PHYSICS 231
Laws of motion
Extra credit quiz

At which point during the flight of the object is:

1) The total speed largest? A  B  C  D
2) The total speed zero? A  B  C  D  E:Never
3) The acceleration zero? A  B  C  D  E:Never

PHY 231
Newton’s Laws

- **First Law:** If the net force exerted on an object is zero the object continues in its original state of motion; if it was at rest, it remains at rest. If it was moving with a certain velocity, it will keep on moving with the same velocity.

- **Second Law:** The acceleration of an object is proportional to the net force acting on it, and inversely proportional to its mass: \( \Sigma F = ma \)

- If two objects interact, the force exerted by the first object on the second is equal but opposite in direction to the force exerted by the second object on the first: \( F_{12} = -F_{21} \)
Forces are quantified in units of *Newton* (N).

\[ 1 \text{ N} = 1 \text{ kgm/s}^2 \]

\[ F = ma \]

- A force is a vector: it has direction.
Question!

In the absence of friction, to keep an object that is traveling with a certain velocity moving with exactly the same velocity over a level floor, one:

- a) does not have to apply any force on the object
- b) has to apply a constant force on the object
- c) has to apply an increasingly strong force on the object
Two Forces that luckily act upon us nearly all the time.

Normal Force: elastic force acting perpendicular to the surface the object is resting on.

Name: $n$

Gravitational Force

1. No net force: remains at rest.
2. $F_g = mg = -n$
3. $F_{mass-ground} = -F_{ground-mass}$

Gravitational Force

$F_g = -mg$ (referred to as weight)

$g = 9.81 \text{ m/s}^2$
3 cases with different mass and size are standing on a floor. Which of the following is true:

a) the normal forces acting on the crates is the same
b) the normal force acting on the blue crate is largest because its mass is largest

c) the normal force on the green crate is largest because its size is largest

d) the gravitational force acting on each of the crates is the same

e) all of the above
Object on a slope

An object starts from rest on a frictionless slope

1) Draw all Forces acting on the red object
2) If $\alpha = 40^\circ$ and the mass of the red object equals 1 kg, what is the resulting acceleration.
A first example

1) Draw all Forces acting on the red object
2) If $\alpha=40^\circ$ and the mass of the red object equals 1 kg, what is the resulting acceleration (no friction).

Balance forces in directions where you expect no acceleration; whatever is left causes the object to accelerate!

$$F_{gL}=mg \cos \alpha$$

$$F_{g//}=mg \sin \alpha$$

$$F_g=mg$$

$$n=-F_{gL}$$

$$ma=F$$

$F=mg \sin \alpha=6.3 \text{N}$

$\alpha=6.3 \text{ m/s}^2$
Some handy things to remember.

Choose your coordinate system in a clever way: Define one axis along the direction where you expect an object to start moving, the other axis perpendicular to it (these are not necessarily the horizontal and vertical direction.)
A ball is rolling from a slope at angle $\alpha$. It is repeated but after decreasing the angle $\alpha$. Compared to the first trial:

a) the normal force on the ball increases
b) the normal force gets closer to the gravitational force
c) the ball rolls slower
d) all of the above
Gravity, mass and weights.

Weight = mass times gravitational acceleration
\[ F_g (N) = M(\text{kg}) \times g(\text{m/s}^2) \]

Newton’s law of universal gravitation:
\[ F_{\text{gravitation}} = \frac{G m_1 m_2}{r^2} \]
\[ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

For objects on the surface of the earth:
\[ m_1 = m_{\text{earth}} = \text{fixed} \]
\[ r = \text{“radius” of earth} = \text{fixed} \]
Weight = \( F_g = G \frac{Mm}{r^2} = mg \)

- \( M \) is the mass of the Earth
- \( m \) is the mass of the object
- \( r \) is the radius of the Earth
- \( g \) is the acceleration due to gravity at the Earth's surface
Measuring mass and weight.

Given that $g_{\text{earth}} = 9.81 \text{ m/s}^2$, $g_{\text{sun}} = 274 \text{ m/s}^2$, $g_{\text{moon}} = 1.67 \text{ m/s}^2$,
what is the mass of a person on the sun and moon if his mass on earth is 70 kg? And what is his weight on each of the three surfaces?

