These questions are approximately equivalent to a single midterm exam. Please note that the numbering is out of order because these problems were taken from various previous exams, answer are given next to the problems.

5. \( \vec{F}_{\text{center}} = m_a \vec{a} \) and \( \vec{F}_{\text{center}} = F_{\text{gravitational}} \). Since \( F_{\text{grav}} = \frac{GMm}{r^2} \) where \( M \) is the mass of the sun and \( m \) is the mass of the object, we find:

\[
ma = \frac{GMm}{r^2}
\]

\[
a = \frac{GM}{r^2}
\]

therefore \( a_c \) does not depend on the mass of the object. **Answer C**

6. \( F_{\text{grav}} = \frac{GMm}{r^2} \) and since \( m_2 > m_1 \) the gravitational force acting on \( m_2 \) is greater \( (r_1=r_2) \). **Answer A**

7. Kepler’s 3\(^{rd}\) law states that \( r^3 = \frac{T^2}{K} \), where \( K \) is a constant. Since \( r_1 = r_2, T_1 = T_2 \), **answer C**

8. Kepler’s 3\(^{rd}\) law states that \( r^3 = \frac{T^2}{K} \), where \( K \) is a constant. Since \( r_3 > r_2, T_3 > T_2 \), **answer A**
6) This is a perfectly inelastic collision:

\[ m_1v_{1i} + m_2v_{2i} = (m_1+m_2)v_f \]

where 1 relates to Dora and 2 to Billy, “i” to just prior to the collision and “f” to after the collision. Use \( m_1 = m_2 \) and \( v_{2i} = 0 \) to find that \( v_f = 40 \text{ mph} \). answer C

7) elastic collision: use conservation of momentum:

\[ m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \]

and the simplified equation for conservation of kinetic energy:

\[ (v_{1i} - v_{2i}) = (v_{2f} - v_{1f}) \]

with \( m_1 = m_2 \) and \( v_{2i} = 0 \), to find that \( v_{2f} = v_{1i} = 40 \text{ mph} \) (using the answer \( v_{1i} = 40 \text{ mph} \) from 6). answer C.
9) momentum is conserved, no matter what type of collision occurs so answer C

10) mechanical energy is conserved while swinging from A to B, so answer C

11) The collision is perfectly inelastic, so some kinetic energy is lost in the collision, answer A

12) Since some kinetic energy has been lost in the collision, the total energy has also been reduced, answer A

13) The kinetic energy of the bowling ball is
0.5mv^2=0.5*7.5*2.1^2=16.54 J

For the ping pong ball:
0.5mv^2=0.5*0.00245*v^2 which must equal 16.54 J
So v=√(16.54/(0.5*0.00245))=
116.2 m/s, answer B
14) $v = \omega r = 12.6 \times 2 = 6.3 \text{ m}$
$\omega = 450 \text{ rev/min}$
$= 450 \times \frac{2\pi}{60} \text{ rad/s}$
$= 47.12 \text{ rad/s}$
So $v = 6.3 \times 47.12 = 296 \text{ m/s}$
$v = \frac{296}{343} = 0.87 \text{ v_{sound} answer E}$

15) $\text{PE}_i + \text{KE}_i = \text{PE}_f + \text{KE}_f$
$\text{PE}_i = -\frac{GM_{\ast}}{r} m/r = 0 \ (r=\infty)$
$\text{KE}_i = 0$
$\text{PE}_f = -\frac{GM_{\ast}}{r} m/r \text{ where } r = r_{\ast}$
$\text{KE}_f = 0.5mv^2 \text{ so:}$
$\text{PE}_f + \text{KE}_f = 0$
$\frac{GM_{\ast}}{r_{\ast}} = 0.5v^2 \text{ (mass of ball cancels out). Use}$
$G = 6.673 \times 10^{-11} \text{Nm}^2/\text{kg}^2$
To find $v = 2.45 \times 10^6 \text{ answer G}$

16) Use the conservation of momentum:
$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
$15.3 - 22.3 = v_{1f} + v_{2f}$
(masses drop out) and the conservation of KE (simplified):
$(v_{1i} - v_{2i}) = (v_{2f} - v_{1f})$
$15.3 + 22.3 = v_{2f} - v_{1f}$ to find
$V_{2f} = 15.3 \text{ m/s (answer B)}$

31. The motion of an object is described by the equation:
$x = (2.3 \text{ m}) \cos(\pi t / 3.7)$,
where $t$ is assumed to be measured in seconds. What is the frequency (in Hz) of the motion?
$\omega = 2\pi f$ so $2\pi f = \pi / 3.7$ and $f = 0.135 \text{ Hz (answer A)}$

Also: $\omega = 2\pi f$ so $2\pi f = \pi / 3.7$ and $f = 0.135 \text{ Hz (answer A)}$
18) \( I = \Delta p = \text{area under } F-t \text{ diagram.} \)

The area is 52 kgm/s (two triangles of 13 kgm/s each and one square of 26 kgm/s).

Also \( \Delta p = p_f - p_i \) since \( p_i = m v_i = 0 \)

So \( m v_f = 52 \) with \( m = 6.5 \) so \( v_f = 8 \text{ m/s answer B} \)

19) Translational equilibrium: \( \Sigma F = 0 \), in the vertical direction:

\[ T_{\text{left}} + T_{\text{right}} - m_{\text{beam}} g - m_{\text{box}} g = 0 \]

so \( 800 + 1000 - 9.8 m_{\text{beam}} - 9.8 m_{\text{box}} = 0 \)

This gives: \( m_{\text{box}} = 184 - m_{\text{beam}} \)

Rotational equilibrium (choose rotation point e.g. on the left-hand side, at the left support wire)

\( \Sigma \tau = 0 \quad \tau = Fd \) where \( d \): distance from rotation point. Note that the grav. force on the beam acts at its center of gravity, 4 m (L/2) from the left support wire.

\[ 0 = T_{\text{left}} - (L/2) * m_{\text{beam}} g - x * m_{\text{box}} g + L T_{\text{right}} = 0 \]

\[ -4 * 9.8 * m_{\text{beam}} - 6 * 9.8 * m_{\text{box}} + 8 * 1000 = 0 \] combine with the above to find:

\[-4 * 9.8 * m_{\text{beam}} - 6 * 9.8 * (184 - m_{\text{beam}}) + 8 * 1000 = 0 \] which gives \( m_{\text{beam}} = 142.7 \text{ kg (answer F)} \)
A stainless steel orthodontic wire is applied to a tooth, as shown in the figure above. The wire has an unstretched length of 30 mm and a cross sectional area of 3 mm$^2$. The wire is stretched 0.1 mm. Young’s modulus for stainless steel is $1.8 \times 10^{11}$ Pa. What is the tension in the wire? (in N)

$Y = \frac{F/A}{\Delta L/L_0}$

$Y = 1.8 \times 10^{11}$ Pa (given)

$A = 3 \text{mm}^2 = 3 \times 0.001 \text{m}^2 = 3 \times 10^{-6} \text{m}^2$

$\Delta L = 0.1 \text{mm} = 1 \times 10^{-4} \text{m}$

$L_0 = 30 \text{mm} = 0.03 \text{m}$

F: tension in wire, you have to solve for it: F = 1800 N (answer E)