PHY231 Review – Final part I
Final exam

BCC N130 8-10 pm Thursday December 14.
If you have:
• a conflict with another exam
• three exams on that day
You can take the make-up exam Tuesday at 1-3 pm in BPS 1410
You must ask permission to join the make-up: send me an email detailing the conflict.

• we might not have spare calculators!! Make sure to bring your own.
• three letter sizes equation sheets (both sides)
Pressure at depth $h$

$$P = P_0 + \rho_{\text{fluid}} \cdot gh$$

$h$: distance between liquid surface and the point where you measure $P$

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**Buoyant force for submerged object**

$$B = \rho_{\text{fluid}} V_{\text{object}} g = M_{\text{fluid}} g = w_{\text{fluid}}$$

The buoyant force equals the weight of the amount of water that can be put in the volume taken by the object.

If object is not moving: $B = w_{\text{object}} = \rho_{\text{object}} g V_{\text{object}}$

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**Buoyant force for floating object**

The buoyant force equals the weight of the amount of water that can be put in the part of the volume of the object that is under water.

$$\rho_{\text{object}} V_{\text{object}} = \rho_{\text{water}} V_{\text{displaced}} \quad h = \frac{\rho_{\text{object}} V_{\text{object}}}{(\rho_{\text{water}} A)}$$
the mass flowing into area 1 ($\Delta M_1$) must be the same as the mass flowing into area 2 ($\Delta M_2$), else mass would accumulate in the pipe).

$\Delta M_1 = \Delta M_2$

$\rho_1 A_1 \Delta x_1 = \rho_2 A_2 \Delta x_2$  \hspace{1cm} (M=$\rho V = \rho A x$)

$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$  \hspace{1cm} (\Delta x=v\Delta t)

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

if $\rho$ is constant (liquid is incompressible)  \hspace{1cm} $A_1 v_1 = A_2 v_2$
Bernoulli’s equation

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

\[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant} \]

\( P \): pressure  \hspace{1cm} \( y \): height

\( \rho \): height  \hspace{1cm} \( g \): gravitational acceleration

\( v \): velocity

\( \frac{1}{2} \rho v^2 \): kinetic Energy per unit volume

\( \rho g y \): potential energy per unit volume
Poiseuille’s Law

How fast does a fluid flow through a tube?

\[ Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L} \]  

(\text{unit: m}^3/\text{s})

\( R \)=coefficient of viscosity  
\( \text{unit: Ns/m}^2 \)  
or poise=0.1 Ns/m^2

Rate of flow \( Q = \Delta v/\Delta t = \)  

**Diagram:**

- \( P_1 \) and \( P_2 \) represent pressures at the ends of the tube.
- \( R \) represents the radius of the tube.
- \( L \) represents the length of the tube.
- \( \Delta v/\Delta t \) represents the flow rate.
- \( \eta \) represents the coefficient of viscosity.
Traveling waves

Transverse wave

Longitudinal wave: movement is in the direction of the wave motion. Example: sound wave
Traveling waves

\[ v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f \]

\( \lambda \): wavelength = length (m) of one oscillation.
\( T \): period = time for one oscillation
\( T = 1/f \) f: frequency (Hz)
sound

Sound Intensity: rate of energy flow through an area

Intensity: $I = \frac{P}{A}$ (J/m$^2$s=W/m$^2$)

The amount of energy passing through a spherical surface at distance $r$ from the source is constant, but the surface becomes larger. $I = \frac{\text{Power}}{\text{Surface}} = \frac{P}{A} = \frac{P}{(4\pi r^2)}$

Speed of sound: $v = \sqrt{\frac{B}{\rho}}$  \( B: \text{bulk modulus} \quad \rho: \text{density} \)

