Vectors and Scalars

- **Scalar**: A quantity specified by its magnitude only
- **Vector**: A quantity specified both by its magnitude and direction.

To distinguish a vector from a scalar quantity, it is usually written with an arrow above it, or in bold to distinguish it from a scalar.

- **Scalar**: $A$
- **Vector**: $\vec{A}$ or $\textbf{A}$
Question

- Are these two vectors the same?
- Are the lengths of these two vectors the same?

Two vectors are equal if both their length and direction are the same!
Vector addition

A + B = B + A
Vector subtraction

\[ A - B = A + (-B) \]
vector addition

\[ 8 \text{ N, } 0^\circ + 6 \text{ N, } 0^\circ = 14 \text{ N, } 0^\circ \]
Vector operations in equations

\[
\begin{pmatrix}
X_{a+b} \\
Y_{a+b}
\end{pmatrix} = \begin{pmatrix}
X_a \\
Y_a
\end{pmatrix} + \begin{pmatrix}
X_b \\
Y_b
\end{pmatrix} = \begin{pmatrix}
X_a + X_b \\
Y_a + Y_b
\end{pmatrix}
\]

\[
\begin{pmatrix}
X_{a-b} \\
Y_{a-b}
\end{pmatrix} = \begin{pmatrix}
X_a \\
Y_a
\end{pmatrix} - \begin{pmatrix}
X_b \\
Y_b
\end{pmatrix} = \begin{pmatrix}
X_a - X_b \\
Y_a - Y_b
\end{pmatrix}
\]

Example:

\[
\begin{pmatrix}
X_{a+b} \\
Y_{a+b}
\end{pmatrix} = \begin{pmatrix}
5 \\
2
\end{pmatrix} + \begin{pmatrix}
-3 \\
2
\end{pmatrix} = \begin{pmatrix}
2 \\
4
\end{pmatrix}
\]
Which route is shorter?

a) Red
b) Black
c) The same
d) Don't know
The length of a vector and its components

Length of vector (use pythagorean theorem):

\[ l = \sqrt{x_a^2 + y_a^2} \]

\[ x_a = l \cos \theta \]

\[ y_a = l \sin \theta \]

\[ \tan \theta = \frac{y_a}{x_a} \]
A man walks 5 km/h. He travels 12 minutes to the east, 30 minutes to the south-east and 36 minutes to the north.
A) What is the displacement of the man?
B) What is the total distance he walked?
Relative motion

Motion is relative to a frame!

A woman in a train moving 50 m/s throws a ball straight up with a velocity of 5 m/s. A second person watches the train pass by and sees the woman through a window. What is the motion of the ball seen from the point of view from the man outside the train?

Motion of the ball in rest-frame of train

Motion of the train

Resulting motion
Boat crossing the river
Question

A boat is trying to cross a 1-km wide river in the shortest way (straight across). Its maximum speed (in still water) is 10 km/h. The river is flowing with 5 km/h.

1) At what angle $\theta$ does the captain have to steer the boat to go straight across?

2) How long does it take for the boat to cross the river?

3) If it doesn’t matter at what point the boat reaches the other side, at what angle should the captain steer to cross in the fastest way?
plane in the wind

Tailwind

Headwind

Crosswind
What does the motion of the object look like according to the pilot?
Displacement in 2D

Often, we replace motion in 2D into horizontal and vertical components.

In vector notation:
\[ \mathbf{r} = \mathbf{x} + \mathbf{y} \]
\((\mathbf{r}, \mathbf{x}, \mathbf{y}: \text{vectors})\)

We can then work on horizontal and vertical components separately.
1d motion

\[ x(t) = x_0 + v_0 t + 0.5at^2 \]
\[ v(t) = v_0 + at \]

decomposition for 2D

2D motion; decompose into horizontal and vertical components

\[ x(t) = x_0 + v_{0x} t + 0.5a_x t^2 \]
\[ v_x(t) = v_{0x} + a_x t \]
\[ y(t) = y_0 + v_{0y} t + 0.5a_y t^2 \]
\[ v(t) = v_{0y} + a_y t \]

general 2D motion

Parabolic motion:

\[ x(t) = x_0 + v_{0x} t \]
\[ v_x(t) = v_{0x} \]
\[ y(t) = y_0 + v_{0y} t - 0.5gt^2 \]
\[ v(t) = v_{0y} - gt \]
\[ g = 9.81 \text{ m/s}^2 \]
While studying motion in 2D one almost always makes a decomposition into horizontal and vertical components of the motion, which are both described in 1D.

