

# Self-Consistent Study of Fission Barriers of Even-Even Superheavy Nuclei

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**Abstract.** Static fission barriers of even-even nuclei with  $100 \leq Z \leq 110$  are investigated using the Skyrme-Hartree-Fock model with particular attention paid to symmetry-breaking effects along the fission path. Effects of reflection-asymmetric and triaxial degrees of freedom on the fission barriers are discussed.

**Keywords:** Skyrme-Hartree-Fock, Fission barriers, Superheavy nuclei

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## INTRODUCTION

Since the middle of the 1960s, studies of superheavy nuclei have provided rich and unique information on the structure of atomic nuclei; for recent reviews see, e.g., Refs. [1, 2, 3, 4, 5, 6]. Recent experimental works [7, 8, 9, 10] illustrate continuous progress in this area.

Spontaneous fission, together with  $\alpha$ -decay, is a major decay channel of superheavy nuclei. Microscopically, the phenomenon of fission can be viewed as many-body tunneling through a potential barrier. Studies of fission barriers allow for a determination of stability of the heaviest nuclei (see, e.g., Refs. [11, 12, 13, 14, 15, 16]).

Recently, a number of theoretical calculations of static fission barriers of nuclei in the actinide and trans-actinide regions have been carried out. These include calculations based on the microscopic-macroscopic treatment [17], the self-consistent approach with the Gogny [18] and Skyrme [19, 20, 21] forces, and also within the relativistic mean-field model [20, 22].

The present paper is a continuation of our previous work [21] with the objective to study static fission barriers of even-even nuclei with  $100 \leq Z \leq 110$  within the Skyrme-Hartree-Fock (SHF) approach. Nuclei from this region are well deformed in their ground states. It is worth noting that their stability is specified by two ‘magic’ neutron numbers:  $N = 152$  for the isotopes of fermium ( $Z = 100$ ) and nobelium ( $Z = 102$ ), and  $N = 162$  for the isotopes of hassium ( $Z = 108$ ) (see, e.g., Refs. [23, 24]). In this study, we investigate the influence of reflection-asymmetric (non-zero octupole mass moment,  $Q_{30} \equiv \langle \hat{Q}_{30} \rangle \neq 0$ ) and triaxial ( $Q_{22} \neq 0$ ) degrees of freedom on static fission paths.

# FISSION BARRIERS IN THE SHF+BCS MODEL

## Description of the model

Calculations have been carried out using the Hartree-Fock+BCS code that solves self-consistent HF equations by using the Cartesian (3D) harmonic oscillator (HO) finite basis [25, 26, 27]. This code makes it possible to break all self-consistent symmetries of the nuclear mean field, including simultaneous breaking of axial and reflection symmetries, which are of particular interest to our study.

We use the energy density functional defined by the Skyrme interaction SLy4 [28] and a seniority pairing force treated in the BCS approximation. The pairing strengths have been adjusted to reproduce the experimental proton (0.803 MeV) and neutron (0.696 MeV) pairing gaps in  $^{252}\text{Fm}$ .

In Ref. [21] we investigated the stability of SHF results to the number of deformed HO states used in the basis. It has been found that reliable calculations can be carried out with 1140 HO states. At the spherical shape, this number corresponds to 17 oscillator shells.

For all the nuclei considered, the self-consistent binding energy ( $E^{tot}$ ) is computed by using a quadratic [29] constraint on the mass quadrupole moment  $Q_{20}$ . Our study covers the prolate shapes in the range of  $Q_{20} = 0-300$  b (barns) that have been binned with the step of 10 b. To fix the position of the center of mass, an additional constraint on the mass dipole moment  $Q_{10} = 0$  has been imposed.