In air: $v = 331 \sqrt{\frac{T}{273 \text{ K}}}$  \( T: \text{temperature in Kelvin} \)
Sound level

\[ \beta = 10 \log\left(\frac{I}{I_0}\right) \quad I_0 = 10^{-12} \text{ W/m}^2 \]

\[ y = \log_{10} x \quad \text{inverse of } x = 10^y \]

\[ \log(ab) = \log(a) + \log(b) \]

\[ \log\left(\frac{a}{b}\right) = \log(a) - \log(b) \]

\[ \log(a^n) = n \log(a) \]

An increase of 10 dB: intensity of the sound is multiplied by a factor of 10.
Doppler effect

\[ f' = f \left( \frac{v + v_{\text{observer}}}{v - v_{\text{source}}} \right) \]

- \( v_{\text{observer}} \): positive if moving towards to source
- \( v_{\text{source}} \): positive if moving towards the observer
we can produce different wave lengths

\[ \lambda_1 = 2L \]
\[ \lambda_2 = L \]
\[ \lambda_3 = \frac{2L}{3} \]
\[ \lambda_4 = \frac{2L}{4} \]
\[ \lambda_5 = \frac{2L}{5} \]

both ends fixed \( \lambda_n = \frac{2L}{n} \) or \( L = n\lambda_n / 2 \)
standing waves

both ends fixed $\lambda_n=2L/n$ or $L=n\lambda_n/2$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{F}{\mu}}$$

$F$: tension in rope
$\mu$: mass per unit length

$$f_1 = \frac{v}{2L}$$
$f_1$: fundamental frequency

$$f_2 = \frac{2v}{2L}$$

$n$th harmonics

$$f_n = \frac{nv}{2L} = nf_1$$
Number of particles: mol

1 mol of particles: $6.02 \times 10^{23}$ particles

Avogadro’s number $N_A = 6.02 \times 10^{23}$ particles per mol

It doesn’t matter what kind of particles:
1 mol is always $N_A$ particles

Weight of 1 mol of atoms

1 mol of atoms: A gram (A: mass number)
Conversions

\[ T_{\text{celsius}} = T_{\text{kelvin}} - 273.5 \]
\[ T_{\text{fahrenheit}} = \frac{9}{5} T_{\text{celcius}} + 32 \]

\[ \Delta L = \alpha L_0 \Delta T \]
\[ \Delta A = \gamma A_0 \Delta T \quad \gamma = 2\alpha \]
\[ \Delta V = \beta V_0 \Delta T \quad \beta = 3\alpha \]

\( \alpha \): coefficient of linear expansion
different for each material

Ideal gas law:

\[ \frac{PV}{T} = nR = Nk_B \]

- \( n \): number of particles in the gas (mol)
- \( R \): universal gas constant \( 8.31 \text{ J/mol} \cdot \text{K} \)
- \( N \): total number of molecules
- \( K_B = 1.38 \times 10^{-23} \text{ J/K} \) (bolzmann’s constant)

\[ \frac{PV}{T} = \text{constant} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \]
$PV = \frac{2}{3} N \left( \frac{1}{2} m \bar{v}^2 \right) \quad$ Microscopic

$PV = Nk_B T \quad$ Macroscopic

$T = \frac{2}{3k_B} \left( \frac{1}{2} m \bar{v}^2 \right) \quad$ Temperature ~ average molecular kinetic energy

$\frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T \quad$ Average molecular kinetic energy

$E_{kin} = \frac{3}{2} Nk_B T = \frac{3}{2} nRT \quad$ Total kinetic energy

$v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} \quad$ rms speed of a molecule

M= Molar mass (kg/mol)
Energy in thermal processes

\[ Q = cm \Delta T \]

- **Q**: heat transfer (J)
- **c**: specific heat (J/kg°C)
- **M**: mass (kg)
- **ΔT**: change in temperature

