Gamow HF/HFB

Self-consistent methods for exotic nuclei

Standard HF/HFB : problems with exotic nuclei

Complex scaled HF : advantages and limitations

Standard interactions incompatibility

Modification of the interaction and method

Unbound HF/HFB ground state definition

Conclusion

Standard HF/HFB : problems with exotic nuclei

- $U_{HF} = U_{HF}(\rho_{\alpha\beta})$.
  $U_{HFB} = U_{HFB}(\rho_{\alpha\beta}, \widetilde{\rho}_{\alpha\beta})$.
  $\widetilde{U}_{HFB} = \widetilde{U}_{HFB}(\rho_{\alpha\beta}, \widetilde{\rho}_{\alpha\beta})$.

- Well bound states : $\rho \to 0$, $\widetilde{\rho} \to 0$ quickly.
  Localized HF/HFB potentials and wave functions.

- Problems
  loosely bound states : $U, \widetilde{U} \to 0$ too slowly.
  unbound states : $U_{HF/HFB}$ undefined as $|\rho|, |\widetilde{\rho}| \to +\infty$.
  Fundamental problem : wrong asymptotics.

Complex scaled HF : advantages and limitations

- Kruppa et al., Phys. Rev. Lett. **79**, 2217 (1997)

- Densities : exterior complex scaling.
  $\rho(R + x \cdot e^{i\theta}) \to 0$, $\theta > \theta_c$.
  Localized potentials and densities in the complex plane.

- Advantages
  Fully self-consistent.
  Standard interactions like Skyrme can be used.

- Limitations
  Complex HF potential : cannot generate a basis $\Rightarrow$ HF only.
  Difficult interpretation of single particle states.
  Slow decrease of $U_{HF}(z)$ in the complex plane.

Standard interactions incompatibility

- The problem lies in the interaction itself.
  Density dependence and translational invariance.

- $\underline{{}^5\text{He} = {}^4\text{He} + 0p_{3/2}}$
  density dependence $\Rightarrow$ self-induced interaction : $|U(r)| \rightarrow +\infty$.
  no density dependence $\Rightarrow$ $U(r) \rightarrow 0$ quickly.

- $\underline{{}^6\text{He} = {}^4\text{He} + 2\text{n}}$
  HF/HFB : $|U_{HF}(r)| \rightarrow 0$ very slowly.
  Cluster picture : $\Psi(2n) = \Psi_{CM}(r_{CM}) \cdot \Psi_{rel}(r_{rel})$.
  $U_{CM}(r_{CM}), U_{rel}(r_{rel}) \rightarrow 0$ quickly.

- Translational invariance demands good asymptotics.
- Independent (quasi-)particles : no translational invariance.

Modification of the interaction

- Best possible : good one body asymptotics.
  $U_{HF}, U_{HFB}, \widetilde{U}_{HFB} \rightarrow 0$ or $\dfrac{Z-1}{r}$ quickly in all cases.

- $V_{int}(\vec{r_1} - \vec{r_2}) \rightarrow V_{int}(\vec{r_1} - \vec{r_2}) \cdot F_{\mu}(r_1 - R_0) \cdot F_{\mu}(r_2 - R_0)$
  F_{μ} : Fermi-like function, $R_0 \propto A^{\frac{1}{3}}$.
  Analogous tranformation for the rest of the interaction.

- V_{int} no longer translationally invariant.
- Not important for heavy nuclei.
- Light nuclei : center of mass excitation removal.

Modification of the method

- $U_{HF}, U_{HFB}, \widetilde{U}_{HFB}$ must be hermitian operators.
  $U_{HF}, U_{HFB}, \widetilde{U}_{HFB} \rightarrow \Re[U_{HF}], \Re[U_{HFB}], \Re[\widetilde{U}_{HFB}]$.
  Chemical potential : $\lambda \rightarrow \Re[\lambda]$.
  Real bound states and positive width.
  Not fully self-consistent.