- The mass is the same on each of the surfaces
- On Earth: \( w = 686.7 \text{ N} \)
- On the Moon: \( w = 116.7 \text{ N} \)
- On the Sun: \( 19180 \text{ N} \)
Extra credit quiz: puck on ice

After being hit by a hockey-player, a puck is moving over a sheet of ice (frictionless). The forces working on the puck are:

a) No force whatsoever
b) the normal force only

c) the normal force and the gravitational force
d) the force that keeps the puck moving
e) the normal force, the gravitational force and the force that keeps the puck moving
Buck Rogers travels from earth to a planet which has a radius that is 2 times larger than that of earth and has a mass 6 times that of earth. By what factor does his weight change relative to earth?

- a) 1/3
- b) 2/3
- c) 1
- d) 1.5
- e) 3

His weight is 1.5 times larger.
Decomposing forces

An object has a mass of 30.0 kg and three forces are acting on it as shown in the figure. What is its acceleration and direction of acceleration?

Total Force: \( F = \sqrt{(114^2 + 80^2)} = 139 \, \text{N} \)

Direction: \( \theta = \tan^{-1}(F_y/F_x) = 35.2^\circ \)

Acceleration: \( a = \frac{F}{m} = \frac{139}{30.0} = 4.65 \, \text{m/s}^2 \)
Tension: pulling a crate with a rope

The magnitude of the force $T$ acting on the crate, is the same as the tension in the rope.

You could measure the tension by inserting a spring-scale...
A block of mass $M$ is hanging from a string. What is the tension in the string?

a) $T=0$

b) $T=Mg$ with $g=9.81 \text{ m/s}^2$

c) $T=0$ near the ceiling and $T=Mg$ near the block

d) $T=M$

e) one cannot tell
Newton's second law and tension

Object 1: \[ \Sigma F = m_1 a, \text{ so } T = m_1 a \]

Object 2: \[ \Sigma F = m_2 a, \text{ so } F_g - T = m_2 a \]

Ignoring frictions, what is the acceleration of the objects?

Combine 1&2 (Tension is the same): \[ a = \frac{m_2 g}{m_1 + m_2} \]
Pulley Problem

What is the tension in the string and what will be the acceleration of the two masses?

For 3.00 kg mass: \( \sum F = ma \)

\[ T - 9.81 \cdot 3.00 = 3.00 \cdot a \]

For 5.00 kg mass: \( \sum F = ma \)

\[ 9.81 \cdot 5.00 - T = 5.00 \cdot a \]

T = 36.8 N

\( a = 2.45 \text{ m/s}^2 \)
Friction are the forces acting on an object due to interaction with the surroundings (air-friction, ground-friction etc).

Two variants:

- **Static Friction**: as long as an external force (F) trying to make an object move is smaller than $f_{s,\text{max}}$, the static friction $f_s$ equals F but is pointing in the opposite direction: no movement!

  $$f_{s,\text{max}} = \mu_s n \quad \mu_s = \text{coefficient of static friction}$$

- **Kinetic Friction**: After F has surpassed $f_{s,\text{max}}$, the object starts moving but there is still friction. However, the friction will be less than $f_{s,\text{max}}$!

  $$f_k = \mu_k n \quad \mu_k = \text{coefficient of kinetic friction}$$
As long as \( F_{\text{applied}} < (f_{s,\text{max}} = \mu_s n) \), the object will not move.

- \( \mu_s \) is the coefficient of static friction
- Once the object moves, the frictional force has a magnitude of \( \mu_k n \)
  - \( \mu_k \) is the coefficient of kinetic friction
A person wants to drag a crate of 20 kg over the floor. If he pulls the crate with a force of 98 N, the crate starts to move. What is $\mu_s$?

**Answer:**

$$F_s = \mu_s N = \mu_s Mg = 20 \times 9.8 \times \mu_s = 196 \mu_s$$

Just start to move: $F_s = F_{\text{pull}} \quad 98 = 196 \mu_s$

so: $\mu_s = 0.5$

After the crate starts moving, the person continues to pull with the same force. Given $\mu_k = 0.4$, what is the acceleration of the crate?

$$\sum F = ma \quad F = 98 - 0.4Mg = 98 - 0.4 \times 20 \times 9.8 = 98 - 78.4 = 19.6 \text{ N}$$

$$19.6 = 20a \quad a = 0.98 \text{ m/s}^2$$
drag forces

• Observed when an object passes through a medium such as water or air.
• the magnitude of the drag force depends on the velocity of the object:

\[ F_{\text{drag}} \propto v^\alpha \]