Heat transfer between 2 objects: \( Q_{\text{cold}} = -Q_{\text{hot}} \)

\[
m_{\text{cold}}c_{\text{cold}}(T_{\text{final}} - T_{\text{cold}}) = -m_{\text{hot}}c_{\text{hot}}(T_{\text{final}} - T_{\text{hot}})\]

\[
T_{\text{final}} = \frac{m_{\text{cold}}c_{\text{cold}}T_{\text{cold}} + m_{\text{hot}}c_{\text{hot}}T_{\text{hot}}}{m_{\text{cold}}c_{\text{cold}} + m_{\text{hot}}c_{\text{hot}}}\]
Phase Change

GAS (high T)

\[ Q = c_{\text{gas}} m \Delta T \]

Gas ↔ liquid

Q = mL_v

Liquid (medium T)

\[ Q = c_{\text{liquid}} m \Delta T \]

Solid (low T)

\[ Q = c_{\text{solid}} m \Delta T \]

Liquid ↔ solid

Q = mL_f
Heat transfer:

1) Convection
2) Conduction
   Rate of energy transfer \( P \)
   \[ P = \frac{Q}{\Delta t} \text{ (unit Watt)} \]
   \[ P = kA(T_h - T_c)/\Delta x = kA\Delta T/\Delta x \]
   \( k \): thermal conductivity
   \( \text{Unit:} \frac{\text{J}}{\text{ms}^0\text{C}} \)

3) Radiation
   \( P = \sigma AeT^4 \): Stefan’s law \( \text{(J/s)} \)
   \( \sigma = 5.6696 \times 10^{-8} \text{ W/m}^2\text{K}^4 \)
   \( A \): surface area
   \( e \): object dependent constant emissivity \((0-1)\)
   \( T \): temperature \( \text{(K)} \)
   \[ P = \sigma Ae(T^4 - T_0^4) \text{ where} \]
   \( T \): temperature of object
   \( T_0 \): temperature of surroundings.
Black body

A black body is an object that absorbs all electromagnetic radiation that falls onto it. They emit radiation, depending on their temperature. If $T<700$ K, almost no visible light is produced (hence a ‘black’ body).

The energy emitted from a black body: $P=\sigma T^4$ with $\sigma=5.67\times10^{-8}$ W/m$^2$K$^4$

$$\lambda_{max} = \frac{b}{T}$$

$b=2.8977685(51)\times10^{-3}$ mK
Wiens displacement constant
Laws of Thermodynamics

\[ W = -P\Delta V = -P(V_f - V_i) \] (in Joule)

\( W \): work done on the gas
  + if \( \Delta V < 0 \)
  - if \( \Delta V > 0 \)

Work: area under P-V diagram

1st law of thermodynamics: \( \Delta U = U_f - U_i = Q + W \)

\( \Delta U \): change in internal energy
\( Q \): energy transfer through heat (+ if heat is transferred to the system)
\( W \): energy transfer through work (+ if work is done on the system)
A: Isovolumetric $\Delta V = 0$
B: Adiabatic $Q = 0$
C: Isothermal $\Delta T = 0$
D: Isobaric $\Delta P = 0$
<table>
<thead>
<tr>
<th>process</th>
<th>$\Delta U$</th>
<th>$Q$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>isobaric</td>
<td>$nC_v\Delta T$</td>
<td>$nC_p\Delta T$</td>
<td>$-P\Delta V$</td>
</tr>
<tr>
<td>adiabatic</td>
<td>$nC_v\Delta T$</td>
<td>0</td>
<td>$\Delta U$</td>
</tr>
<tr>
<td>isovolumetric</td>
<td>$nC_v\Delta T$</td>
<td>$\Delta U$</td>
<td>0</td>
</tr>
<tr>
<td>isothermal</td>
<td>0</td>
<td>$-W$</td>
<td>$-nRT\ln(V_f/V_i)$</td>
</tr>
<tr>
<td>general</td>
<td>$nC_v\Delta T$</td>
<td>$\Delta U-W$</td>
<td>(PV Area) negative if V expands</td>
</tr>
</tbody>
</table>

Ideal (monatomic) gas (only case looked at here…)

$C_v = 3/2 \, R$
$C_p = 5/2 \, R$
Engines and fridges..