- Good approximation.
  Exact potentials unlikely to have large imaginary parts.
  1p-1h excitations very small with HF for closed shell nuclei.

- Newly defined $U_{HF}, U_{HFB}, \widetilde{U}_{HFB}$: well-defined basis generated.

Unbound HF/HFB ground state definition (1)

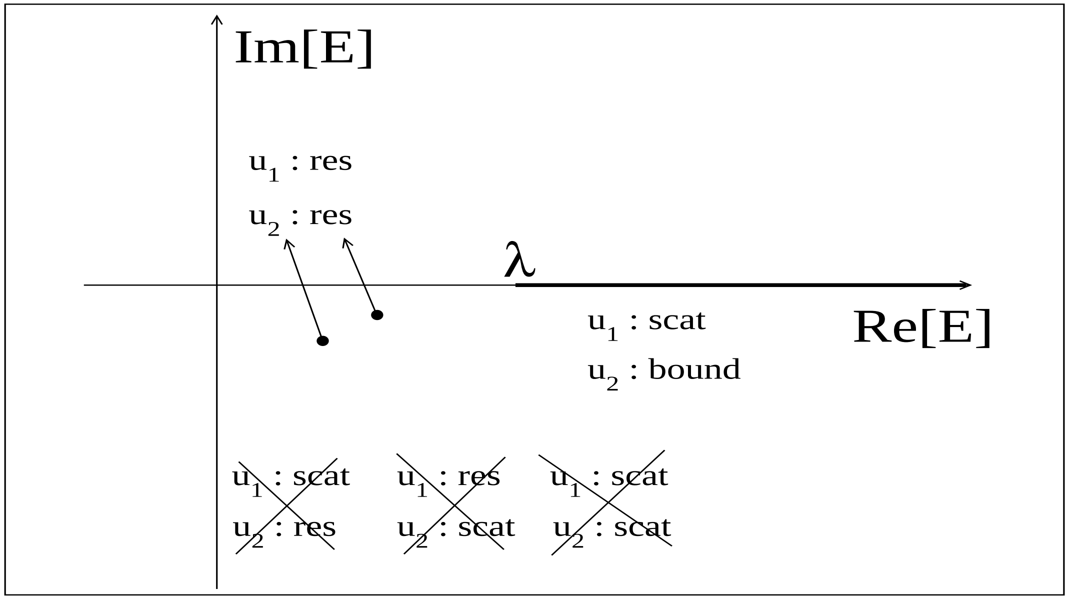
- Bound states : variational principle ⇒ -$E_{HF/HFB}$ maximal.

- Unbound states : not enough.
  Many body scattering states with $E < E_{GS}$.
  Asymptotics must be imposed.

- Asymptotics : pure outgoing wave function behavior.
  $\rho(R + x \cdot e^{i\theta}), \widetilde{\rho}(R + x \cdot e^{i\theta}) \rightarrow 0$, $\theta > \theta_c$.
  HF : only bound and resonant states are occupied.
  HF+BCS : Same condition.
  Scattering states occupied ⇒ scattering BCS state.

- No scattering asymptotics cancellation as in shell model.
  No many body Berggren completeness relation used.

Unbound HF/HFB ground state definition (2)



- u_2 cannot be a scattering state : $|\rho(R + x \cdot e^{i\theta})| \rightarrow +\infty$.
- $\theta > 0$: $|\widetilde{\rho}(R + x \cdot e^{i\theta})| \rightarrow +\infty$ for $\Re[E_{u_1(scat)}] < \lambda$, $E_{u_1(scat)} \sim \lambda$.

  Contour dependence $\Rightarrow$ HFB scattering state.

  No contour before $E = \lambda$.

  Many body Berggren completeness relation : QRPA.

Conclusion

- HFB can be defined theoretically with $\lambda > 0$.
- Complex scaled HF : HF only.
- Main problem : many body asymptotics.
- Interaction and method have to be modified.
- Same interaction for HFB and GSM : direct comparison possible.