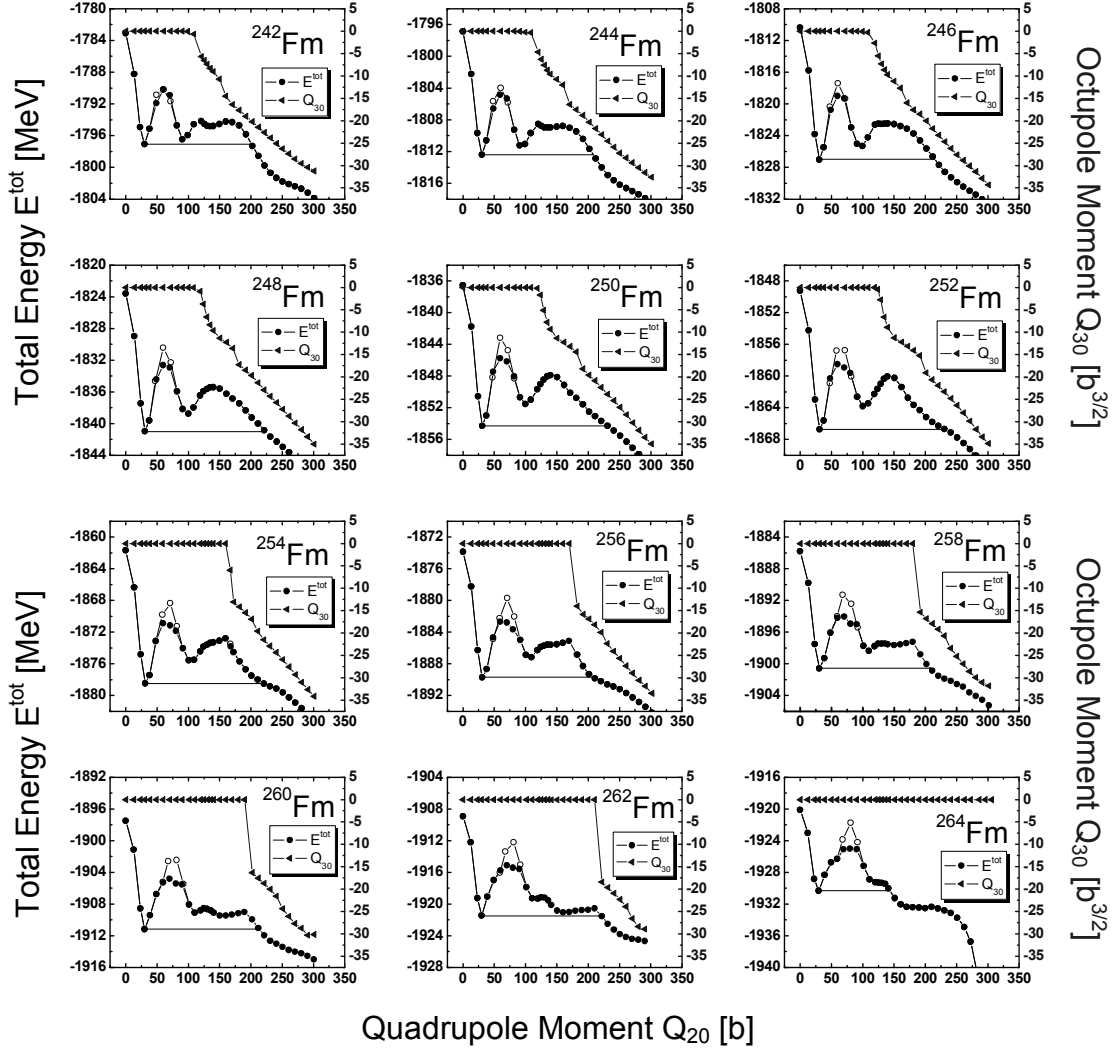
## Static fission barriers

In Figs. 1 and 2 the total binding energies ( $E^{tot}$ ) and the mass octupole moments ( $Q_{30}$ ) calculated for the even-even fermium and nobelium isotopes, respectively, are plotted as functions of the mass quadrupole moment  $Q_{20}$ . All calculated static barriers exhibit a similar, two-humped pattern with the inner barrier higher than the outer one. The calculated barrier widths and heights are well correlated with the ‘magic’ neutron number  $N=152$ . One can see that the barriers in  $^{252}\text{Fm}$  and  $^{254}\text{No}$  reach their local maxima. For the heavier isotopes with  $N > 152$ , the outer barrier collapses and practically disappears for  $N=162$  (Fm isotopes) and  $N=160$  (No isotopes).

As one can see, the disappearance of the outer barrier is related to the transition from the reflection-symmetric fission path ( $Q_{30} = 0$ ) to the reflection-asymmetric fission path ( $Q_{30} \neq 0$ ). For the heavier Fm and No isotopes, such a transition occurs at greater values of  $Q_{20}$ . In the extreme cases of  $^{264}\text{Fm}$  (when a fission into the two doubly magic  $^{132}\text{Sn}$  nuclei is expected) and  $^{264}\text{No}$ , the static fission paths are predicted to be totally reflection-symmetric.