**Clockwise loop: engine**: heat is supplied to let the gas do work

**Counterclockwise loop: refrigerator**: heat is extracted by doing work on the gas

Work performed in a cycle: area of loop in PV diagram

Engine: work on gas is negative

Refridgerator: work on gas is positive
The efficiency is determined by how much of the heat you supply to the engine is turned into work instead of being lost as waste.
Reverse direction: the fridge

Heat is expelled to outside

Heat reservoir $T_h$

A piston compresses the coolant

$Q_h$

Work is done

Work

The fridge is cooled

Cold reservoir $T_c$

Coefficient of performance

$\text{COP}_{\text{cooling}} = |Q_c|/W$

$Q_c$: amount of energy extracted

From the cold reservoir

$W$: work performed by the device
Heat Pump

heat is expelled to outside

heat reservoir $T_h$

a piston compresses the coolant

$Q_h$

work is done

work

the fridge is cooled

cold reservoir $T_c$

Coefficient of performance

$\text{COP}_{\text{heating}} = |Q_h|/W$

$Q_h$: amount of energy rejected into the hot reservoir

$W$: work performed by the device
2\textsuperscript{nd} law: It is impossible to construct an engine that, operating in a cycle produces no other effect than the absorption of energy from a reservoir and the performance of an equal amount of work: we cannot get 100% efficiency

General engine:
\[ W = |Q_h| - |Q_c| \]

efficiency: \[ W / |Q_h| \]
\[ e = 1 - |Q_c| / |Q_h| \]

efficiency = 1 - \( T_{\text{cold}} / T_{\text{hot}} \) cannot only!!

The Carnot engine is the most efficient way to operate an engine based on hot/cold reservoirs because the process is \textbf{reversible}: it can be reversed without loss or dissipation of energy. Unfortunately, a perfect Carnot engine cannot be built.
The (loss of) ability to do work: entropy

entropy: \( \Delta S = \frac{Q_R}{T} \) \text{ R refers to a reversible process}  
The equation ONLY holds for a reversible process.

equation: \( \Delta S = \frac{Q}{T} \)

example: Carnot engine:  
Hot reservoir: \( \Delta S_{\text{hot}} = -\frac{Q_{\text{hot}}}{T_{\text{hot}}} \) (entropy is decreased)  
Cold reservoir: \( \Delta S_{\text{cold}} = \frac{Q_{\text{cold}}}{T_{\text{cold}}} \)

We saw: efficiency for a general engine: \( e = 1 - \frac{Q_{\text{cold}}}{Q_{\text{hot}}} \)  
efficiency for a Carnot engine: \( e = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \)  
So for a Carnot engine: \( \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{Q_{\text{cold}}}{Q_{\text{hot}}} \)  
and thus: \( \frac{Q_{\text{hot}}}{T_{\text{hot}}} = \frac{Q_{\text{cold}}}{T_{\text{cold}}} \)

Total change in entropy: \( \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = 0 \)

For a Carnot engine, there is no change in entropy
Midterm 1, problem 1: example on different version

4 pt A rocket, undergoing uniform acceleration from rest, experiences a displacement of 850 m in 3.7 s. What is its acceleration? (in m/s\(^2\))

1. A \(\bigcirc\) 52.78       B \(\bigcirc\) 70.20       C \(\bigcirc\) 93.37
   D \(\bigcirc\) 124.18       E \(\bigcirc\) 165.16       F \(\bigcirc\) 219.66
   G \(\bigcirc\) 292.15       H \(\bigcirc\) 388.55

\(x(0) = 0\) and \(v(0) = 0\)
\(x(t = 3.7) = 850 = 0.5a(3.7)^2\)
So \(a = 850/(0.5*3.7^2) = 124.18\) m/s\(^2\)

