Brief Overview Midterm II
**Types of collisions**

<table>
<thead>
<tr>
<th>Inelastic collisions</th>
<th>Elastic collisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Momentum is conserved</td>
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</tr>
<tr>
<td>( m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} )</td>
<td>• No energy is lost in the collision: KE conserved</td>
</tr>
<tr>
<td>• Some energy is lost in the collision: KE not conserved</td>
<td>Conservation of momentum: ( m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} )</td>
</tr>
<tr>
<td>• Perfectly inelastic: the objects stick together.</td>
<td>Conservation of KE: ( \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 )</td>
</tr>
<tr>
<td>Conservation of momentum: ( m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f )</td>
<td>( (v_{1i} - v_{2i}) = (v_{2f} - v_{1f}) )</td>
</tr>
</tbody>
</table>

\[ \text{Momentum } p = mv \]
\[ F = \Delta p / \Delta t \]

**Impulse (the change in momentum)** \( \Delta p = F \Delta t \)
Simple harmonic motion

Amplitude (A): maximum distance from equilibrium (unit: m)
Period (T): Time to complete one full oscillation (unit: s)
Frequency (f): Number of completed oscillations per second (unit: 1/s = 1 Herz [Hz])

\[ x(t) = A \cos(\omega t) = A \cos(2\pi ft) = A \cos(2\pi t/T) \]
\[ \omega = 2\pi f = 2\pi / T \]
\[ \omega: \text{“angular frequency”} \quad f = 1/T \]
\[ x_{\text{harmonic}}(t) = A \cos(\omega t) \]

\[ \omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \]

\[ v_{\text{harmonic}}(t) = -\omega A \sin(\omega t) \]

for the spring \( \omega = \sqrt{\frac{k}{m}} \)

\[ a_{\text{harmonic}}(t) = -\omega^2 A \cos(\omega t) \]
pendulum vs spring

<table>
<thead>
<tr>
<th>parameter</th>
<th>spring</th>
<th>pendulum</th>
</tr>
</thead>
<tbody>
<tr>
<td>restoring force $F$</td>
<td>$F=-kx$</td>
<td>$F=-(mg/L)s$</td>
</tr>
<tr>
<td>period $T$</td>
<td>$T=2\pi\sqrt{(m/k)}$</td>
<td>$T=2\pi\sqrt{(L/g)}$ *</td>
</tr>
<tr>
<td>frequency $f$</td>
<td>$f=\sqrt{(k/m)/(2\pi)}$</td>
<td>$f=\sqrt{(g/L)/(2\pi)}$</td>
</tr>
<tr>
<td>angular frequency</td>
<td>$\omega=\sqrt{(k/m)}$</td>
<td>$\omega=\sqrt{(g/L)}$</td>
</tr>
</tbody>
</table>

*$T = 2\pi \sqrt{\frac{m}{mg/L}} = 2\pi \sqrt{\frac{L}{g}}$
\[ 360^\circ = 2\pi \text{ rad} = 6.28... \ \text{ rad} \]
\[ \theta \ (\text{rad}) = \frac{\pi}{180^\circ} \theta \ (\text{deg}) \]

\[ \bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta \theta}{\Delta t} \]

\[ \bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta \omega}{\Delta t} \]

\[ \theta(t) = \theta(0) + \omega(0)t + \frac{1}{2} \alpha t^2 \]
\[ \omega(t) = \omega(0) + \alpha t \]
\[ v = \omega r \]
\[ a = \alpha r \]

**Centripetal acceleration:**
\[ a_c = v^2/r = \omega^2 r \]

- For an object moving over a circular orbit at constant angular velocity:
\[ \Sigma F = ma_c = mv^2/r = m\omega^2 r \]

\[ F = G \frac{m_1 m_2}{r^2} \]

\[ \text{PE}_{\text{gravity}} = -GM_{\text{Earth}} m/r \]
First cosmic speed: speed of a satellite on a low-lying circular orbit ($r_{s(\text{satellite})}=r_{p(\text{planet})}$): $v_1 = \sqrt{gr}$

where $g=Gm_{\text{planet}}/r_{\text{planet}}^2$

Second cosmic speed: speed needed to break free from a planet: $v_1 = \sqrt{(2GM_p/R_p)} = \sqrt{(2gr)}$

Synchronous orbit of a satellite: rotation period of satellite is the same as rotation period of the planet:

$\omega=2\pi/T$ with $T$ is the period

Use $F_g=ma_c$ so $Gm_{\text{planet}}m_{\text{sat}}/r^2=m_{\text{sat}}\omega^2r$ (note $r\neq r_{\text{planet}}$!)