• \( \alpha \) varies, depending on the case.
no drag

with drag
General strategy

- If not given, make a drawing of the problem.
- Put all the relevant forces in the drawing, object by object.
  - Think about the axis
  - Think about the signs
- Decompose the forces in direction parallel to the motion and perpendicular to it.
- Write down Newton’s law for forces in the parallel direction and perpendicular direction.
- Solve for the unknowns.
- Use in further equations if necessary
- Check whether your answer makes sense.
A 2000 kg sailboat is pushed by the tide of the sea with a force of 3000 N to the East. Because of the wind in its sail it feels a force of 6000 N toward to North-West direction. What is the magnitude and direction of the acceleration?

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to tide:</td>
<td>3000 N</td>
<td>0 N</td>
</tr>
<tr>
<td>Due to wind:</td>
<td>$6000 \cos(135) = -4243$</td>
<td>$6000 \sin(135) = +4243$</td>
</tr>
<tr>
<td>Sum:</td>
<td>$-1243$ N</td>
<td>$4243$ N</td>
</tr>
</tbody>
</table>

Magnitude of resulting force: $F_{\text{sum}} = \sqrt{(-1243)^2 + (4243)^2} = 4421$ N

Direction: $\angle = \tan^{-1}(4243/-1243) = 106^0$ (calc: $-73^0$, add 180$^0$)

$F = ma$ so $a = F/m = 4421/2000 = 2.21 \text{ m/s}^2$
A mass of 1 kg is hanging from a rope as shown in the figure. If the angle between the 2 supporting wires is 90 degrees, what is the tension in each rope?

Object is stationary, so:

\[ 2T\cos(45) - 9.81 = 0 \]

so, \[ T = 6.9 \text{ N} \]
**Problem**

A) If $\mu_s = 1.0$, what is the angle $\alpha$ for which the block just starts to slide?

B) The block starts moving. Given that $\mu_k = 0.5$, what is the acceleration of the block?

A) Parallel direction: \( mgsin\alpha - \mu_s n = 0 \) \((\Sigma F = ma)\)

Perpendicular direction: \( mgcos\alpha - n = 0 \) so \( n = mgcos\alpha \)

Combine: \( mgsin\alpha - \mu_s mgcos\alpha = 0 \)

\( \mu_s = \frac{sin\alpha}{cos\alpha} = tan\alpha = 1 \) so \( \alpha = 45^o \)

B) Parallel direction: \( mgsin(45^o) - \mu_k mgcos(45^o) = ma \) \((\Sigma F = ma)\)

\( g(\frac{1}{2}\sqrt{2} - \frac{1}{4}\sqrt{2}) = a \) so \( a = g\frac{1}{4}\sqrt{2} \)
All the forces come together...

If \( a = 3.30 \, \text{m/s}^2 \) (the 12 kg block is moving downward), what is the value of \( \mu_k \)?

For the 7 kg block parallel to the slope:

\[
T - mg \sin \alpha - \mu_k mg \cos \alpha = ma
\]

For the 12 kg block:

\[
Mg - T = Ma
\]

Solve for \( \mu_k \)

\[
\mu_k = \frac{-M (g - a) + mg \sin \alpha + ma}{-mg \cos \alpha} = 0.25
\]
A small block is put on a frictionless slope. Attached on the block is an airblower that blows air along the direction of the slope and which keeps the block from slipping down the slope. Which of the following is true:

a) The force by the airblower on the block equals the total gravitational force on the block

b) The force by the airblower on the block is smaller than the total gravitational force on the block

c) The force by the airblower on the block is larger than the total gravitational force on the block
A cart is placed on a slope that makes an angle $\alpha$ with the horizontal. An airblower is placed on the cart, creating a force in the direction up the slope. If the cart and blower have a combined weight of 10 kg and $\alpha=10^0$, what is the force exerted by the blower (ignore friction) if the cart is stationary?