Variation:
A rocket starting from rest undergoes uniform acceleration of 52 m/s\(^2\) for a distance of 900 m. What is the final velocity?

\[
x(t) = x_o + v_o t + \frac{1}{2}at^2
\]

\[
v(t) = v_0 + at
\]
A pitcher throws a ball horizontally with a speed of 41 m/s to a catcher 17.7 m away. When the ball is caught its height has decreased by: (in m)

\[ x(t) = x_0 + v_{0x}t \]
\[ v_x(t) = v_{0x} \]
\[ y(t) = y_0 + v_{0y}t - 0.5gt^2 \]
\[ v_y(t) = v_{0y} - gt \quad g = 9.81 \text{ m/s}^2 \]

\[ x(t) = x_0 + v_{0x}t \]
\[ 17.7 = 0 + 41t \]
\[ t = 0.432 \text{ s} \]
\[ y(t) - y_0 = v_{0y}t - 0.5gt^2 \]
\[ \Delta y = 0 - 0.5 \times 9.81 \times 0.432^2 = -0.91 \text{ m} \]

Variation: A pitcher throws a ball horizontally to a catcher 17.7 m away. The ball is caught at a height 0.8 m below the height from which the ball was thrown. What was the initial speed of the ball?
A boat heads directly across a river 250 m wide. It takes it 500 seconds to cross, but it ends up 200 m downstream. What is its speed relative to the water in m/s? That is, how fast is the boat?

The engine power is used to go across only (no correction for the current in the river), so to determine the speed relative to the water, I just need to consider the direction across the river:

\[ V = \frac{\text{width of river}}{\text{time to cross}} = \frac{250}{500} = 0.5 \text{ m/s} \]

Alternate formulations:
• give boat speed, time and ask for downstream distance
• give speed of boat and speed of river, ask for distance downstream
• give speed of boat and speed of river, ask for total distance
• etc etc
At the top of a cliff 100 m high, Raoul throws a rock upward with velocity 15.0 m/s. How much later should he drop a second rock from rest so both rocks arrive simultaneously at the bottom of the cliff?

a. 5.05 s  
b. 3.76 s  
c. 2.67 s  
d. 1.78 s

\[ y(t) = y_0 + v_{0y} t - 0.5gt^2 \]
\[ v_y(t) = v_{0y} - gt \quad g = 9.81 \text{ m/s}^2 \]
4) A 5.0 Kg sledge moves on ice with a speed of 3.0 m/s when it encounters a level surface with kinetic coefficient of friction equal to 0.15. Determine the distance it travels before stopping.
   a) 0.5 m  
   b) 3.1 m  
   c) 5.0 m  
   d) 6.6 m  
   e) 30 m
6) A 0.25 kg block is dropped onto a vertical spring with spring constant $k=2.5 \times 10^2$ N/m. The block becomes attached to the spring and compresses it by 12 cm before momentarily stopping. Determine the height, relative to the point of maximum compression, from which the block was dropped.
   a) 0.12 m
   b) 0.73 m
   c) 1.5 m
   d) $1.5 \times 10^2$ m
   e) None of the above
A train has a mass of $7.67 \times 10^6$ kg and is moving at a speed of 81.1 km/hr. The engineer applies the brakes, which results in a net backward force of $1.47 \times 10^6$ N on the train. The brakes are held on for a time period of 18.6 s.

3 pt What is the new speed of the train?

(in m/s)

54. A $1.49 \times 10^1$
   B $1.68 \times 10^1$
   C $1.90 \times 10^1$
   D $2.14 \times 10^1$
   E $2.42 \times 10^1$
   F $2.74 \times 10^1$
   G $3.09 \times 10^1$
   H $3.49 \times 10^1$

3 pt How far does the train travel during this period?

