\gamma^2[\xi^2 + b^2/\gamma^2]^{3/2}}\right\} \times \exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi + \sqrt{\xi^2 + b^2/\gamma^2}\right)\right\}. \quad (4)$$

Where ξ is defined as:

$$\xi = vt - z$$

Now Considering the Expansion of the Plasma

Now considering (2):

$$-\nabla^2 \boldsymbol{A}^{(0)} + \partial_t^2 \boldsymbol{A}^{(0)} + \sigma \partial_t \boldsymbol{A}^{(0)} = \boldsymbol{j}, \quad (1)$$

$$-\nabla^2 \boldsymbol{A}^{(1)} + \partial_t^2 \boldsymbol{A}^{(1)} + \sigma \partial_t \boldsymbol{A}^{(1)} = \sigma \boldsymbol{u} \times \boldsymbol{B}^{(0)}. \quad (2)$$

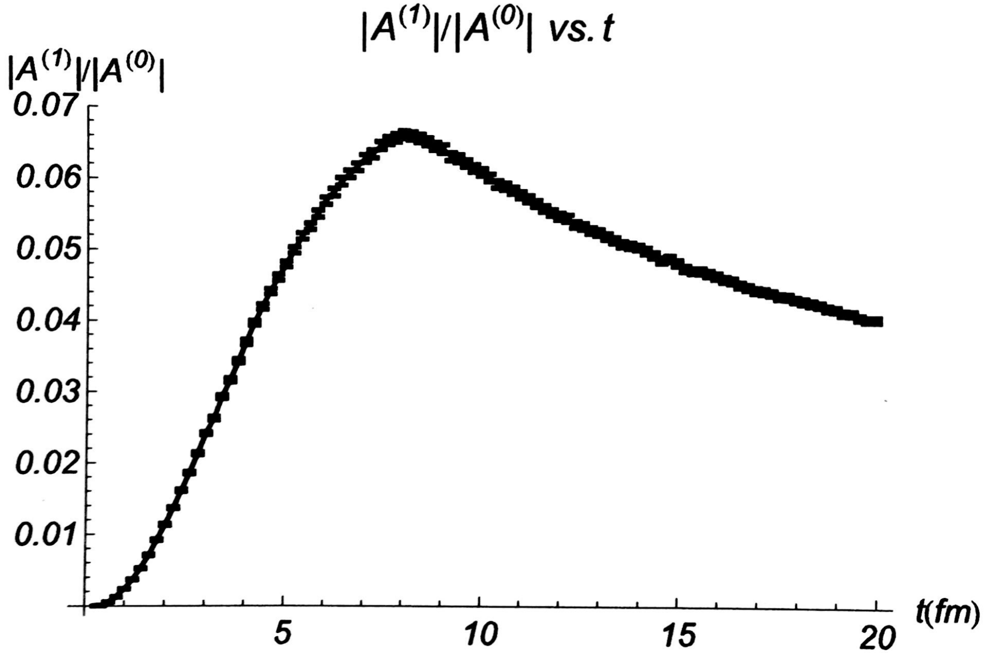
The solution again using Green's functions is the following:

$$\boldsymbol{A}^{(1)}(\boldsymbol{r},t|\tilde{\boldsymbol{r}}) = \sigma \int_{\tau}^{t+} dt' \int d^3r' G(\boldsymbol{r},t|\boldsymbol{r}',t')\, \boldsymbol{u}(\boldsymbol{r}',t') \times \boldsymbol{B}^{(0)}(\boldsymbol{r}' - \tilde{\boldsymbol{r}},t'). \quad (5)$$

Flow velocity $\boldsymbol{u}$, in the lab frame is: $\boldsymbol{u}(r,t) = z/t$

Technically, for both equations, you have to consider the sum of two Green's functions, one for the original pulse, and one for what is known as the wake. The wake accounts for the currents induced into the plasma. It was shown that these wake effects are not significant at small times, so they are neglected here. However, these effects dominate at large times.

Results



Comparing the expanding plasma vector potential to the static plasma vector potential, we see that by accounting for the expansion of the plasma, we only see a $< 10\%$ correction.

Reference: https://doi.org/10.1103/PhysRevC.97.044906