$$F_{\text{blower}} = F_{g/\parallel} \quad \text{(direction along the slope)}$$

$$F_{\text{blower}} = mg \sin \alpha = 10 \times 9.8 \times \sin(10^0) = 17 \text{ N}$$
A homogeneous block of 10 kg is hanging with 2 ropes from a ceiling. The tension in each of the ropes is:

a) 0 N
b) 49 N
c) 98 N
d) 196 N
e) don’t know
elevator blues

An object of 1 kg is hanging from a spring scale in an elevator. What is the weight read from the scale if the elevator:

a) moves with constant velocity
b) accelerates upward with 3 m/s²
c) accelerates downward with 3 m/s²
d) decelerates with 3 m/s² while moving up

Before starting this:
1) Imagine standing in an elevator yourself
2) the scale reading is equal to the tension $T$

$\sum F=ma$ so... $T-mg=ma$ and thus... $T=m(a+g)$

a) $a=0$ $T=1\times9.8=9.8$ N
b) $a=+3$ m/s² $T=1(3+9.8)=12.8$ N (heavier)
c) $a=-3$ m/s² $T=1(-3+9.8)=6.8$ N (lighter)
d) $a=-3$ m/s² $T=6.8$ N
elevator blues

What if the elevator is in free-fall?
example

A 1000-kg car is pulling a 300 kg trailer. Their acceleration is 2.15 m/s². Ignoring friction, find:

a) the net force on the car
b) the net force on the trailer
c) the net force exerted by the trailer on the car
d) the resultant force exerted by the car on the road

\[ F_{\text{engine}} = m_{\text{total}} a = 1300 \times 2.15 = 2795 \text{ N} \]

\[ F_{ct} = m_{\text{trailer}} \times 2.15 = 645 \text{ N}, \text{ so } F_{tc} = -645 \text{ N} \]

a) \[ \sum F_{\text{car}} = 2795 - 645 = 2150 \quad F_{\text{car}} = m_{\text{car}} \times 2.15 = 2150 \]

b) \[ \sum F_{\text{trailer}} = F_{tc} = 645 \text{ N} \]

c) \[ F_{tc} = -645 \text{ N} \]

d) \[ F_{\text{total}} = \sqrt{(2150^2 + (-9800)^2)} = 1E+04 \text{ N} \]

\[ mg = 1000 \times 9.8 \]
A force $F$ (10N) is exerted on the red block (1 kg). The coef. of kinetic friction between the red block and the blue one is 0.2. If the blue block (10kg) rests on a frictionless surface, what will be its acceleration?

\[
F_{\text{friction}} = \mu_k n = \mu_k mg = 0.2 \times 9.8 = 1.96 \text{ N (to the left)}
\]
\[
F_{\text{red-blue}} = -F_{\text{blue-red}} \text{ so force on blue block} = 1.96 \text{ N (to the right)}
\]
\[
F = ma \text{ so } 1.96 = 10a \quad a = 0.196 \text{ m/s}^2
\]
A person pulls on the robe, with force \((T_c)\) 100 N. The mass \(M\) move up with constant velocity. Given that the pulleys are weightless, what is the mass \(M\)?

\[ T_A = T_B = T_c = 100 \text{N because all in the same rope} \]

The vertical component of \(T_A\) equals \(T_A \cos(20) = +94 \text{ N (Up)}\)

The vertical component of \(T_B\) equals \(T_B \cos(20) = +94 \text{ N (Up)}\)

Total upward force on pulley: \(94 + 94 = 188 \text{ N}\)

Downward force on pulley: \(T_D = Mg = M \times 9.81\)

\(M\): constant velocity, so no acceleration: \(\Sigma F = 0\)

So (downward force) = (upward force): \(M \times 9.81 = 188 \text{N}\) \(M = 19.2 \text{ kg}\)
Is there a value for the static friction of surface A for which these masses do not slide? If so, what is it?

0.5 kg mass: \( \sum F = ma \) (only vertical)
\[
T - mg = ma \quad T - 0.5g = 0.5a
\]

1 kg mass: \( \sum F = ma \) (parallel to the slope)
\[
-F_{g//} - T + F_{friction} = ma \\
-mg\sin(q) - T + \mu_s mg\cos(q) = ma \\
-3.35 - T + 9.2\mu_s = a
\]

No sliding: \( a = 0 \), so \( T = 0.5g = 4.9 \) (from 0.5kg mass equation)
\[
-3.35 - 4.9 + 9.2\mu_s = 0 \\
\mu_s = 0.9
\]