• Remember that the object can accelerate in one direction, but remain at the same speed in the other direction.
• Remember that after decomposition of 2D motion into horizontal and vertical components, you should investigate both components to understand the complete motion of a particle.
• After decomposition into horizontal and vertical directions, treat the two directions independently.
Parabolic motion

\[ v_x = v_0 \cos \theta \]
\[ v_y = v_0 \sin \theta - 2g \]

\[ v_x = v_0 \cos \theta \]
\[ v_y = v_0 \sin \theta - 3g \]

\[ v_x = v_0 \cos \theta \]
\[ v_y = v_0 \sin \theta - 4g \]
Parabolic motion

Where is the velocity...
1) highest?
2) lowest?

Assume height of catapult is negligible to the maximum height of the stone.

\[ X(t) = X_0 + V_0 \cos \theta t \]
\[ Y(t) = Y_0 + V_0 \sin \theta t - \frac{1}{2} gt^2 \]

\[ X = X_0 \]
\[ Y = Y_0 \]
Question

- A hunter aims at a bird that is some distance away and flying very high (i.e. consider the vertical position of the hunter to be 0), but he misses. If the bullet leaves the gun with a speed of $v_0$ and friction by air is negligible, with what speed $v_f$ does the bullet hit the ground after completing its parabolic path?
At the moment the hunter fires, the monkey drops off the branch. What happens?

a) hit
b) bullet goes over the monkey
c) bullet goes under the monkey
d) no idea
Another example

- A football player throws a ball with initial velocity of 30 m/s at an angle of $30^\circ$ degrees w.r.t. the ground. How far will the ball fly before hitting the ground? And what about $60^\circ$? And at what angle is the distance thrown maximum?
Pop and Drop

For A:
- \( V_y = -0.5gt^2 \)
- \( V_x = 0 \)

For B:
- \( V_y = -0.5gt^2 \)
- \( V_x = V_0 \)

For A:
- \( X_y = X_0 - 0.5gt^2 \)
- \( X_x = 0 \)

For B:
- \( X_y = X_0 - 0.5gt^2 \)
- \( X_x = V_0 t \)
And another...

A boy standing on top of a building throws a small ball from a height of $H_1 = 37.0$ m. (See figure.) The ball leaves with a speed of 10.1 m/s, at an angle of 59.0 degrees from the horizontal, and lands on a building with a height of $H_2 = 11.0$ m. *Calculate how far the ball goes.*
A careless driver.

A man driving in his sports car finishes his drink and throws the can out of his car through the sun roof. Assuming that air friction is negligible and his throw is straight up, what happens?
Quiz (for extra credit)

Galileo and Newton stand on top of the tower of Pisa. Galileo drops a stone of mass 2 kg straight down (no initial velocity), while Newton throws a similar stone with an initial horizontal velocity of 3 m/s. Which stone will hit the ground first? (you can ignore any friction effects).

a) The stone thrown by Galileo
b) The stone thrown by Newton
c) Both stones arrive at the same time
d) It is impossible to say.
remember...

acceleration is the change in velocity per unit of time

velocity can change in two ways:
1. change of the magnitude, i.e. change in speed
2. change in direction

Both types of changes indicate acceleration.

In other words: if an object changes direction but not its speed, the object is still accelerating.
Centripetal acceleration in circular motion

\[ \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t} \]

The change in velocity is not the change in speed but in direction.

\[ \sin(\Delta \theta/2) = \frac{\Delta s/2}{r} \]

\[ \sin(\Delta \theta/2) = \frac{\Delta v/2}{v} \]

\[ (\Delta s/2)/r = (\Delta v/2)/v \]

\[ \Delta v = \Delta s \times \frac{v}{r} \]

\[ \frac{\Delta v}{\Delta t} \]

\[ v \approx \Delta s/\Delta t \]

\[ a_c = \frac{v^2}{r} \]
An object that is undergoing circular motion is continuously accelerating.