$r=(Gm_{\text{planet}}/\omega^2)^{1/3}$

Kepler's 3rd law: $T^2/r^3=K_s$ (constant: $2.97\times10^{-19} \, s^2/m^3 \, \text{sun}$)
Torque $\tau = F_{\text{perpendicular}} \cdot d \ (\text{Nm})$

Center of gravity:

$$x_{CG} = \frac{\sum_{i} m_{i} x_{i}}{\sum_{i} m_{i}} \quad y_{CG} = \frac{\sum_{i} m_{i} y_{i}}{\sum_{i} m_{i}}$$

Translational equilibrium: $\Sigma F = ma = 0$  
The center of gravity does not move!

Rotational equilibrium: $\Sigma \tau = 0$  
The object does not rotate

Mechanical equilibrium: $\Sigma F = ma = 0 \ & \Sigma \tau = 0$  
No movement!
\[ \tau = I \alpha \]

Moment of inertia \( I \):
\[ I = (\sum m_i r_i^2) \]

Rotational kinetic energy
\[ KE_r = \frac{1}{2} I \omega^2 \]

Conservation of energy for rotating object:
\[ [PE + KE_t + KE_r]_{\text{initial}} = [PE + KE_t + KE_r]_{\text{final}} \]
Angular momentum

$$\tau = I \alpha = I \left( \frac{\omega - \omega_0}{\Delta t} \right) = \frac{I \omega - I \omega_0}{\Delta t}$$

$$L \equiv I \omega$$

$$\tau = \frac{L - L_0}{\Delta t} = \frac{\Delta L}{\Delta t}$$

if $$\sum \tau = 0$$ then $$\Delta L = 0$$

Conservation of angular momentum

If the net torque equals zero, the angular momentum $$L$$ does not change

$$I_i \omega_i = I_f \omega_f$$
**Stress**: Tells something about the force causing the deformation:

Force per unit area causing the deformation

**Strain**: Measure of the degree of deformation

Measure of the amount of deformation

\[ Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} \]

tensile stress: \( \frac{F}{A} \) \([\text{N/m}^2 = \text{Pascal (Pa)}]\)

tensile strain: \( \frac{\Delta L}{L_0} \)

\[ Y = \frac{F}{A \Delta L} = \frac{FL_0}{A \Delta L} \]
\[ S \equiv \frac{\text{shear stress}}{\text{shear strain}} \]

shear stress : \( \frac{F}{A} \) \( \text{[N/m}^2 \text{ = Pascal (Pa)]} \)

shear strain : \( \frac{\Delta x}{h} \)

\[ S = \frac{F/A}{\Delta x/h} = \frac{Fh}{A\Delta x} \]

\[ B \equiv \frac{\text{volume stress}}{\text{volume strain}} \]

volume stress : \( \frac{\Delta F}{A} \) \( \text{[N/m}^2 \text{ = Pascal (Pa)]} \)

volume strain : \( \frac{\Delta V}{V_0} \)

\[ B = -\frac{\Delta F/A}{\Delta V/V_0} = -\frac{\Delta P}{\Delta V/V_0} \]

\[ P = \text{pressure} \]
\[ \rho = \frac{M}{V} \quad (kg/m^3) \]

\[ \rho_{\text{specific}} = \rho_{\text{material}} / \rho_{\text{water}(4^\circ C)} \]

Pressure \( P = F/A \) (N/m\(^2\)=Pa)

**Pascal’s principle:** a change in pressure applied to a fluid that is *enclosed* is transmitted to the whole fluid and all the walls of the container that hold the fluid.
Pressure at depth $h$

$$P = P_0 + \rho_{\text{fluid}} gh$$

$h$: distance between liquid surface and the point where you measure $P$