The reduction of the outer barrier plays a crucial role in the standard interpretation of the experimentally known rapid decrease of the spontaneous fission half-lives in the heavy Fm and No isotopes.

In Figs. 1 and 2, the influence of the triaxial asymmetry (for  $Q_{22} \neq 0$ ) on the height of the first fission barrier is shown as a difference between the open and solid circles. The

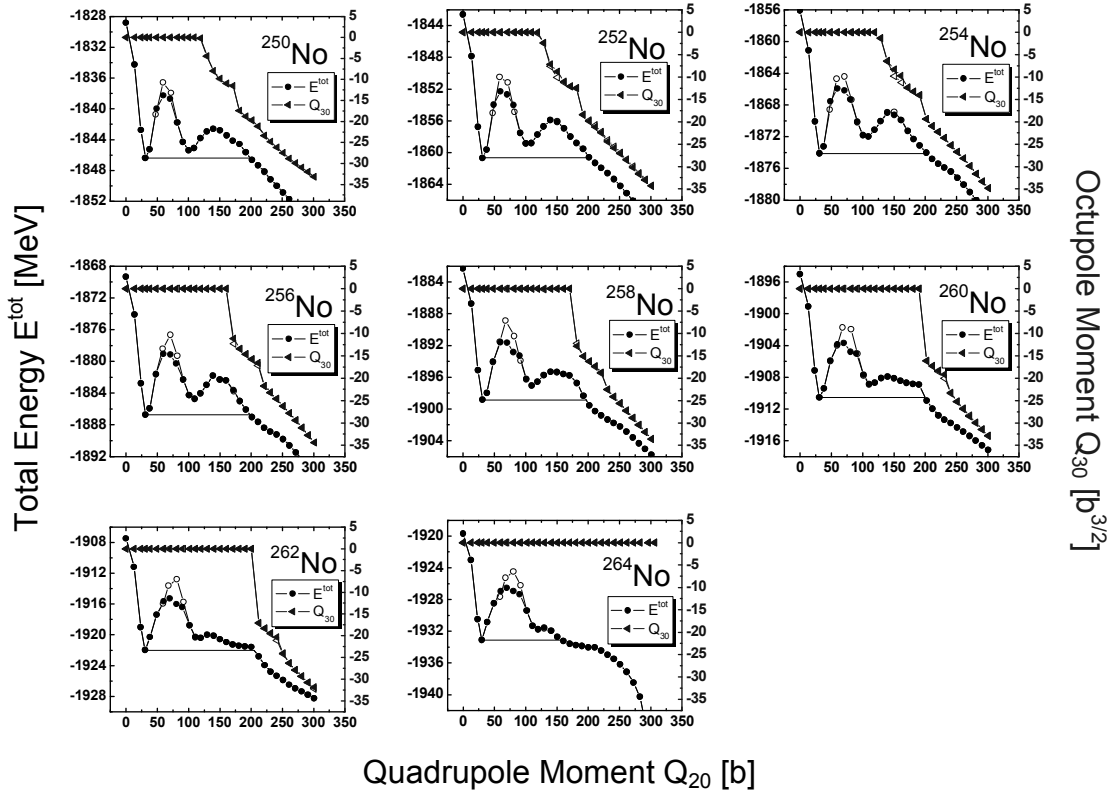


**FIGURE 1.** Total binding energies  $E^{tot}$  (solid circles) and mass octupole moments  $Q_{30}$  (solid triangles) calculated along the lowest static fission paths for the even-even fermium ( $Z=100$ ) isotopes with  $N=142$ – $164$ . The results corresponding to axial shapes are marked by open circles.

effect of triaxiality increases with the neutron number, reaching peak values of about 3 MeV.

The total binding energies and mass hexadecapole moments ( $Q_{40}$ ) calculated along the static fission paths for the even-even rutherfordium ( $Z=104$ ), seaborgium ( $Z=106$ ), hassium ( $Z=108$ ), and darmstadtium ( $Z=110$ ) isotopes are shown in Figs. 3 and 4. We have found that almost all of these nuclei have purely reflection-symmetric paths with  $Q_{30}=0$ . Only in the case of  $^{254}\text{Rf}$ ,  $^{256}\text{Rf}$ , and  $^{258}\text{Sg}$  have the reflection-asymmetric paths been predicted (results not shown). The effects of the triaxial degree of freedom are again represented by the differences between open and solid circles.