(in m)

55. A $3.86 \times 10^2$
   B $5.13 \times 10^2$
   C $6.83 \times 10^2$
   D $9.08 \times 10^2$
   E $1.21 \times 10^3$
   F $1.61 \times 10^3$
   G $2.14 \times 10^3$
   H $2.84 \times 10^3$
An artillery shell is launched on a flat, horizontal field at an angle of \( \alpha = 38.1^\circ \) with respect to the horizontal and with an initial speed of \( v_0 = 338 \text{ m/s} \). What is the horizontal distance covered by the shell after 5.57 s of flight?

\[(\text{in m})\]

56. A \( \bigcirc \) \( 1.48 \times 10^3 \) \hspace{1cm} B \( \bigcirc \) \( 1.73 \times 10^3 \) \hspace{1cm} C \( \bigcirc \) \( 2.03 \times 10^3 \)

D \( \bigcirc \) \( 2.37 \times 10^3 \) \hspace{1cm} E \( \bigcirc \) \( 2.78 \times 10^3 \) \hspace{1cm} F \( \bigcirc \) \( 3.25 \times 10^3 \)

G \( \bigcirc \) \( 3.80 \times 10^3 \) \hspace{1cm} H \( \bigcirc \) \( 4.45 \times 10^3 \)

What is the height of the shell at this moment?

\[(\text{in m})\]

57. A \( \bigcirc \) \( 5.71 \times 10^2 \) \hspace{1cm} B \( \bigcirc \) \( 7.59 \times 10^2 \) \hspace{1cm} C \( \bigcirc \) \( 1.01 \times 10^3 \)

D \( \bigcirc \) \( 1.34 \times 10^3 \) \hspace{1cm} E \( \bigcirc \) \( 1.79 \times 10^3 \) \hspace{1cm} F \( \bigcirc \) \( 2.37 \times 10^3 \)

G \( \bigcirc \) \( 3.16 \times 10^3 \) \hspace{1cm} H \( \bigcirc \) \( 4.20 \times 10^3 \)
A 3.28 kg sphere makes a perfectly inelastic collision with a second sphere that is initially at rest. The composite system moves with a speed equal to one third the original speed of the 3.28 kg sphere. What is the mass of the second sphere?

(in kg)

58. A ○ 2.79  B ○ 3.15  C ○ 3.56  D ○ 4.02
   E ○ 4.55  F ○ 5.14  G ○ 5.81  H ○ 6.56
An athlete, swimming at a constant speed, covers a distance of 137 m in a time period of 2.15 minutes. The drag force exerted by the water on the swimmer is 58.0 N. Calculate the power the swimmer must provide in overcoming that force.

(in W)

\begin{align*}
&\text{A} \circ 1.61 \times 10^1 \\
&\text{B} \circ 2.02 \times 10^1 \\
&\text{C} \circ 2.52 \times 10^1 \\
&\text{D} \circ 3.15 \times 10^1 \\
&\text{E} \circ 3.94 \times 10^1 \\
&\text{F} \circ 4.93 \times 10^1 \\
&\text{G} \circ 6.16 \times 10^1 \\
&\text{H} \circ 7.70 \times 10^1
\end{align*}
A cheetah can run at approximately 100 km/hr and a gazelle at 80.0 km/hr. If both animals are running at full speed, with the gazelle 70.0 m ahead, how long before the cheetah catches its prey?

a. 12.6 s  
b. 25.2 s  
c. 6.30 s  
d. 10.7 s
A plane is moving due north, directly towards its destination. Its airspeed is 200 mph. A constant breeze is blowing from west to east at 30 mph. At what rate is the plane moving north?

a. 198 mph  
b. 193 mph  
c. 188 mph  
d. 180 mph
A 9.0-kg hanging weight is connected by a string over a pulley to a 5.0-kg block sliding on a flat table. If the coefficient of sliding friction is 0.20, find the tension in the string.

a. 19 N  
b. 24 N  
c. 32 N  
d. 38 N