One can see that the hexadecapole moments increase along the static paths. It is worth noting that a similar behavior of  $Q_{40}$  is also seen in the Fm and No isotopes discussed

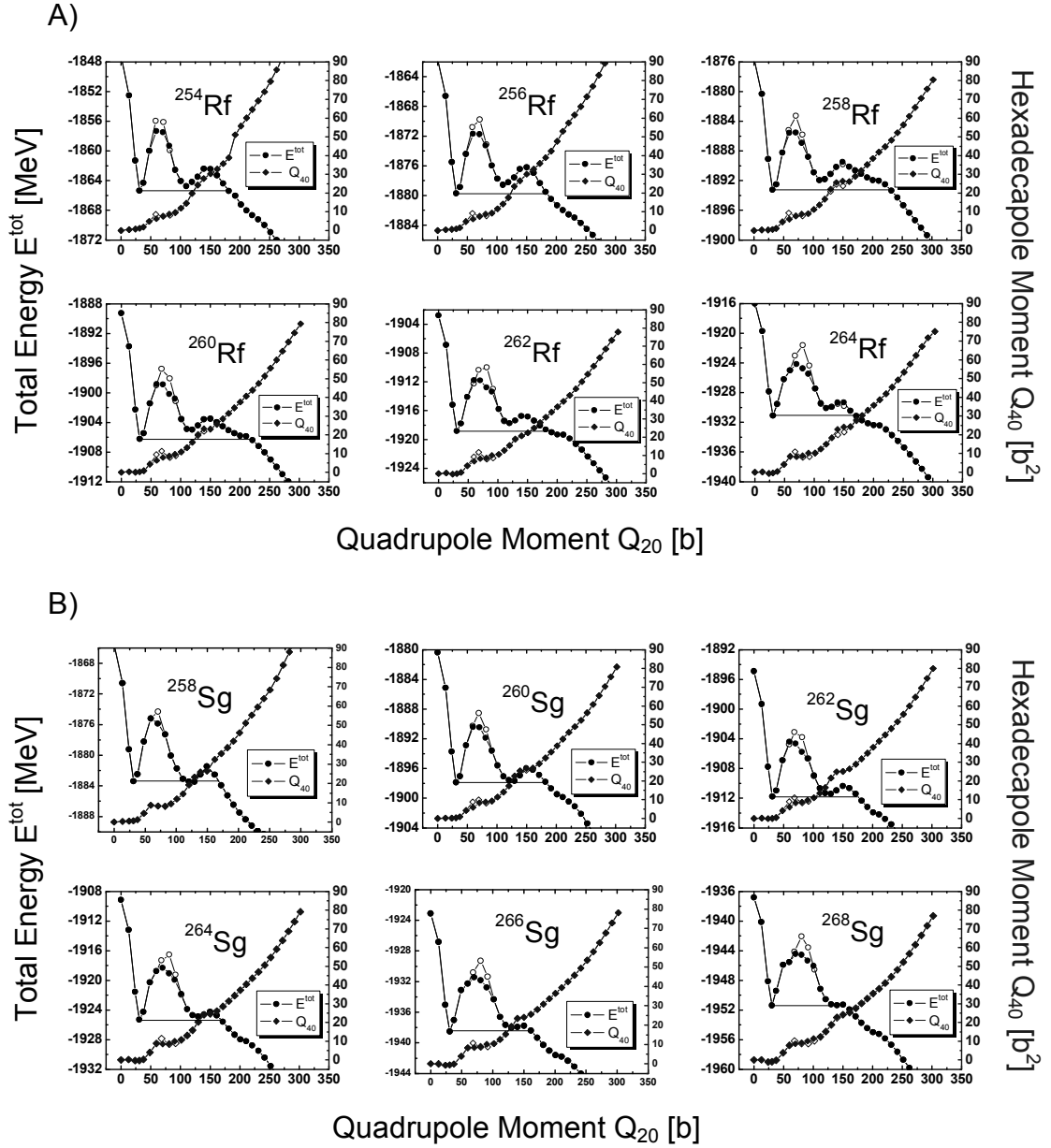


**FIGURE 2.** Similar to Fig. 1 except for the even-even nobelium ( $Z=102$ ) isotopes with  $N=148-162$ .

above. However, in contrast to smooth changes of  $Q_{40}$  as a function of  $Q_{20}$  seen in Figs. 3 and 4, sudden jumps of  $Q_{30}$  along the lowest static fission paths appear for the heavier Fm and No isotopes.

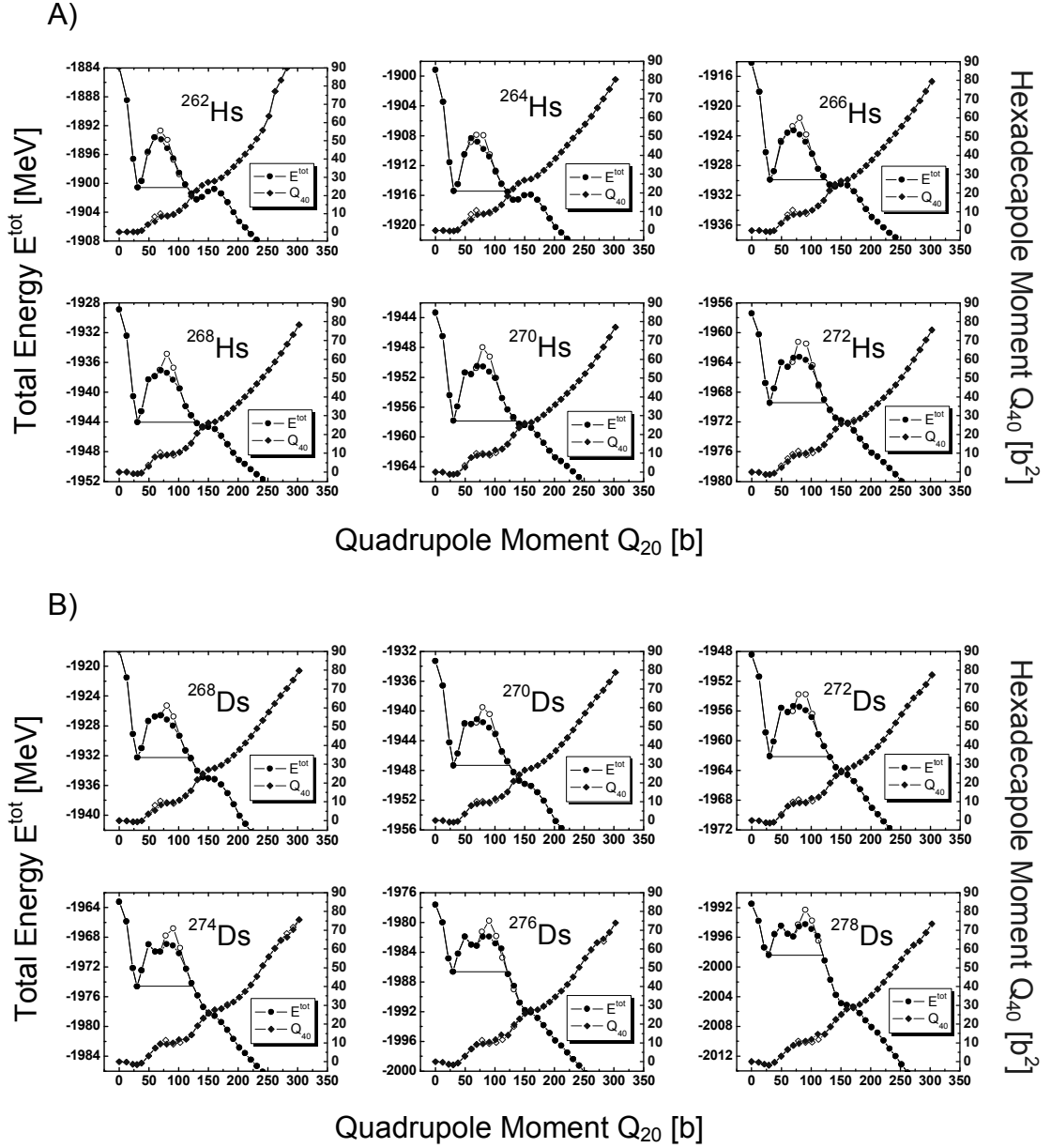
When the proton number increases from  $Z=100$  to 110, one can notice two effects: (i) the disappearance of the outer fission barrier, and (ii) the decrease of the inner barrier height. The first effect can be seen in Figs. 3B and 4 which display one-humped, narrow barriers obtained for Sg, Hs, and Ds. The second effect is particularly evident for the heaviest  $^{276,278}\text{Ds}$  isotopes, where the barriers are twice as low as that in  $^{252}\text{Fm}$ .

The even-even superheavy nuclei considered in this paper are schematically indicated in Fig. 5. Nuclei exhibiting the mass-asymmetry along the fission path are represented by gray, while those having symmetric fission paths are shown as dark gray. In the same figure, the experimentally known even-even nuclei are shown as black squares and diamonds. (The diamonds correspond to superheavy elements produced in hot-fusion reactions.) The deformed and postulated (spherical) ‘magic’ numbers are indicated by light gray.



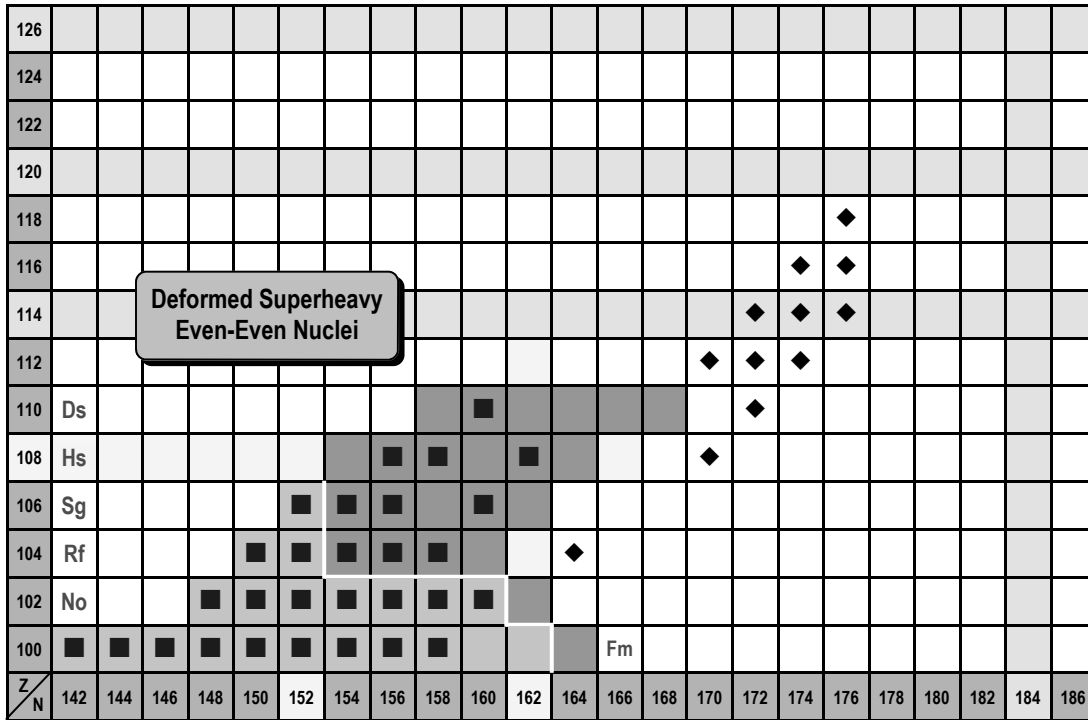
## CONCLUSIONS

We have applied the self-consistent SHF+BCS method with the Skyrme parametrization SLy4 to study static fission barriers of the even-even superheavy elements with  $100 \leq Z \leq 110$ . All calculations have been done taking into account the non-axial and reflection-symmetry-breaking shapes along the fission paths.



**FIGURE 4.** Similar to Fig. 3 except for the even-even hassium ( $Z=108$ ) isotopes with  $N=154$ – $164$  (A) and darmstadtium ( $Z=110$ ) isotopes with  $N=158$ – $168$  (B).

For all the investigated superheavy nuclei, the appearance of triaxial distortion reduces the height of the first barrier by up to 3 MeV. It is also found that the breaking of intrinsic reflection-symmetry appears for  $^{258}\text{Sg}$ ,  $^{254,256}\text{Rf}$ , and for the Fm and No isotopes (excluding  $^{264}\text{Fm}$  and  $^{264}\text{No}$ ). In all other cases purely reflection-symmetric fission paths are obtained.



**FIGURE 5.** The portion of the nuclear chart showing even-even superheavy nuclei with  $Z \geq 100$  considered in the present study.

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